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Triple Simultaneous Fourier Series Equations Involving Heat Polynomials

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Abstract: ***In this paper, we have considered the triple series equations involving heat polynomials of the first kind and second kind & solved the two sets of series equations.***

1. Introduction

In this problem, the equations are the generalization of dual series equations considered by Pathak [1]. He pointed out that the dual series equations involving generalized Laguerre polynomials considered by Lowndes [2], Srivastava [3] and panda [4] were the special cases of the equations considered by him and solution derived for the Laguerre case, was also different from those obtained by previous authors. We have considered the triple series equations, involving heat polynomials $P_{n\upsilon}(x,-t)$, of the first kind and second kind.

2. Triple Series Equations Involving Heat Polynomials

Here we shall solve the two sets of series equations involving heat polynomials.

(i) Triple Series Equations of the First Kind

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\,\sigma}(x,-t)=f_1(x,t),\qquad 0\le x<a \tag{2.1}$$

$$\sum_{n=0}^{\infty}\frac{t^{-n}\ell^{n}A_n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)}P_{n+p,\,\nu}(x,-t)=f_2(x,t),\qquad a\le x<b \tag{2.2}$$

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\,\sigma}(x,-t)=f_3(x,t),\qquad b\le x<\infty \tag{2.3}$$

(ii) Triple Series Equations of the Second kind

$$\sum_{n=0}^{\infty}\frac{t^{-n}\ell^{n}B_n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)}P_{n+p,\,\nu}(x,-t)=g_1(x,t),\qquad 0\le x<a \tag{2.4}$$

$$\sum_{n=0}^{\infty}\frac{B_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\,\sigma}(x,-t)=g_2(x,t),\qquad a\le x<b \tag{2.5}$$

$$\sum_{n=0}^{\infty}\frac{t^{-n}\ell^{n}B_n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)}P_{n+p,\,\nu}(x,-t)=g_3(x,t),\qquad b\le x<\infty \tag{2.6}$$

where $f_i(x,\ t)$ and $g_i(x,\ t),\ (i=1,\ 2,\ 3)$ are prescribed functions for $t\ge\ell>0$ and $A_n,\ B_n$ are to be determined and $P_{n,\ \nu}(x,\ t)$ is heat polynomial.

3. Preliminary Results

In this course of analysis, we shall use the following results:

(i) Orthogonality Result for Heat Polynomials

$$\int_0^{\infty}W_{m,\ \nu}(x,\ t)P_{n,\ \nu}(x,-t)d\Omega(x)=\frac{\delta_{mn}}{k_n}, \tag{3.1}$$

where
$$d\Omega(x)=2^{\frac{1}{2}-\nu}\left[\Gamma\left(\nu+\frac{1}{2}\right)\right]^{-1}x^{2\nu}dx \tag{3.2}$$

and
$$k_n=\Gamma\left(\nu+\frac{1}{2}\right)\Big/2^{4n}\,n!\,\Gamma\left(\nu+\frac{1}{2}+n\right) \tag{3.3}$$

(ii) The Series

$$S(x,\ \xi,\ t)=2^{\frac{1}{2}-\sigma}\sum_{n=0}^{\infty}\frac{(\ell/t)^n\,\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\ \nu}(x,\ -t)W_{n+p,\ \sigma}(\xi,\ t)}{2^{4(n+p)}(n+p)!\,\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)} \tag{3.4}$$

$$=\frac{x^{-2\nu+1}\xi^{-2\sigma+1}\ell^{\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*\int_0^{\infty}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{\nu-\sigma+m-1}dy \tag{3.5}$$

$$=\frac{x^{-2\nu+1}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_{\infty}(\xi,\ x,\ y) \tag{3.6}$$

where

$$a^*=\frac{\Gamma\left(\mu+\frac{1}{2}+n+p\right)\ell^n t^{-(n+m)}2^{2(1-m)}}{\Gamma\left(\sigma-m+\frac{1}{2}+n+p\right)} \tag{3.7}$$

$$\eta(y)=y^{2(\sigma-m)}e^{y^2/4t}$$

$$\omega=\min(\xi,\ x),\qquad \nu-\sigma+m>0$$

If h(y) is strictly monotonically increasing and differentiable function in the interval (a, b) and h'(y) ≠ 0 in this interval.

