Contra qs$\mathcal{I}$- Continuous Functions in Ideal Bitopological Spaces

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Abstract: ***In this paper, we apply the notion of qs$\mathcal{I}$-open sets and qs$\mathcal{I}$-continuous functions to present and study a new class of functions called contra qs$\mathcal{I}$-continuous functions in ideal bitopological spaces.***

Keywords: Ideal bitopological space, qs$\mathcal{I}$-open sets, qs$\mathcal{I}$-continuous functions contra qs$\mathcal{I}$-continuous functions

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1. Preliminaries

In 1961 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2[5] The study of quasi open sets in bitopological spaces was initiated by Datta[1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [9] if it is a union of a τ_1-open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp., τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A. It is denoted by qcl(A). The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A)[1].

In 1963 N. Levine [8] introduced the concept of semi open sets in topology. A subset A of a topological space (X, τ) is called semi open if there exists an open set O in X such that $O \subset A \subset Cl(O)$. Every open set is semi open but the converse may not be true. In 1985, Maheshwari, Chae and Thakur[10] introduced quasi semi open sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi semi open[10] if it is a union of a τ_1-semi open set and a τ_2-semi open set. Complement of a quasi semi open set is called quasi semi closed. Every τ_1-semi open (τ_2-semiopen, quasi open) set is quasi semi open but the converse may not be true. Any union of quasi semi open sets of X is a quasi semi open set in X. The intersection of all quasi semi closed sets which contains A is called quasi semi closure of A. It is denoted by qscl(A). The union of quasi semi open subsets of A is called quasi semi interior of A. It is denoted by qsInt(A)

In 1996 Dontchev[2] introduced a new class of functions called contra-continuous functions. A function f: X → Y to be contra continuous if the pre image of every open set of Y is closed in X. The study of ideal topological spaces was initiated by Kuratowski [7] and Vaidyanathaswamy [13]. An Ideal $\mathcal{I}$ on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: $A \in \mathcal{I}$ and $B \subset A \Rightarrow B \in \mathcal{I}$ and $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$. If $\mathcal{P}(X)$ is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ is called the local mapping[6] of A with respect to τ and $\mathcal{I}$ and is defined as follows: $A^*(\tau, \mathcal{I})$ (in short A^*) = $\{x \in X \mid U \cap A \notin \mathcal{I}, \forall U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$[4]. Given an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) the quasi local mapping[3] of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by A^*_q (τ_1, τ_2, $\mathcal{I}$) (in short A^*_q) is defined as follows: $A^*_q(\tau_1, \tau_2, \mathcal{I}) = \{x \in X \mid U \cap A \notin \mathcal{I}, \forall$ quasi open set U containing x$\}$.

A subset A of an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) is said to be q$\mathcal{I}$- open [3] if $A \subset qInt\, A^*_q$. A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) → (Y, σ_1, σ_2) is called q$\mathcal{I}$- continuous[3] if $f^{-1}(V)$ is q$\mathcal{I}$-open in X for every quasi open set V of Y. Recently the authors of this paper[11] defined qs$\mathcal{I}$- open sets and qs$\mathcal{I}$-continuous mappings in ideal bitopological spaces.

Definition1.1. [11] Given an ideal bitopological space (X, τ_1, τ_2 $\mathcal{I}$) the quasi semi local mapping of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by $A^*_{qs}(\tau_1, \tau_2, \mathcal{I})$ (more generally as A^*_{qs}) is defined as $A^*_{qs}(\tau_1, \tau_2, \mathcal{I}) = \{x \in X \mid U \cap A \notin \mathcal{I}, \forall$ quasi semi-open set U containing x$\}$

Definition1.2. [11] A subset A of an ideal bitopological space (X, τ_1, τ_2, $\mathcal{I}$) is qs$\mathcal{I}$- open if $A \subset qsInt(A^*_{qs})$ and qs$\mathcal{I}$-closed if its complement is qs$\mathcal{I}$- open. If the set A is qs$\mathcal{I}$-open and qs$\mathcal{I}$-closed, then it is called qs$\mathcal{I}$-clopen

Remark1.1. [11] Every q$\mathcal{I}$- open set is qs$\mathcal{I}$- open but the converse is not true

Remark1.2. [11] The concepts of qs$\mathcal{I}$- open sets and quasi semi open sets are independent.

Definition1.3.[11] A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) → (Y, σ_1, σ_2) is called a qs$\mathcal{I}$- continuous if $f^{-1}(V)$ is a qs$\mathcal{I}$- open set in X for every quasi open set V of Y

Remark1.3. [11] Every q$\mathcal{I}$- continuous mapping is qs$\mathcal{I}$-continuous but the converse is not true

Definition1.4.[11] In an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ the quasi $*$-semi closure of A of X denoted by $qscl^*(A)$ is defined by $qscl^*(A) = A \cup A^*_{qs}$

Definition1.5.[11] A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be a qs$\mathcal{I}$- neighbourhood of a point $x \in X$ if $\exists$ a qs$\mathcal{I}$- open set O such that $x \in O \subset A$

Definition1.6.[11] Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and $x \in X$. Then x is called a qs$\mathcal{I}$- interior point of A if $\exists$ V a qs$\mathcal{I}$- open set in X such that $x \in V \subset A$.
The set of all qs$\mathcal{I}$- interior points of A is called the qs$\mathcal{I}$-interior of A and is denoted by qs$\mathcal{I}$Int(A).

