Contra qs$\mathcal{I}$- Continuous Functions in Ideal Bitopological Spaces

Mandira Kar[1], S. S. Thakur[2]

[1]Department of Mathematics, St. Aloysius College, Jabalpur (M.P.) 482001 India

[2]Department of Applied Mathematics, Government Engineering College, Jabalpur (M.P.) 482011 India

Abstract: ***In this paper, we apply the notion of qs$\mathcal{I}$-open sets and qs$\mathcal{I}$-continuous functions to present and study a new class of functions called contra qs$\mathcal{I}$-continuous functions in ideal bitopological spaces.***

Keywords: Ideal bitopological space, qs$\mathcal{I}$-open sets, qs$\mathcal{I}$-continuous functions contra qs$\mathcal{I}$-continuous functions

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1. Preliminaries

In 1961 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2[5] The study of quasi open sets in bitopological spaces was initiated by Datta[1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [9] if it is a union of a τ_1-open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp., τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A. It is denoted by qcl(A). The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A)[1].

In 1963 N. Levine [8] introduced the concept of semi open sets in topology. A subset A of a topological space (X, τ) is called semi open if there exists an open set O in X such that O$\subset$ A $\subset$Cl(O). Every open set is semi open but the converse may not be true. In 1985, Maheshwari, Chae and Thakur[10] introduced quasi semi open sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi semi open[10] if it is a union of a τ_1-semi open set and a τ_2-semi open set. Complement of a quasi semi open set is called quasi semi closed. Every τ_1-semi open (τ_2-semiopen, quasi open) set is quasi semi open but the converse may not be true. Any union of quasi semi open sets of X is a quasi semi open set in X. The intersection of all quasi semi closed sets which contains A is called quasi semi closure of A. It is denoted by qscl(A). The union of quasi semi open subsets of A is called quasi semi interior of A. It is denoted by qsInt(A)

In 1996 Dontchev[2] introduced a new class of functions called contra-continuous functions. A function f: X $\to$ Y to be contra continuous if the pre image of every open set of Y is closed in X. The study of ideal topological spaces was initiated by Kuratowski [7] and Vaidyanathaswamy [13]. An Ideal $\mathcal{I}$ on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: A $\in$. $\mathcal{I}$ and B $\subset$ A $\Rightarrow$ B $\in$. $\mathcal{I}$ and A $\in$. $\mathcal{I}$ and B $\in$. $\mathcal{I}$ $\Rightarrow$ A$\cup$B $\in$. $\mathcal{I}$. If $\mathcal{P}$(X) is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*$:$\mathcal{P}$(X) $\to$ $\mathcal{P}$(X) is called the local mapping[6] of A with respect to τ and $\mathcal{I}$ and is defined as follows: $A^*(\tau, \mathcal{I})$ (in short A^*) = {x$\in$X |U $\cap$ A $\notin$ $\mathcal{I}$, $\forall$ U$\in$ τ(x)} where τ(x) = {U$\in$ τ | x$\in$U}[4]. Given an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) the quasi local mapping[3] of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by A^*_q (τ_1, τ_2, $\mathcal{I}$) (in short A^*_q) is defined as follows: $A^*_q(\tau_1, \tau_2, \mathcal{I})$ = {x$\in$X |U $\cap$ A $\notin$ $\mathcal{I}$,$\forall$ quasi open set U containing x}.

A subset A of an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) is said to be q$\mathcal{I}$- open [3] if A $\subset$ qInt A^*_q. A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) $\to$ (Y, σ_1, σ_2) is called q$\mathcal{I}$- continuous[3] if f^{-1}(V) is q$\mathcal{I}$-open in X for every quasi open set V of Y. Recently the authors of this paper[11] defined qs$\mathcal{I}$- open sets and qs$\mathcal{I}$-continuous mappings in ideal bitopological spaces.