Then solutions of the integral equations

$$f(x)=\int_a^x\frac{\Phi(y)}{[h(x)-h(y)]^{\alpha}}dy\qquad 1<\alpha<1 \tag{3.8}$$

and

$$f(x)=\int_x^b\frac{\Phi(y)}{[h(y)-h(x)]^{\alpha}}dy\quad 0<\alpha<1 \tag{3.9}$$

are given by

$$\Phi(y)=\frac{\sin(\alpha\pi)}{\pi}\frac{d}{dy}\int_a^y\frac{h'(x)f(x)dx}{[h(x)-h(y)]^{1-\alpha}}, \qquad (3.10)$$

$$\Phi(y)=-\frac{\sin(\alpha\pi)}{\pi}\frac{d}{dy}\int_y^b\frac{h'(x)f(x)dx}{[h(x)-h(y)]^{1-\alpha}}, \qquad (3.11)$$

The Solution

4.1 Equations of the First Kind

In order to solve the triple series equations of the first kind, we set

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\,\sigma}(x,\,-t)=\tilde{\mathbf{N}}(x,\,t),\qquad a<x<b \qquad (4.1.1)$$

Using orthogonality relation (3.1) in equations (2.1), (2.3) and (4.1.1), we get

$$A_n=\frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)}{2^{4(n+p)}(n+p)!\,\Gamma\left(\sigma+\frac{1}{2}+n+p\right)}$$

$$\left[\int_0^a f_1(x,\,t)+\int_a^b\tilde{\mathbf{N}}(x,t)+\int_b^\infty f_3(x,t)\right]W_{n+p,\,\sigma}(x,\,t)d\Omega(x) \qquad (4.1.2)$$

Substituting this expression for A_n in equation (2.2), we have

$$\sum_{n=0}^{\infty}\frac{t^{-n}\ell^n\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\,\nu}(x,\,-t)}{2^{4(n+p)}(n+p)!\,\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}$$

$$\left[\int_0^a f_1(\xi,\,t)+\int_a^b\tilde{\mathbf{N}}(\xi,t)+\int_b^\infty f_3(\xi,t)\right]W_{n+p,\,\sigma}(\xi,\,t)d\Omega(\xi)$$

$$=f_2(x,\,t),\qquad a<x<b \qquad (4.1.3)$$

Putting the value of $d\Omega(\xi)$ from (3.3) in equation (4.1.3) and changing the order of integration and summation, we get

$$\left[\int_0^a\xi^{2\sigma}f_1(\xi,\,t)d\xi+\int_a^b\xi^{2\sigma}\tilde{\mathbf{N}}(\xi,t)d\xi+\int_b^\infty\xi^{2\sigma}f_3(\xi,t)\right]\times 2^{2^{\frac{1}{2}-\sigma}}$$

$$\sum_{n=0}^{\infty}\frac{\left(\frac{\ell}{t}\right)^n\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\,\nu}(x,\,-t)}{2^{4(n+p)}(n+p)!\,\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}W_{n+p,\,\sigma}(\xi,\,t)$$

$$=f_2(x,\,t),\qquad a<x<b \qquad (4.1.4)$$

Using summation result (3.4), equation (4.1.4) becomes

$$\int_0^a\xi^{2\sigma}f_1(\xi,\,t)S(x,\,\xi,\,t)d\xi+\int_a^b\xi^{2\sigma}\tilde{\mathbf{N}}(\xi,t)S(x,\xi,t)d\xi$$

$$+\int_b^\infty\xi^{2\sigma}f_3(\xi,\,t)S(x,\,\xi,\,t)d\xi=f_2(x,\,t),\qquad a<x<b \qquad (4.1.5)$$