Definition1.7.[11] Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and $x \in X$. Then x is called a qs$\mathcal{I}$-cluster point of A, if $V \cap A \neq \emptyset$ for every qs$\mathcal{I}$- open set V in X. The set of all qs$\mathcal{I}$- cluster points of A denoted by qs$\mathcal{I}$cl(A) is called the qs$\mathcal{I}$- closure of A .

Definition1.9.[12] A mapping $f: (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2)$ is called q$\mathcal{I}$- irresolute if $f^{-1}(V)$ is a q$\mathcal{I}$- open set in X for every quasi open set V of Y.

Definition1.8.[12] A mapping $f: (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2)$ is called qs$\mathcal{I}$- irresolute if $f^{-1}(V)$ is a qs$\mathcal{I}$- open set in X for every quasi semi open set V of Y.

2. Contra qs$\mathcal{I}$-continuous functions

Definition 2.1. A function $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is called contra qs$\mathcal{I}$- continuous if $f^{-1}(V)$ is qs$\mathcal{I}$-closed in X for each quasi open set V in Y.
Theorem 2.1. For a function $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:
a) f is contra qs$\mathcal{I}$-continuous .
b) For every quasi closed subset F of Y, $f^{-1}(F)$ is qs$\mathcal{I}$-open in X.
c) For each $x \in$. X and each quasi closed subset F of Y with $f(x) \in$.F, there exists a qs$\mathcal{I}$-open subset U of X with $x \in$. U such that $f(U) \subset F$.

Proof: (a) $\Rightarrow$ (b) and (b) $\Rightarrow$ (c) are obvious.
(c) $\Rightarrow$ (b) Let F be any quasi closed subset of Y. If $x \in$. f^{-1}(F) then $f(x) \in F$, and there exists a qs$\mathcal{I}$- open subset U_x of X with $x \in U_x$ such that $f(U_x) \subset F$. Therefore, $f^{-1}(F) = \cup \{U_x: x \in. f^{-1}(F)\}$. Hence we get $f^{-1}(F)$ is qs$\mathcal{I}$-open.[11]

Remark 2.1. Every contra qs$\mathcal{I}$-continuous function is contra q$\mathcal{I}$-continuous, but the converse need not be true

Remark 2.2. The concepts of qs$\mathcal{I}$ -continuity and contra qs$\mathcal{I}$ -continuity are independent of each other

Theorem 2.2. If a function $f: (X, \tau 1, \tau 2, I) \to (Y, \sigma_1, \sigma_2)$ is contra qs$\mathcal{I}$-continuous and Y is regular, then f is qs$\mathcal{I}$-continuous

Proof: Let $x \in X$ and let V be a quasi open subset of Y with $f(x) \in$.V Since Y is regular, there exists an quasi open set W in Y such that $f(x) \in W \subset cl(W) \subset V$. Since f is contra qs$\mathcal{I}$-continuous, by Theorem 2.1.there exists a qs$\mathcal{I}$-open set U in X with $x \in U$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. Hence f is qs$\mathcal{I}$-continuous [11].

Remark 2.3. If f is contra qs$\mathcal{I}$-continuous and Y is regular, then f need not be contra qs$\mathcal{I}$-continuous

Definition 2.2. A topological space $(X, \tau 1, \tau 2, I)$ is said to be qs$\mathcal{I}$ -connected if X is not the union of two disjoint non-empty qs$\mathcal{I}$-open subsets of X.

Theorem 2.3. If $f: (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is a contra qs$\mathcal{I}$-continuous function from a qs$\mathcal{I}$-connected space X onto any space Y , then Y is not a discrete space.

Proof: Suppose that Y is discrete. Let A be a proper non-empty quasi clopen set in Y. Then $f^{-1}(A)$ is a proper non-empty qs$\mathcal{I}$- clopen subset of X, which contradicts the fact that X is qs$\mathcal{I}$-connected.

Theorem 2.4. A contra qs$\mathcal{I}$-continuous image of a qs$\mathcal{I}$-connected space is connected.

Proof: Let $f: (X, \tau 1, \tau 2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a contra qs$\mathcal{I}$-continuous function from a qs$\mathcal{I}$-connected space X onto a space Y . Assume that Y is disconnected. Then $Y = A \cup B$, where A and B are non-empty quasi clopen sets in Y with $A \cap B = \emptyset$. Since f is contra qs$\mathcal{I}$-continuous , we have that $f^{-1}(A)$ and $f^{-1}(B)$ are qs$\mathcal{I}$-open non-empty sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. This means that X is not semi-I-connected, which is a contradiction. Then Y is connected.

