Putting the value of $d\Omega(\xi)$ from (3.2) and changing the order of integration and summation, we get

$$\left[\int_0^a \xi^{2\sigma}\Psi_1(\xi,\, t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\, t)d\xi+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\, t)d\xi\right]\times 2^{1/2-\sigma}$$

$$\sum_{n=0}^{\infty}\frac{\left(\frac{\ell}{t}\right)^n\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\,\nu}(x,-t)}{2^{4(n+p)}(n+p)!\,\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}W_{n+p,\,\sigma}(\xi,\, t)$$

$$=\begin{cases} g_1(x,\, t), & 0\le x<a\\ g_3(x,\, t), & b<x<\infty\end{cases}\qquad (4.2.6)$$

Using summation results (3.4) in equation (4.2 (4.2.7), we get **(4.2.7)**

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,\, t)S(x,\, \xi,\, t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\, t)S(x,\, \xi,\, t)d \qquad (4.2.8)$$

$$=\begin{cases} g_1(x,\, t), & 0\le x<a\\ g_3(x,\, t), & b<x<\infty\end{cases}\qquad (4.2.9)$$

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,\, t)S(x,\, \xi,\, t)d\xi+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\, t)S(x,\, \xi,\, t)d\xi$$

$$=\begin{cases} G(x,\, t), & 0\le x<a \qquad (4.2.10)\\ H(x,\, t), & b\le x<\infty\end{cases}\qquad (4.2.11)$$

where $G(x,\, t)=g_1(x,\, t)-\int_a^b \xi^{2\sigma}g_2(\xi,\, t)S(x,\, \xi,\, t)d\xi$

(4.2.12)

$$H(x,\, t)=g_3(x,\, t)-\int_a^b \xi^{2\sigma}g_2(\xi,\, t)S(x,\, \xi,\, t)d\xi \qquad (4.2.13)$$

Now using the notation given by (3.6) in equation (4.2.10), we get

$$\int_0^x \xi^{2\sigma}\Psi_1(\xi,\, t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*s_\xi(x,\, \xi,\, t)\right\}d\xi$$

$$+\int_x^a \xi^{2\sigma}\Psi_1(\xi,\, t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*s_x(x,\, \xi,\, t)\right\}d\xi$$

$$+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\, t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*s_x(x,\, \xi,\, t)\right\}d\xi$$

$$=G(x,\, t)\qquad 0\le x<a \qquad (4.2.14)$$

$$\int_0^x \xi e^{-\xi^2/4t}\Psi_1(\xi,\, t)s_\xi(x,\, \xi,\, t)d\xi+\int_x^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\, t)s_x(x,\, \xi,\, t)d\xi$$

$$+\int_b^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)s_x(x,\, \xi,\, t)d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G(x,\, t)\qquad 0\le x<a \qquad (4.2.15)$$

Now putting the value of summation in terms of integral from (3.5), we obtain

$$\int_0^x \xi e^{-\xi^2/4t}\Psi_1(\xi,\, t)\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_x^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\, t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_b^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

(4.2.16)

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G(x,\, t)\qquad 0\le x<a$$

Inverting the (4.2.7) n, we have

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_y^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,\, t),}{\left(\xi^2-y^2\right)^{1-m}}d\xi$$

$$+\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G(x,\, t)\qquad 0\le x<a \qquad (4.2.17)$$

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\left[\overline{\psi_1(y)}+\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\right]$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G \quad (4.2.9) \quad <a \qquad (4.2.18)$$

where (4.2.10)

$$\overline{\psi_1(y)}=\int_y^a \frac{\xi e \quad (4.2.11) \quad t)}{\left(\xi^2-y^2\right)^{\;n}}d\xi \qquad (4.2.19)$$

With the help of equations (3.8) and (3.10) solving equation (4.218), as

(4.2.13)

$$\eta(y)\left[\overline{\psi_1(y)}+\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\right]$$

$$=\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_0^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\times\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G(x,\, t)\qquad 0\le x<a \qquad (4.2.20)$$

$$G_1(y,\, t)=\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_0^y \frac{2x^{2\nu}G(x,\, t),}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}dx \qquad (4.2.21)$$

Now equation (4.2.21) can be rewritten as

$$\eta(y)\overline{\psi_1(y)}=G_1(y,\, t)-\eta(y)\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \qquad (4.2.22)$$