$$\Rightarrow\int_a^b\xi^{2\sigma}f_3\tilde{\mathbf{N}}(\xi,t)S(x,\xi,t)d\xi=f_2(x,\,t),\qquad a<x<b \qquad (4.1.6)$$

where

$$F(x,\,t)=f_2(x,\,t)-\int_0^a\xi^{2\sigma}f_1(\xi,\,t)S(x,\,\xi,\,t)d\xi-\int_b^\infty\xi^{2\sigma}f_3(\xi,\,t)S(x,\,\xi,\,t)d\xi \qquad (4.1.7)$$

Now using the notation given by equation (3.6) in equation (4.1.6), we get

$$\int_a^b\xi^{2\sigma}\tilde{\mathbf{N}}(\xi,t)\left\{\frac{\xi^{-2\sigma+1}e^{-\xi^2/4t}a^*}{\Gamma(m)\Gamma(\nu-\sigma+m)x^{-1+2\nu}}S_x(x,\xi,t)\right\}d\xi,$$

$$=F(x,\,t),\qquad a<x<b \qquad (4.1.8)$$

$$\frac{a^*}{\Gamma(m)\Gamma(\nu-\sigma+m)}\int_a^b\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)S_x(x,\xi,t)d\xi$$

$$=x^{2\nu-1}F(x,\,t),\qquad a<x<b \qquad (4.1.9)$$

$$\int_a^x\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)S_\xi(x,\,\xi,\,t)d\xi+\int_x^b\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)S_x(x,\,\xi,\,t)d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{-2\nu+1}}F(x,\,t) \qquad (4.1.10)$$

Now putting the value of summation in terms of integral from (3.5), we obtain

$$\int_a^x\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)\int_0^\xi\eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}\alpha\xi dy$$

$$+\int_x^b\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)\int_0^x\eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{-2\nu+1}}F(x,\,t)\qquad a<x<b \qquad (4.1.11)$$

Inverting the order of integration, we get

$$\int_a^x\frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_y^b\frac{\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{-2\nu+1}}F(x,\,t)$$

$$-\int_0^a\frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_a^b\frac{\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \qquad (4.1.12)$$

Assuming

$$\overline{\tilde{\mathbf{N}}(y)}=\int_y^b\frac{\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \qquad (4.1.13)$$

Now equation (4.1.13) reduces to the following form

$$\int_a^x\frac{\eta(y)\overline{\tilde{\mathbf{N}}(y)}}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{-2\nu+1}}F(x,t)$$

$$-\int_0^a\frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_a^b\frac{\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi,\quad a<x<b \qquad (4.1.14)$$

With the help of equations (3.8) and (3.10) we can solve the above equation as,

$$\eta(y)\overline{\tilde{\mathbf{N}}}=\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}\int_a^y\frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\left[\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{-2\nu+1}}F(x,\,t)-\int_0^a\frac{\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\right.$$

$$\left.\times\int_a^b\frac{\xi e^{-\xi^2/4t}\tilde{\mathbf{N}}(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right]\qquad a<x<b \qquad (4.1.15)$$

$$\eta(y)\overline{\tilde{\mathbf{N}}(y)}=F_1(y,t)-\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}\int_a^y\frac{2xd.x.}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\times\left[\int_0^a \frac{\eta(x)dx}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\int_a^b \frac{\xi e^{-\xi^2/4t}\tilde{N}(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right],\ a<x<b \qquad \textbf{(4.1.16)}$$

where,

$$F_1(y,t)=\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_a^y \frac{2x^{2\nu}F(x,t)}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}dx \qquad \textbf{(4.1.17)}$$

