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Travelling Salesman Problem (TSP) Using Fuzzy Quantifier

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Abstract: ***In this paper we have preferred the fuzzy quantifier and ranking method for finding the initial solution, also Nearest Neighbour Method (NN-Method) is applied for finding an optimal solution for Travelling Salesman Problem (TSP). This method requires least iterations to reach the optimality. Results are given to check the validity of the proposed method.***

Keywords: TSP, Fuzzy quantifier, ranking function, NN-method, optimal solution.

1. Introduction

Travelling Salesman Problem (TSP) is a well known, popular and extensively studied problem in the field of conditional optimization and attracts computer scientists, mathematicians and others. Its statement is deceptively simple, but yet it remains one of the most challenging problems in operational research. It also an optimization problem of finding a shortest closed tour that visits all the given cities. It is known as a classical NP-Complete problem, which has extremely large search space and is very difficult to solve.

The definition of a TSP is : given N cities, if a salesman starting from his home city is to visit each city exactly once and then return home, find the order of a tour such that the total distances (cost) travelled is minimum. Cost can be distance, time, money, energy, etc. TSP is an NP-hard Problem and researchers especially mathematicians and scientists have been studying to develop efficient solving methods since 1950's. Because it is so easy to describe and so difficult to solve. The travelling salesman problem is widespread in engineering applications. It has been employed in designing hardware devices and radio electronic devices, in communications in the architecture of computational networks etc. In addition, some industrial problems such as machine scheduling, cellular manufacturing and frequency assignment problems can be formulated as a TSP. In this paper we investigate a TSP with fuzzy cost $\tilde{C}_{ij}$ represented by fuzzy quantifier which are replaced by triangular or trapezoidal fuzzy numbers.

2. Preliminaries

In 1965, Zadeh introduced the concept of a fuzzy set as a mathematical way of representing impreciseness in real world problems.

2.1 Definition

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined by

$\tilde{A} = \{< x, \mu_{\tilde{A}}(x) > / x \epsilon X\}$ where $\mu_{\tilde{A}}(x) : X \longrightarrow [0,1]$

is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \epsilon \tilde{A}$.

2.2 Definition

A fuzzy set $\tilde{A}$ on R is convex iff for any $x_1, x_2 \epsilon X$, the membership function of $\tilde{A}$ satisfies the inequality

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}; 0 \leq \lambda \leq 1.$$

2.3 Definition

A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set if there exists at least one $x \epsilon X$ such that $\mu_{\tilde{A}}(x) = 1$.

2.4 Definition

A fuzzy set $\tilde{A}$, is defined on the universal set of real number R, is said to be a fuzzy number, if its membership function has the following characteristics

(i) $\tilde{A}$, is convex
(ii) $\tilde{A}$, is normal and
(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

2.5 Definition

A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \leq x \leq b \\ 1 \; x = b \\ \frac{c-x}{c-b}, b \leq x \leq c \end{cases}$$

2.6 Definition

The α −cut set of a fuzzy set $\tilde{A}$ is a crisp set defined by
$\tilde{A}_\alpha = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\}$.

2.7 Definition

For a convex fuzzy $\tilde{a}$, the Robust's Ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\propto^L, a_\propto^U)\, d \propto$$

where $(a_\propto^L, a_\propto^U)$ is the $\propto$-level cut of the fuzzy number $\tilde{a}$.

Robust's ranking technique which satisfy compensation, linearity and additive properties and provides results which are consist human intuition.

3. Fuzzy Quantifier

Fuzzy quantifiers are fuzzy numbers that take part in fuzzy propositions. Fuzzy quantifiers that characterize linguistic terms such as about 10, much more than 100, atleast about 5 of the first kind and almost all, about half, most and so on of the second kind.

One of them is the form
P: There are Q_i's in I such that $v(i)$ of v is F,

where $v(i)$ of v is a variable , Q is a fuzzy number expressing linguistic term, F is a fuzzy set of variable v.

4. Nearest Neighbour Method

Step 1: Locate the smallest element in the cost matrix (break ties arbitrarily) circle it, and include the corresponding link in the itinerary.
Step 2: If the newly circled element is C_{pq}, replace all other elements in the pth row and all other element in the qth column, as well as the transposed element C_{pq}, by a prohibitively large number.
Step 3: Locate the smallest uncircled element in the latest cost matrix. Tentatively adjoin its corresponding link to the (incomplete) itinerary. If the resulting itinerary is feasible, circle the designated cost and go to step 5.
Step 4: If the resulting itinerary is infeasible, remove the latest link from the itinerary and replace its corresponding cost by a prohibitively large number. Go to step 3.
Step 5: Determine whether the itinerary is complete. If so, accept it as the near-optimal one. If not, go to step 2.

