for some smooth function α and β on M, then we say Trans-Sasakian structure is of type (α,β).

On a Trans-Sasakian manifold, it can be shown that [10]

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi) \tag{2.7}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y) \tag{2.8}$$

$$\Phi(X,Y) = g(\phi X, Y) \tag{2.9}$$

Where Φ is fundamental 2-form.

$$S(X,\xi) = 2n(\alpha^2 - \beta^2 - \xi\beta)\eta(X) \tag{2.10}$$

$$S(\phi X, \phi Y) = S(X,Y) - 2n(\alpha^2 - \beta^2 - \xi\beta)\eta(X)\eta(Y) \tag{2.11}$$

In a Trans-Sasakian manifold the curvature tensor [9] is defined as

$$\begin{aligned} R(X,Y)\xi = {} & (\alpha^2 - \beta^2)[\eta(Y)X - \eta(X)Y] + \\ & 2\alpha\beta[\eta(Y)\phi X - \eta(X)\phi Y] + \\ & (Y\alpha)\phi X - (X\alpha)\phi Y - (Y\alpha)\phi X \\ & + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y \end{aligned} \tag{2.12}$$

$$R(\xi, X)\xi = (\alpha^2 - \beta^2 - \xi\beta)[\eta(X)\xi - X] \tag{2.13}$$

Now for the quarter-symmetric non metric connection D of the Trans-Sasakian manifold is satisfied the following conditions [8].

$$(D_X g)(Y,Z) = -\{\eta(Y)g(\phi X, Z) + \eta(Z)g(\phi X, Y)\} \tag{2.14}$$

$$D_X Y = \nabla_X Y + \eta(Y)\phi X \tag{2.15}$$

$$\begin{aligned} \overline{R}(X,Y)Z = {} & R(X,Y)Z + 2\beta\eta(Z)g(\phi X, Y)\xi \\ & + \alpha[\eta(X)Y - \eta(Y)X]\eta(Z) + \\ & \alpha\{g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y\} \\ & + \beta\{g(X,Z)\phi Y - g(Y,Z)\phi X\} \end{aligned} \tag{2.16}$$

$$\begin{aligned} \overline{S}(X,Y) = {} & S(X,Y) - (n-1)\alpha\eta(X)\eta(Y) \\ & + \alpha g(\phi X, \phi Y) + \beta(\phi Y, X) \end{aligned} \tag{2.17}$$

$$\bar{r} = r \tag{2.18}$$

We know that the Trans-Sasakian structure of type $(0,0)$, $(0,\beta)$ and $(\alpha,0)$ are called Cosympletic, β-Kenmostu and the α-Sasakian manifold respectively.

3. Some Results and Discussion

We note that

$$\overline{K}(X,Y,Z,U) = g(\overline{R}(X,Y)Z,U) \tag{3.1}$$

$$K(X,Y,Z,U) = g(R(X,Y)Z,U) \tag{3.2}$$

Theorem1: In the quarter-symmetric non metric connection of Trans-Sasakian manifold, we have the following relations:

$$\overline{K}(X,Y,Z,U) + \overline{K}(Y,X,Z,U) = 0 \tag{3.3}$$

$$\begin{aligned} & \overline{K}(X,Y,Z,U) - \overline{K}(Z,U,X,Y) = \\ & \alpha[\eta(X)\eta(U)g(Z,Y) - \eta(Y)\eta(Z)g(X,U)] \\ & + \beta[g(X,U)g(\phi Z, Y) - g(Y,Z)g(\phi X, U)] \\ & + 2\beta\begin{bmatrix} \eta(Z)\eta(U)g(\phi X, Y) - \\ \eta(X)\eta(Y)g(\phi Z, U) \\ + g(X,Z)g(\phi U, Y) \end{bmatrix} \end{aligned} \tag{3.4}$$

Proof. From (2.16) and (3.1), we have

$$\begin{aligned} & \overline{K}(X,Y,Z,U) = \\ & K(X,Y,Z,U) + 2\beta\eta(Z)\eta(U)g(\phi Y, X) \\ & \alpha\eta(Z)\{\eta(X)g(Y,U) - \eta(Y)g(X,U)\} + \\ & \alpha\{g(\phi X, Z)g(\phi Y, U) - g(\phi Y, Z)g(\phi X, U)\} \\ & + \beta\{g(X,Z)g(\phi Y, U) - g(Y,Z)g(\phi X, U)\} \end{aligned} \tag{3.5}$$

Now interchanging X and Y in (3.5) and using the equation $K(X,Y,Z,U) + K(Y,X,Z,U) = 0$,
We obtain (3.3).

Similarly from (2.16), (3.5) and the equation $K(X,Y,Z,U) = K(Z,U,X,Y)$,
we have (3.4).