Changing the order of integration in equation (4.1.16), we obtain

$$\eta(y)\overline{\tilde{N}(y)}=F_1(y,t)-\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\int_0^a \eta(z)dz$$

$$\times\frac{d}{dy}\int_a^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}\left(x^2-z^2\right)^{1+\sigma-\nu-m}}$$

$$\times\int_a^b \frac{\xi e^{-\xi^2/4t}\tilde{N}(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi,\quad a<x<b \qquad \textbf{(4.1.18)}$$

Using the result given by

$$\frac{d}{dy}\int_a^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}\left(x^2-z^2\right)^{1+\sigma-\nu-m}}=\frac{\left(a^2-z^2\right)^{\nu+m-\sigma}}{\left(y^2-z^2\right)\left(y^2-a^2\right)^{\nu+m-\sigma}} \qquad \textbf{(4.1.19)}$$

in equation (4.1.18), we get

$$\eta(y)\overline{\tilde{N}(y)}=F_1(y,t)-\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi}{\pi}\int_0^a \eta(z)dz$$

$$\times\frac{\left(a^2-z^2\right)^{\nu+m-\sigma}}{\left(y^2-z^2\right)\left(y^2-a^2\right)^{\nu+m-\sigma}}$$

$$\times\int_a^b \frac{\xi e^{-\xi^2/4t}\tilde{N}(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi,\quad a<x<b \qquad \textbf{(4.1.20)}$$

Now using the results (3.9) and (3.11), we solve the integral equation (4.1.13) as follows

$$\xi e^{-\xi^2/4t}\tilde{N}(\xi,t)=-\frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^b \frac{2.y\overline{\tilde{N}(y)}}{\left(y^2-\xi^2\right)^m}dy \qquad \textbf{(4.1.21)}$$

With the help of equation (4.21) we obtain

$$\int_a^b \frac{\xi e^{-\xi^2/4t}\tilde{N}(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi=\frac{\mathrm{Sin}(1-m)\pi}{\pi\left(a^2-z^2\right)^{1-m}}\int_a^b \frac{2x\overline{\tilde{N}(x)}dx}{\left(x^2-a^2\right)^m\left(x^2-z^2\right)} \qquad \textbf{(4.1.22)}$$

Putting the above value from (4.1.22) to equation (4.1.20), we get

$$\eta(y)\overline{\tilde{N}(y)}=F_1(y,t)-\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi\mathrm{Sin}(1-m)\pi}{\pi^2\left(y^2-a^2\right)^{\nu+m-\sigma}}$$

$$\times\int_0^a \frac{\left(a^2-z^2\right)^{\nu+2m-\sigma}\eta(z)}{\left(y^2-z^2\right)}dz\int_a^b \frac{2x\overline{\tilde{N}(x)}}{\left(x^2-a^2\right)^m\left(x^2-z^2\right)} \qquad \textbf{(4.1.23)}$$

Changing the order of integration we get

$$\eta(y)\overline{\tilde{N}(y)}=F_1(y,t)-\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi\mathrm{Sin}(1-m)\pi}{\pi^2\left(y^2-a^2\right)^{\nu+m-\sigma}}$$

$$\times\int_a^b \frac{2x\overline{\tilde{N}(x)}}{\left(x^2-a^2\right)^m}dx\int_0^a \frac{\eta(z)\left(a^2-z^2\right)^{\nu+2m-\eta}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz \qquad \textbf{(4.1.24)}$$

Let

$$A(x,y)=\frac{\mathrm{Sin}(1+\sigma-\nu-m)\pi\mathrm{Sin}(1-m)\pi}{\pi^2\left(y^2-a^2\right)^{\nu+m-\sigma}}\frac{2x}{\left(x^2-a^2\right)^m}$$

$$\times\int_0^a \frac{\eta(z)\left(a^2-z^2\right)^{\nu+2m-\sigma}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz \qquad \textbf{(4.1.25)}$$

Using this expression in (4.1.24), we get

$$\eta(y)\overline{\tilde{N}(y)}=F_1(y,t)-\int_a^b A(x,y)\overline{\tilde{N}(x)}\,dx,\quad a<x<b \qquad \textbf{(4.1.26)}$$

Equation (4.1.26) is a Fredholm integral equation which determines $\overline{\tilde{N}(y)}\Phi(\xi,t)$ is then obtained from equation (4.1.21) and therefore the coefficient A_n can be found from equation(4.1.2), which satisfy the equations (2.1) to (2.3).