Definition1.1. [11] Given an ideal bitopological space (X, τ_1, τ_2 $\mathcal{I}$) the quasi semi local mapping of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by $A^*_{qs}(\tau_1, \tau_2, \mathcal{I})$ (more generally as A^*_{qs}) is defined as $A^*_{qs}(\tau_1, \tau_2, \mathcal{I})$ = {x$\in$X |U $\cap$ A $\notin$ $\mathcal{I}$,$\forall$ quasi semi-open set U containing x}

Definition1.2. [11] A subset A of an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) is qs$\mathcal{I}$- open if $A \subset$ qsInt(A^*_{qs}) and qs$\mathcal{I}$-closed if its complement is qs$\mathcal{I}$- open. If the set A is qs$\mathcal{I}$-open and qs$\mathcal{I}$-closed, then it is called qs$\mathcal{I}$-clopen

Remark1.1. [11] Every q$\mathcal{I}$- open set is qs$\mathcal{I}$- open but the converse is not true

Remark1.2. [11] The concepts of qs$\mathcal{I}$- open sets and quasi semi open sets are independent.

Definition1.3.[11] A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) $\to$ (Y, σ_1 ,σ_2) is called a qs$\mathcal{I}$- continuous if f^{-1}(V) is a qs$\mathcal{I}$- open set in X for every quasi open set V of Y

Remark1.3. [11] Every q$\mathcal{I}$- continuous mapping is qs$\mathcal{I}$-continuous but the converse is not true

Definition1.4.[11] In an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ the quasi $*$-semi closure of A of X denoted by $qscl^*(A)$ is defined by $qscl^*(A) = A \cup A^*_{qs}$

Definition1.5.[11] A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be a qs$\mathcal{I}$- neighbourhood of a point $x \in X$ if $\exists$ a qs$\mathcal{I}$- open set O such that $x \in O \subset A$

Definition1.6.[11] Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and $x \in X$. Then x is called a qs$\mathcal{I}$- interior point of A if $\exists$ V a qs$\mathcal{I}$- open set in X such that $x \in V \subset A$.
The set of all qs$\mathcal{I}$- interior points of A is called the qs$\mathcal{I}$-interior of A and is denoted by qs$\mathcal{I}$Int(A).

Definition1.7.[11] Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and $x \in X$. Then x is called a qs$\mathcal{I}$-cluster point of A, if $V \cap A \neq \emptyset$ for every qs$\mathcal{I}$- open set V in X. The set of all qs$\mathcal{I}$- cluster points of A denoted by qs$\mathcal{I}$cl(A) is called the qs$\mathcal{I}$- closure of A .

Definition1.9.[12] A mapping $f: (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2)$ is called q$\mathcal{I}$- irresolute if $f^{-1}(V)$ is a q$\mathcal{I}$- open set in X for every quasi open set V of Y.

Definition1.8.[12] A mapping $f: (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2)$ is called qs$\mathcal{I}$- irresolute if $f^{-1}(V)$ is a qs$\mathcal{I}$- open set in X for every quasi semi open set V of Y.

2. Contra qs$\mathcal{I}$-continuous functions

Definition 2.1. A function $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is called contra qs$\mathcal{I}$- continuous if f^{-1} (V) is qs$\mathcal{I}$-closed in X for each quasi open set V in Y.

Theorem 2.1. For a function $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:

a) f is contra qs$\mathcal{I}$-continuous .
b) For every quasi closed subset F of Y, f^{-1} (F) is qs$\mathcal{I}$-open in X.
c) For each $x \in$. X and each quasi closed subset F of Y with $f(x) \in$.F, there exists a qs$\mathcal{I}$-open subset U of X with $x \in$. U such that $f(U) \subset F$.