Using the results (3.9) and (3.11), we solve the integral equation (4.2.19), as follows

$$\xi e^{-\xi^2/4t}\Psi_1(\xi,\, t)=-\frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^a \frac{2y\overline{\psi_1(y)}}{\left(y^2-\xi^2\right)^m}dy \qquad (4.2.23)$$

Similarly, let us

$$\overline{\psi_2(y)}=\int_y^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\, t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \qquad (4.2.24)$$

then we have

$$\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t) = -\frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_{\xi}^{\infty}\frac{2y\overline{\psi_2(y)}}{\left(y^2-\xi^2\right)^m}dy \qquad (4.2.25)$$

With the help of (4.2.24) and (4.2.25), we obtain

$$\int_0^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{\mathrm{Sin}(1-m)\pi}{\pi\left(-y^2\right)^{-m}}\int_0^a \frac{2x\overline{\psi_1(x)}}{\left(x^2\right)^m\left(x^2-y^2\right)}dx \qquad (4.2.26)$$

$$\int_b^{\infty} \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{\mathrm{Sin}(1-m)\pi}{\pi\left(b^2-y^2\right)^{-m}}\int_b^{\infty} \frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)}dx \qquad (4.2.27)$$

Using equation (4.2) in (4.2.23), we get

$$\eta(y)\overline{\psi_1(y)} = G_1(y,\ t) - \frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}$$

$$\int_b^{\infty}\frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)}dx \qquad (4.2.28)$$

Now equation (4.2) reduces to

$$\eta(y)\overline{\psi_1(y)} = G_1(y,\ t) - \int_b^{\infty} B(x,\ y)\overline{\psi_2(x)}dx, \quad 0 \le x < a \qquad (4.2.29)$$

where

$$B(x,\ y) = \frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}\frac{2x}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)} \qquad (4.2.30)$$

Again starting from equation (4.2.11) we have

$$+\int_0^a \xi^{2\sigma}\Psi_1(\xi,\ t)S(x,\ \xi,\ t)d\xi + \int_b^{\infty}\xi^{2\sigma}\Psi_2(\xi,\ t)S(x,\ \xi,\ t)d\xi = H(x,\ t) \qquad b < x < \infty \qquad (4.2.31)$$

Using the notation given by (3.6) in above equation, we get

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_{\xi}(x,\ \xi,\ t)\right\}d\xi$$

$$+\int_b^x \xi^{2\sigma}\Psi_2(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_{\xi}(x,\ \xi,\ t)\right\}d\xi$$

$$+\int_x^{\infty} \xi^{2\sigma}\Psi_2(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_{x}(x,\ \xi,\ t)\right\}d\xi$$

$$= H(x,\ t) \qquad b < x < \infty \qquad (4.2.32)$$

$$\int_0^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)S_{\xi}(x,\ \xi,\ t)d\xi + \int_b^x \xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)S_{\xi}(x,\ \xi,\ t)d\xi$$

$$+\int_x^{\infty}\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)S_x(x,\ \xi,\ t)d\xi$$

$$= \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}H(x,\ t), \qquad 0 < x < \infty \qquad (4.2.33)$$

Putting the value of summation in terms of integral from (3.5), we have

$$\int_0^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)\int_0^{\xi}\eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_b^x \xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)\int_0^{\xi}\eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_x^{\infty} \xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)\int_0^{x}\eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$= \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}H(x,\ t), \qquad b < x < \infty \qquad (4.2.34)$$

Inverting the order of integration of equation (4.2.34), we get

$$\int_b^x \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_y^{\infty}\frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}H(x,\ t)$$

$$-\int_0^a \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_y^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)}{\left(\xi^2-y^2\right)^{-m}}d\xi$$

$$-\int_0^b \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_b^{\infty}\frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi, \quad b < x < \infty \qquad (4.2.35)$$

Using the expression given by equation (4.2.19) and (4.2.25) in equation (4.2.35, we get) (4.2.28)

$$\int_b^x \frac{\eta(y)\overline{\Psi_2(y)}dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}} = \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}H(x,\ t) - \int_0^a \frac{\eta(y)\overline{\Psi_1(y)}dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}$$

$$-\int_0^b \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_b^{\infty}\frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi, \qquad b < x < \infty \qquad (4.2.36)$$