Particular Case

If we put a = 0 in equations (2.1), (2.2) and (2.3) then they reduce to the dual series equations and it can easily be shown that the above solution agree with that obtained earlier for dual series equations [3].

4.2 Equations of the Second Kind

In order to solve the triple series equations of the second kind, we set

$$\sum_{n=0}^{\infty}\frac{B_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\sigma}(x,t)=\begin{cases}\Psi_1(x,t), & 0\le x<a\\ \Psi_2(x,t), & b\le x<\infty\end{cases} \qquad \textbf{(4.2.2)}$$

Using orthogonality relation (3.1), in equations (2.5) and (4.2.2), we get

$$B_n=\frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)}\left[\int_0^a \Psi_1(x,t)\right.$$

$$\left.+\int_a^b g_2(x,t)+\int_b^\infty \Psi_2(x,t)\right]W_{n+p,\sigma}(x,t)\,d\Omega(x) \qquad \textbf{(4.2.3)}$$

Substituting above expression for B_n in equations (2.4) and (2.6), we have

$$\sum_{n=0}^{\infty}\frac{t^{-n}\ell^n\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\nu}(x-t)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}$$

$$\left[\int_0^a \Psi_1(\xi,t)+\int_a^b g_2(\xi,t)+\int_b^\infty \Psi_2(\xi,t)\right]W_{n+p,\sigma}(\xi,t)\,d\Omega(\xi)$$

$$=\begin{cases}g_1(x,t), & 0\le x<a \qquad \textbf{(4.2.4)}\\ g_3(x,t), & b<x<\infty \qquad \textbf{(4.2.5)}\end{cases}$$

Putting the value of $d\Omega(\xi)$ from (3.2) and changing the order of integration and summation, we get

$$\left[\int_0^a \xi^{2\sigma}\Psi_1(\xi,\ t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\ t)d\xi+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\ t)d\xi\right]\times 2^{1/2-\sigma}$$

$$\sum_{n=0}^{\infty}\frac{\left(\frac{\ell}{t}\right)^n\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\ \nu}(x,-t)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}W_{n+p,\ \sigma}(\xi,\ t)$$

$$=\begin{cases}g_1(x,\ t), & 0\le x<a\\ g_3(x,\ t), & b<x<\infty\end{cases} \quad (4.2.6)$$

Using summation results (3.4) in equation (4.2 (4.2.7), we get **(4.2.7)**

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,\ t)S(x,\ \xi,\ t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d \quad (4.2.8)$$

$$=\begin{cases}g_1(x,\ t), & 0\le x<a\\ g_3(x,\ t), & b<x<\infty\end{cases} \quad (4.2.9)$$

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,\ t)S(x,\ \xi,\ t)d\xi+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$=\begin{cases}G(x,\ t), & 0\le x<a\\ H(x,\ t), & b\le x<\infty\end{cases} \quad (4.2.10)$$

(4.2.11)

where $G(x,\ t)=g_1(x,\ t)-\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$

(4.2.12)

$$H(x,\ t)=g_3(x,\ t)-\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d\xi \quad (4.2.13)$$

Now using the notation given by (3.6) in equation (4.2.10), we get

$$\int_0^x \xi^{2\sigma}\Psi_1(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^* s_\xi(x,\ \xi,\ t)\right\}d\xi$$

$$+\int_x^a \xi^{2\sigma}\Psi_1(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^* s_x(x,\ \xi,\ t)\right\}d\xi$$

$$+\int_b^\infty \xi^{2\sigma}\Psi_2(\xi,\ t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^* s_x(x,\ \xi,\ t)\right\}d\xi$$

$$=G(x,\ t) \qquad 0\le x<a \quad (4.2.14)$$

$$\int_0^x \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)s_\xi(x,\ \xi,\ t)d\xi+\int_x^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)s_x(x,\ \xi,\ t)d\xi$$

$$+\int_b^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)s_x(x,\ \xi,\ t)d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}G(x,\ t) \qquad 0\le x<a \quad (4.2.15)$$