Proof: (a) $\Rightarrow$ (b) and (b) $\Rightarrow$ (c) are obvious.
(c) $\Rightarrow$ (b) Let F be any quasi closed subset of Y. If $x \in$. f^{-1} (F) then $f(x) \in F$, and there exists a qs$\mathcal{I}$- open subset U_x of X with $x \in U_x$ such that $f(U_x) \subset F$. Therefore, $f^{-1}(F) = \cup \{U_x : x \in. f^{-1}(F)\}$. Hence we get f^{-1} (F) is qs$\mathcal{I}$-open.[11]

Remark 2.1. Every contra qs$\mathcal{I}$-continuous function is contra q$\mathcal{I}$-continuous, but the converse need not be true

Remark 2.2. The concepts of qs$\mathcal{I}$ -continuity and contra qs$\mathcal{I}$ -continuity are independent of each other

Theorem 2.2. If a function $f: (X, \tau1, \tau2, I) \to (Y, \sigma_1, \sigma_2)$ is contra qs$\mathcal{I}$-continuous and Y is regular, then f is qs$\mathcal{I}$-continuous

Proof: Let $x \in X$ and let V be a quasi open subset of Y with $f(x) \in$.V Since Y is regular, there exists an quasi open set W in Y such that $f(x) \in W \subset cl(W) \subset V$. Since f is contra qs$\mathcal{I}$-continuous, by Theorem 2.1.there exists a qs$\mathcal{I}$-open set U in X with $x \in U$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. Hence f is qs$\mathcal{I}$-continuous [11].

Remark 2.3. If f is contra qs$\mathcal{I}$-continuous and Y is regular, then f need not be contra qs$\mathcal{I}$-continuous

Definition 2.2. A topological space $(X, \tau1, \tau2, I)$ is said to be qs$\mathcal{I}$ -connected if X is not the union of two disjoint non-empty qs$\mathcal{I}$-open subsets of X.

Theorem 2.3. If $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is a contra qs$\mathcal{I}$-continuous function from a qs$\mathcal{I}$-connected space X onto any space Y , then Y is not a discrete space.

Proof: Suppose that Y is discrete. Let A be a proper non-empty quasi clopen set in Y. Then $f^{-1}(A)$ is a proper non-empty qs$\mathcal{I}$- clopen subset of X, which contradicts the fact that X is qs$\mathcal{I}$-connected.

Theorem 2.4. A contra qs$\mathcal{I}$-continuous image of a qs$\mathcal{I}$-connected space is connected.

Proof: Let $f: (X, \tau1, \tau2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a contra qs$\mathcal{I}$-continuous function from a qs$\mathcal{I}$-connected space X onto a space Y . Assume that Y is disconnected. Then $Y = A \cup B$, where A and B are non-empty quasi clopen sets in Y with $A \cap B = \emptyset$. Since f is contra qs$\mathcal{I}$-continuous , we have that $f^{-1}(A)$ and $f^{-1}(B)$ are qs$\mathcal{I}$-open non-empty sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. This means that X is not semi-I-connected, which is a contradiction. Then Y is connected.

Definition 2.3. A space $(X, \tau1, \tau2, \mathcal{I}))$ is said to be qs$\mathcal{I}$-normal if each pair of non-empty disjoint quasi closed sets can be separated by disjoint qs$\mathcal{I}$-open sets.

Definition 2.4. A space (Y, σ_1, σ_2) is said to be ultra normal if each pair of non-empty disjoint quasi closed sets can be separated by disjoint quasi clopen sets.

Theorem 2.5. If $f: (X, \tau1, \tau2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is contra qs$\mathcal{I}$-continuous, closed and one-one, Y is ultra normal, then X is qs$\mathcal{I}$- normal.

Proof: Let C_1 and C_2 be disjoint quasi closed subsets of X. Since f is closed and one-one $f(C_1)$ and $f(C_2)$ are disjoint quasi closed subsets of Y. But Y is ultra normal, so $f(C_1)$ and $f(C_2)$ are separated by disjoint quasi clopen sets V_1 and V_2, respectively.

Since f is contra qs$\mathcal{I}$- continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are qs$\mathcal{I}$- open, where $C_1 \subseteq f^{-1}(V_1)$, $C_2 \subseteq f^{-1}(V_2)$ and $f^{-}(V_1) \cap f^{-1}(V_2) = \emptyset$. Hence, X is qs$\mathcal{I}$-normal.