Definition 2.3. A space $(X, \tau 1, \tau 2, \mathcal{I}))$ is said to be qs$\mathcal{I}$-normal if each pair of non-empty disjoint quasi closed sets can be separated by disjoint qs$\mathcal{I}$-open sets.

Definition 2.4. A space (Y, σ_1, σ_2) is said to be ultra normal if each pair of non-empty disjoint quasi closed sets can be separated by disjoint quasi clopen sets.

Theorem 2.5. If $f: (X, \tau 1, \tau 2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is contra qs$\mathcal{I}$-continuous, closed and one-one, Y is ultra normal, then X is qs$\mathcal{I}$- normal.

Proof: Let C_1 and C_2 be disjoint quasi closed subsets of X. Since f is closed and one-one $f(C_1)$ and $f(C_2)$ are disjoint quasi closed subsets of Y. But Y is ultra normal, so $f(C_1)$ and $f(C_2)$ are separated by disjoint quasi clopen sets V_1 and V_2, respectively.

Since f is contra qs$\mathcal{I}$- continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are qs$\mathcal{I}$- open, where $C_1 \subseteq f^{-1}(V_1)$, $C_2 \subseteq f^{-1}(V_2)$ and $f^{-}(V_1) \cap f^{-1}(V_2) = \emptyset$. Hence, X is qs$\mathcal{I}$-normal.

Definition 2.5. A space (X, $\tau 1$, $\tau 2$, $\mathcal{I}$) is said to be qs$\mathcal{I}$- compact if every qs$\mathcal{I}$- opencover of X has a finite subcover.

Definition 2.6. A mapping f: (X, τ_1, τ_2, $\mathcal{I}$) $\rightarrow$ (Y, σ_1, σ_2) is called contra qs$\mathcal{I}$- irresolute if f^{-1}(V) is a qs$\mathcal{I}$- closed set in X for every quasi semi open set V of Y.

Remark 2.5. Contra qs$\mathcal{I}$- irresoluteness and qs$\mathcal{I}$ -irresoluteness are independent

Definition 2.7 A function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is called quasi- irresolute if f^{-1}(V) is qs$\mathcal{I}_1$- open in X for each qs$\mathcal{I}_2$- open set V of Y.

Definition 2.8. A function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is called contra quasi- irresolute if f^{-1}(V) is qs$\mathcal{I}_1$- closed in X for each qs$\mathcal{I}_2$- open set V of Y.
The following two remarks are evident from the definition

Remark 2.5. Contra quasi- irresoluteness and quasi- irresoluteness are independent

Remark 2.6. Contra quasi- irresolute function is contra qs$\mathcal{I}$- continuous, but the converse is not true

Theorem 2.6. A function f: f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is quasi-irresolute if and only if the inverse image of each qs$\mathcal{I}_2$-closed set in Y is qs$\mathcal{I}_1$-open in X.

Theorem 2.7. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) and g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) Then,

1. gof is contra quasi- irresolute if g is quasi -irresolute and f is contra quasi- irresolute.
2. gof is contra quasi- irresolute if g is contra quasi- irresolute and f is quasi -irresolute.

Theorem 2.8. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) and g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) Then,

1. gof is contra qs$\mathcal{I}$- continuous if g is continuous and f is contra qs$\mathcal{I}$- continuous.
2. gof is contra qs$\mathcal{I}$- continuous if g is qs$\mathcal{I}$- continuous and f is contra quasi- irresolute

The next theorem follows from the fact that a function f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) is qs$\mathcal{I}$- open [11] if for each quasi open set U of X, f(U) is qs$\mathcal{I}$- open in Y .

Theorem 2.9. Let f: (X, τ_1, τ_2, $\mathcal{I}_1$) $\rightarrow$ (Y, σ_1, σ_2, $\mathcal{I}_2$) be onto, quasi- irresolute and qs$\mathcal{I}$- open and let g: (Y, σ_1, σ_2, $\mathcal{I}_2$) $\rightarrow$ (Z, ρ_1, ρ_2, $\mathcal{I}_3$) be any function. Then gof is contra qs$\mathcal{I}$- continuous if and only if g is contra qs$\mathcal{I}$ -continuous.

Proof: Necessary: Let gof be contra qs$\mathcal{I}$- continuous and C a quasi closed subset of Z. Then $(gof)^{-1}$(C) is a qs$\mathcal{I}$- open subset of X. Thus f^{-1} (g^{-1} (C)) is qs$\mathcal{I}$- open in X. Since f is qs$\mathcal{I}$- open, f(f^{-1} (g^{-1} (C))) is qs$\mathcal{I}$ -open subset of Y. So g^{-1} (C) is qs- open in Y. Therefore, g is contra qs$\mathcal{I}$ -continuous.
Sufficient: Obvious.

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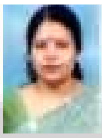

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