Now putting the value of summation in terms of integral from (3.5), we obtain

$$\int_0^x \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_x^a \xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_b^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

(4.2.16)

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}G(x,\ t) \qquad 0\le x<a$$

Inverting the (4.2.7) n, we have

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_y^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,\ t),}{\left(\xi^2-y^2\right)^{1-m}}d\xi$$

$$+\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}G(x,\ t) \qquad 0\le x<a \quad (4.2.17)$$

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}dy\left[\overline{\psi_1(y)}+\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\right]$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}G \quad (4.2.9) \quad <a \quad (4.2.18)$$

where (4.2.10)

$$\overline{\psi_1(y)}=\int_y^a \frac{\xi e \quad (4.2.11) \quad t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \quad (4.2.19)$$

With the help of equations (3.8) and (3.10) solving equation (4.218), as

(4.2.13)

$$\eta(y)\left[\overline{\psi_1(y)}+\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\right]$$

$$=\frac{\operatorname{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_0^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\times\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^* x^{1-2\nu}}G(x,\ t) \qquad 0\le x<a \quad (4.2.20)$$

$$G_1(y,\ t)=\frac{\operatorname{Sin}(1+\sigma-\nu-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_0^y \frac{2x^{2\nu}G(x,\ t),}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}dx \quad (4.2.21)$$

Now equation (4.2.21) can be rewritten as

$$\eta(y)\overline{\psi_1(y)}=G_1(y,\ t)-\eta(y)\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \quad (4.2.22)$$

Using the results (3.9) and (3.11), we solve the integral equation (4.2.19), as follows

$$\xi e^{-\xi^2/4t}\Psi_1(\xi,\ t)=-\frac{\operatorname{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^a \frac{2y\overline{\psi_1(y)}}{\left(y^2-\xi^2\right)^m}dy \quad (4.2.23)$$

Similarly, let us

$$\overline{\psi_2(y)}=\int_y^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \quad (4.2.24)$$

then we have

$$\xi e^{-\xi^2/4t}\Psi_2(\xi,t) = -\frac{Sin(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^\infty \frac{2y\overline{\psi_2(y)}}{\left(y^2-\xi^2\right)^m}dy \quad (4.2.25)$$

With the help of (4.2.24) and (4.2.25), we obtain

$$\int_0^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{Sin(1-m)\pi}{\pi\left(-y^2\right)^{-m}}\int_0^a \frac{2x\overline{\psi_1(x)}}{\left(x^2\right)^m\left(x^2-y^2\right)}dx \quad (4.2.26)$$

$$\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{Sin(1-m)\pi}{\pi\left(b^2-y^2\right)^{-m}}\int_b^\infty \frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)}dx \quad (4.2.27)$$

Using equation (4.2) in (4.2.23), we get

$$\eta(y)\overline{\psi_1(y)} = G_1(y,t) - \frac{Sin(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}$$

$$\int_b^\infty \frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)}dx \quad (4.2.28)$$

Now equation (4.2) reduces to

$$\eta(y)\overline{\psi_1(y)} = G_1(y,t) - \int_b^\infty B(x,y)\overline{\psi_2(x)}dx, \quad 0 \le x < a \quad (4.2.29)$$

where

$$B(x,y) = \frac{Sin(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}\frac{2x}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)} \quad (4.2.30)$$

Again starting from equation (4.2.11) we have

$$+\int_0^a \xi^{2\sigma}\Psi_1(\xi,t)S(x,\xi,t)d\xi + \int_b^\infty \xi^{2\sigma}\Psi_2(\xi,t)S(x,\xi,t)d\xi = H(x,t) \qquad b < x < \infty \quad (4.2.31)$$