With the help of equations (3.8) and (3.10) above equation can be solved as

$$\eta(y)\overline{\Psi_2(y)} = \frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\times\left[\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}H(x,\ t) - \int_0^a \frac{\overline{\Psi_1(z)}\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\right.$$

$$\left.-\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\int_b^{\infty}\frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right], \quad b < x < \infty \qquad (4.2.37)$$

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}$$

$$\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}\times\left[\int_0^a \frac{\overline{\Psi_1(z)}\eta(z)}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}dz\right.$$

$$\left.+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\times\int_b^{\infty}\frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right], \ b < x < \infty \qquad (4.2.38)$$

where

$$H_1(y,\ t) = \frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_b^y \frac{2x^{2\nu}H(x,\ t)}{\left(y^2-x^2\right)^{\nu-\sigma+m}}dx \qquad (4.2.39)$$

Now using the equation (4.2.23) in equation (4.2.39), we obtain

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi}$$

$$\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}}\times\left\{\int_0^a \frac{G_1(x,\ t)}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}dz\right.$$

(4.2.40)

$$-\int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} \times \int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi$$

$$\left.+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} \times \int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi\right\},\ b<x<\infty$$

Breaking the last term of (4.2.41) in to part, we have

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}}$$

$$\times\left\{\int_0^a \frac{G_1(z,\ t)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} - \int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\right.$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi + \int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi + \int_a^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\left.\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi\right\},\qquad b<x<\infty \qquad \textbf{(4.2.41)}$$

Now changing the order of integration, equation (4.2.41) becomes

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}$$

$$\times\left\{\int_0^a G_1(z,\ t)dz \times \frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\right.$$

$$+\int_a^b \eta(z)dz.\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\left.\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}} d\xi\right\},\qquad b<x<\infty \qquad \textbf{(4.2.42)}$$

We know that

$$\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}} = \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}} \qquad \textbf{(4.2.43)}$$

Using the results (4.2.43) and (4.2.28), in equation (4.2.43), we get

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}$$

$$\left\{\int_0^a G_1(z,\ t)dz \times \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}}\right.$$

$$+\int_a^b \eta(z)dz \times \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$\left.\frac{Sin(1-m)\pi}{\pi\left(b^2-z^2\right)^{-m}} \times \int_b^\infty \frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-z^2\right)}dx\right\},\ b<x<\infty \qquad \textbf{(4.2.44)}$$

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz$$

$$-\frac{Sin(1-\nu+\sigma-m)\pi Sin(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz$$

$$\int_b^\infty \frac{2x\overline{\Psi_2(x)}}{\left(x^2-b^2\right)^m}dx,\qquad b<x<\infty \qquad \textbf{(4.2.45)}$$

Changing the order of integration of the last term of equation (4.2.45), we get

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz$$

$$-\frac{Sin(1-\nu+\sigma-m)\pi Sin(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_b^\infty \frac{2(x)\overline{\Psi_2(x)}}{\left(x^2-b^2\right)^m}dx,$$

$$\times\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz,\quad b<x<\infty \qquad \textbf{(4.2.46)}$$

Now equation (4.2.46) can be reduced to the following form

$$\eta(y)\overline{\Psi_2(y)} + \int_b^\infty C(x,\ y)\overline{\Psi_2(x)}dx = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz,\qquad b<x<\infty \qquad \textbf{(4.2.47)}$$

where C(x, y) is the symmetric kernel given as

$$C(x,\ y) = \frac{Sin(1-\nu+\sigma-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\frac{2x}{\left(x^2-b^2\right)^m}\times\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz \qquad \textbf{(4.2.48)}$$

Equation (4.2.48) is a fredholm integral equation of the second kind which determines $\psi_2(y)$. After that, $\psi_2(\xi,\ t)$ and $\psi_1(\xi,\ t)$ can be found form equations (4.2.26) and (4.2.24) respectively. Finally, the coefficients B_n which satisfy the triple series equations (2.4) to (2.6) are given by equation (4.2.3).

Particular Case

If we let $b\to\infty$ in equations (2.4), (2.5) and (2.6) they reduce to the dual series equations and this solution for the triple series equations of the second kind agree with that obtained earlier of dual series [1].

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