Definition 2.5. A space (X, $\tau 1$, $\tau 2$, $\mathcal{I}$) is said to be qs$\mathcal{I}$-compact if every qs$\mathcal{I}$- opencover of X has a finite subcover.

Definition 2.6. A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) $\rightarrow$ (Y, σ_1, σ_2) is called contra qs$\mathcal{I}$- irresolute if f^{-1}(V) is a qs$\mathcal{I}$- closed set in X for every quasi semi open set V of Y.

Remark 2.5. Contra qs$\mathcal{I}$- irresoluteness and qs$\mathcal{I}$ -irresoluteness are independent

Definition 2.7 A function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is called quasi- irresolute if f^{-1}(V) is qs$\mathcal{I}_1$- open in X for each qs$\mathcal{I}_2$- open set V of Y.

Definition 2.8. A function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is called contra quasi- irresolute if f^{-1}(V) is qs$\mathcal{I}_1$- closed in X for each qs$\mathcal{I}_2$- open set V of Y.
The following two remarks are evident from the definition

Remark 2.5. Contra quasi- irresoluteness and quasi-irresoluteness are independent

Remark 2.6. Contra quasi- irresolute function is contra qs$\mathcal{I}$-continuous, but the converse is not true

Theorem 2.6. A function f: f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is quasi-irresolute if and only if the inverse image of each qs$\mathcal{I}_2$-closed set in Y is qs$\mathcal{I}_1$-open in X.

Theorem 2.7. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) and g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) Then,

1. gof is contra quasi- irresolute if g is quasi -irresolute and f is contra quasi- irresolute.
2. gof is contra quasi- irresolute if g is contra quasi-irresolute and f is quasi -irresolute.

Theorem 2.8. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) and g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) Then,

1. gof is contra qs$\mathcal{I}$- continuous if g is continuous and f is contra qs$\mathcal{I}$- continuous.
2. gof is contra qs$\mathcal{I}$- continuous if g is qs$\mathcal{I}$- continuous and f is contra quasi- irresolute

The next theorem follows from the fact that a function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is qs$\mathcal{I}$- open [11] if for each quasi open set U of X, f(U) is qs$\mathcal{I}$- open in Y .

Theorem 2.9. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) be onto, quasi- irresolute and qs$\mathcal{I}$- open and let g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) be any function. Then gof is contra qs$\mathcal{I}$-continuous if and only if g is contra qs$\mathcal{I}$ -continuous.

Proof: Necessary: Let gof be contra qs$\mathcal{I}$- continuous and C a quasi closed subset of Z. Then $(gof)^{-1}$(C) is a qs$\mathcal{I}$- open subset of X. Thus f^{-1} (g^{-1} (C)) is qs$\mathcal{I}$- open in X. Since f is qs$\mathcal{I}$- open, f(f^{-1} (g^{-1} (C))) is qs$\mathcal{I}$ -open subset of Y. So g^{-1} (C) is qs- open in Y. Therefore, g is contra qs$\mathcal{I}$ -continuous.
Sufficient: Obvious.

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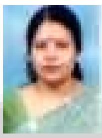

Mandira Kar has completed her M. Sc & M. Phil (Mathematics) degree from RDVV, Jabalpur in 1983 and 1984 respectively. She has worked as Lecturer from 1984-87 in St. Joseph's Convent, as Asst. Prof. Govt. P.G. College, Chindwara from 1987-1992. Presently she is Prof. in St. Aloysius College, Jabalpur. She has presented more than 45 Research papers in International and National conferences. She has also been the Resource person in 06 National Conferences and UGC scheme (coaching) NET Exam. She has been a Visiting Faculty of FOMS, CMM, Jabalpur. She has 20 publications to her credit. Her specialization is Ideal Bitopology. She is the Assistant Editor of Global Research Journal on Mathematics and Science Education ISSN 2278-0769. She is also a Guest lecturer in esteemed institutions. She is the Member of many Academic bodies.