Using the notation given by (3.6) in above equation, we get

$$\int_0^a \xi^{2\sigma}\Psi_1(\xi,t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_\xi(x,\xi,t)\right\}d\xi$$

$$+\int_b^x \xi^{2\sigma}\Psi_2(\xi,t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_\xi(x,\xi,t)\right\}d\xi$$

$$+\int_x^\infty \xi^{2\sigma}\Psi_2(\xi,t)\left\{\frac{x^{1-2\nu}\xi^{-2\sigma+1}e^{-\xi^2/4t}}{\Gamma(m)\Gamma(\nu-\sigma+m)}a^*S_x(x,\xi,t)\right\}d\xi$$

$$= H(x,t) \qquad b < x < \infty \quad (4.2.32)$$

$$\int_0^a \xi e^{-\xi^2/4t}\Psi_1(\xi,t)S_\xi(x,\xi,t)d\xi + \int_b^x \xi e^{-\xi^2/4t}\Psi_2(\xi,t)S_\xi(x,\xi,t)d\xi$$

$$+\int_x^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,t)S_x(x,\xi,t)d\xi$$

$$= \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}H(x,t), \qquad 0 < x < \infty \quad (4.2.33)$$

Putting the value of summation in terms of integral from (3.5), we have

$$\int_0^a \xi e^{-\xi^2/4t}\Psi_1(\xi,t)\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_b^x \xi e^{-\xi^2/4t}\Psi_2(\xi,t)\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_x^\infty \xi e^{-\xi^2/4t}\Psi_2(\xi,t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$= \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}H(x,t), \qquad b < x < \infty \quad (4.2.34)$$

Inverting the order of integration of equation (4.2.34), we get

$$\int_b^x \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_y^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi = \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}H(x,t)$$

$$-\int_0^a \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_y^a \frac{\xi e^{-\xi^2/4t}\Psi_1(\xi,t)}{\left(\xi^2-y^2\right)^{-m}}d\xi$$

$$-\int_0^b \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi, \quad b < x < \infty \quad (4.2.35)$$

Using the expression given by equation (4.2.19) and (4.2.25) in equation (4.2.35, we get) (4.2.28)

$$\int_b^x \frac{\eta(y)\overline{\Psi_2(y)}dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}} = \frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}H(x,t) - \int_0^a \frac{\eta(y)\overline{\Psi_1(y)}dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}$$

$$-\int_0^b \frac{\eta(y)dy}{\left(x^2-y^2\right)^{1+\sigma-\nu-m}}\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi, \qquad b < x < \infty \quad (4.2.36)$$

With the help of equations (3.8) and (3.10) above equation can be solved as

$$\eta(y)\overline{\Psi_2(y)} = \frac{Sin(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}}$$

$$\times\left[\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}H(x,t) - \int_0^a \frac{\overline{\Psi_1(z)}\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\right.$$

$$\left.-\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right], \quad b < x < \infty \quad (4.2.37)$$

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,t) - \frac{Sin(1+\sigma-\nu-m)\pi}{\pi}\frac{d}{dy}$$

$$\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{-\sigma+\nu+m}} \times \left[\int_0^a \frac{\overline{\Psi_1(z)}\eta(z)}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}}dz\right.$$

$$\left.+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1+\sigma-\nu-m}} \times \int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right], \ b < x < \infty \quad (4.2.38)$$

where

$$H_1(y,t) = \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_b^y \frac{2x^{2\nu}H(x,t)}{\left(y^2-x^2\right)^{\nu-\sigma+m}}dx \quad (4.2.39)$$

Now using the equation (4.2.23) in equation (4.2.39), we obtain

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}$$

$$\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}} \times \left\{\int_0^a \frac{G_1(x,t)}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}dz\right. \quad (4.2.40)$$

$$-\int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} \times \int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi$$

$$+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} \times \int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi \Bigg\},\ b<x<\infty$$