5. Result: TSP- A Logistic Problem

Consider the following Travelling Salesman Problem. Here the distance travelled (cost) are represented by fuzzy quantifiers which characterize the linguistic variables are replaced by fuzzy numbers. The problem is then solved by using the proposed method.

$$\tilde{C}_{ij} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ \left(\begin{array}{c} Nil \\ Substantial \\ Above\ average \\ Somewhat\ low \end{array}\right. & \begin{array}{c} Substantially\ low \\ Nil \\ Substantially\ high \\ High \end{array} & \begin{array}{c} Low \\ Below\ average \\ Nil \\ Somewhat\ high \end{array} & \left.\begin{array}{c} Above\ average \\ High \\ Average \\ Nil \end{array}\right) \end{array}$$

Solution

The linguistic variables showing the qualitative data is converted into quantitative data using the following table.
The linguistic variables are represented by triangular fuzzy numbers.

Table 1

Nil(Prohibited/not permitted)	∞
Substantially low	(-2,1,4)
Somewhat low	(-2,3,8)
low	(1,4,7)
Below average	(2,5,8)
Average	(5,6,7)
Above average	(6,7,8)
Substantial	(6,10,14)
High	(7,8,9)
Somewhat high	(8,9,10)
Substantially high	(10,12,14)

Now from table 1 we have

$$\tilde{C}_{ij} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array} \begin{array}{cccc} D1 & D2 & D3 & D4 \\ \left[\begin{array}{c} \infty \\ (6,10,14) \\ (6,7,8) \\ (-2,3,8) \end{array}\right. & \begin{array}{c} (-2,1,4) \\ \infty \\ (10,12,14) \\ (7,8,9) \end{array} & \begin{array}{c} (1,4,7) \\ (2,5,8) \\ \infty \\ (8,9,10) \end{array} & \left.\begin{array}{c} (6,7,8) \\ (7,8,9) \\ (5,6,7) \\ \infty \end{array}\right] \end{array}$$

Now, using Robust Ranking technique the above fuzzy TSP is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} \infty & 1 & 4 & 7 \\ 10 & \infty & 5 & 8 \\ 7 & 12 & \infty & 6 \\ 3 & 8 & 9 & \infty \end{pmatrix} \end{array}$$

and it is given in the following figure.

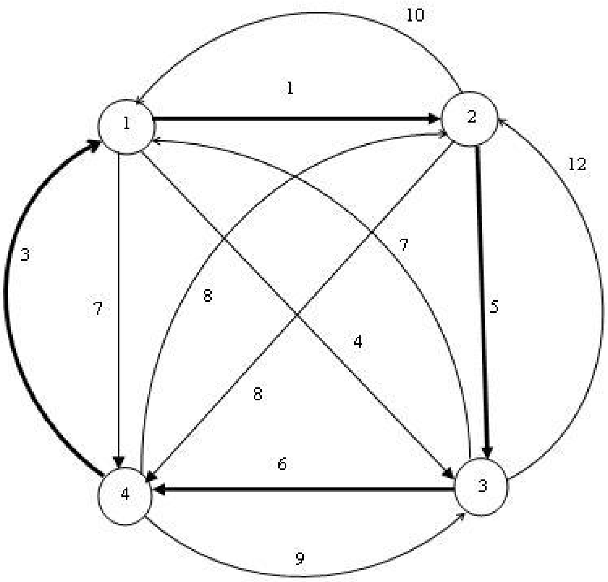

Figure 1: TSP – A Logistic problem

After applying NN Method, the fuzzy optimal total cost is X_{12}= 1, X_{23} = 5, X_{34} = 6, X_{41} = 3

i.e., R(-2,1,4) + R (2,5,8) + R(5,6,7) + R(-2,3,8) = R(3,15,27)
= 1 + 5 + 6 + 3
= Rs. 15

Therefore the near-optimal solution is
$1 \longrightarrow 2, 2 \longrightarrow 3, 3 \longrightarrow 4, 4 \longrightarrow 1$ (or)
$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 1$

$$i.e., O_1 \longrightarrow D_2, O_2 \rightarrow D_3, O_3 \longrightarrow D_4, O_4 \longrightarrow D_4$$

which is complete and it is given in the above figure.

6. Conclusion

In this paper the travelling costs are considered as fuzzy quantifiers that characterize linguistic variables are represented by triangular fuzzy numbers. Moreover, the fuzzy TSP has been transformed into crisp TSP using Robust's Ranking indices. We proposed a Nearest Neighbour Method to find an optimal solution for fuzzy TSP. Numerical example is solved using the proposed method and the results are obtained optimally. Thus the proposed method carries systematic procedure and very easy to understand. It can be extended to assignment problem and transportation problems to get optimal solution. The proposed method is an important tool for the decision makers when they are handling various types of logistic problems to make decision optimally.

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