Breaking the last term of (4.2.41) in to part, we have

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}}$$

$$\times\Bigg\{\int_0^a \frac{G_1(z,\ t)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}} - \int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi + \int_0^a \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi + \int_a^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\Bigg\},\qquad b<x<\infty \quad \textbf{(4.2.41)}$$

Now changing the order of integration, equation (4.2.41) becomes

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}$$

$$\times\Bigg\{\int_0^a G_1(z,\ t)dz \times \frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$+\int_a^b \eta(z)dz.\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}}$$

$$\times\int_b^\infty \frac{\xi e^{-\xi^2/4t}\Psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\Bigg\},\qquad b<x<\infty \quad \textbf{(4.2.42)}$$

We know that

$$\frac{d}{dy}\int_b^y \frac{2xdx}{\left(y^2-x^2\right)^{\nu-\sigma+m}\left(x^2-z^2\right)^{1-\nu+\sigma-m}} = \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}} \quad \textbf{(4.2.43)}$$

Using the results (4.2.43) and (4.2.28), in equation (4.2.43), we get

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi}$$

$$\Bigg\{\int_0^a G_1(z,\ t)dz \times \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$+\int_a^b \eta(z)dz \times \frac{\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$\frac{Sin(1-m)\pi}{\pi\left(b^2-z^2\right)^{-m}} \times \int_b^\infty \frac{2x\overline{\psi_2(x)}}{\left(x^2-b^2\right)^m\left(x^2-z^2\right)}dx\Bigg\},\ b<x<\infty \quad \textbf{(4.2.44)}$$

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz$$

$$-\frac{Sin(1-\nu+\sigma-m)\pi Sin(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz$$

$$\int_b^\infty \frac{2x\overline{\Psi_2(x)}}{\left(x^2-b^2\right)^m}dx,\qquad b<x<\infty \quad \textbf{(4.2.45)}$$

Changing the order of integration of the last term of equation (4.2.45), we get

$$\eta(y)\overline{\Psi_2(y)} = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz$$

$$-\frac{Sin(1-\nu+\sigma-m)\pi Sin(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_b^\infty \frac{2(x)\overline{\Psi_2(x)}}{\left(x^2-b^2\right)^m}dx,$$

$$\times\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz,\quad b<x<\infty \quad \textbf{(4.2.46)}$$

Now equation (4.2.46) can be reduced to the following form

$$\eta(y)\overline{\Psi_2(y)} + \int_b^\infty C(x,\ y)\overline{\Psi_2(x)}dx = H_1(y,\ t) - \frac{Sin(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$\int_0^a \frac{G_1(z,\ t)\left(b^2-z^2\right)^{\nu-\sigma+m}}{\left(y^2-z^2\right)}dz,\quad b<x<\infty \quad \textbf{(4.2.47)}$$

where C(x, y) is the symmetric kernel given as

$$C(x,\ y) = \frac{Sin(1-\nu+\sigma-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\frac{2x}{\left(x^2-b^2\right)^m}\times\int_a^b \frac{\eta(z)\left(b^2-z^2\right)^{\nu-\sigma+2m}}{\left(y^2-z^2\right)\left(x^2-z^2\right)}dz \quad \textbf{(4.2.48)}$$

Equation (4.2.48) is a fredholm integral equation of the second kind which determines $\overline{\psi_2(y)}$. After that, $\psi_2(\xi,\ t)$ and $\psi_1(\xi,\ t)$ can be found form equations (4.2.26) and (4.2.24) respectively. Finally, the coefficients B_n which satisfy the triple series equations (2.4) to (2.6) are given by equation (4.2.3).

Particular Case

If we let $b\rightarrow\infty$ in equations (2.4), (2.5) and (2.6) they reduce to the dual series equations and this solution for the triple series equations of the second kind agree with that obtained earlier of dual series [1].

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