Theorem2: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. Then for all X, Y, Z, $\xi \in TM$, we have

$$\overline{R}(\xi, X)\xi = (\alpha^2 - \beta^2 - \xi\beta - \alpha)(\eta(X)\xi - X) \tag{3.6}$$

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$$\begin{aligned}&\bar{R}(X,Y)\xi = \\ &(\alpha^2-\beta^2-\alpha)[\eta(Y)X-\eta(X)Y] \\ &+\beta(2\alpha-1)[\eta(Y)\phi X-\eta(X)\phi Y] \\ &+(Y\alpha)\phi X-(X\alpha)\phi Y+(Y\beta)\phi^2 X \\ &-(X\beta)\phi^2 Y+2\beta g(\phi X,Y)\xi\end{aligned} \tag{3.7}$$

$$\bar{S}(X,\xi)=[\alpha(2-n)+\beta]\eta(X) \tag{3.8}$$

$$\bar{S}(\phi X,\phi Y)=2\alpha g(X,Y) \tag{3.9}$$

Proof. By using (2.13) and (2.16), we get (3.6).
Again from (2.12) and (2.16), we obtain the result (3.7).
Similarly from (2.10) and (2.17), we have (3.8).
At last by using (2.11) and (2.17), we get (3.9).

4. Some Curvature Tensor

Definition.4.1: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. The projective curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

$$\begin{aligned}&\bar{P}(X,Y)Z = \\ &\bar{R}(X,Y)Z-\frac{1}{(n-1)}[\bar{S}(Y,Z)X-\bar{S}(X,Z)Y]\end{aligned} \tag{4.1}$$

From (2.16) and (2.17), we obtain

$$\begin{aligned}&\bar{P}(X,Y)Z = \\ &P(X,Y)Z+2\beta\eta(Z)g(\phi X,Y)\xi+ \\ &\alpha\{g(\phi Y,Z)\phi X-g(\phi X,Z)\phi Y\}+ \\ &\beta\{g(X,Z)\phi Y-g(Y,Z)\phi X\} \\ &-\frac{1}{(n-1)}\begin{bmatrix}\alpha g(Y,Z)X-\alpha\eta(Y)\eta(Z)X \\ -\alpha g(X,Z)Y-\alpha\eta(X)\eta(Z)Y\end{bmatrix} \\ &-\frac{1}{(n-1)}[\beta g(\phi X,Z)Y-\beta g(\phi Y,Z)X]\end{aligned} \tag{4.2}$$

Again from (4.2), we get

$$\bar{P}(X,Y)Z+\bar{P}(Y,X)Z=0 \tag{4.3}$$

Hence we can state the following theorem:

Theorem3: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D, then the projective curvature tensor of M^n is skew-symmetric with respect to quarter-symmetric non metric connection D is skew-symmetric in X and Y.

Definition 4.2: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. The conformal curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

$$\begin{aligned}&\bar{C}(X,Y,Z,U)= \\ &\bar{K}(X,Y,Z,U)- \\ &\frac{1}{(n-1)}\begin{bmatrix}g(Y,Z)\bar{S}(X,U)-g(X,Z)\bar{S}(Y,U)+ \\ g(X,U)\bar{S}(Y,Z)-g(Y,U)\bar{S}(X,Z)\end{bmatrix}+ \\ &\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]\end{aligned} \tag{4.4}$$

From (2.16), (2.17), (3.1) and (4.4), we get

$$\begin{aligned}&\bar{C}(X,Y,Z,U)= \\ &g(R(X,Y)Z,U)+2\beta\eta(Z)\eta(U)g(\phi X,Y) \\ &+\alpha[\eta(X)g(Y,U)-\eta(Y)g(X,U)]\eta(Z) \\ &+\alpha[g(\phi Y,Z)g(\phi X,U)-g(\phi X,Z)g(\phi Y,U)] \\ &+\beta[g(X,Z)g(\phi Y,U)-g(Y,Z)g(\phi X,U)]- \\ &\frac{1}{(n-2)}\begin{bmatrix}g(Y,Z)\bar{S}(X,U)-g(X,Z)\bar{S}(Y,U)+ \\ g(X,U)\bar{S}(Y,Z)-g(Y,U)\bar{S}(X,Z)\end{bmatrix}+ \\ &\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]\end{aligned} \tag{4.5}$$

Now from (4.5), we get

$$\bar{C}(X,Y,Z,U)+\bar{C}(Y,X,Z,U)=0 \tag{4.6}$$

Hence we can state the following theorem:

Theorem4: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D, then the conformal curvature tensor of M^n is skew-symmetric with respect to quarter-symmetric non metric connection D.

Definition.4.3: Let M be an n dimensional Trans-Sasakian manifold with the Riemannian connection ∇. The concircular curvature tensor of M with respect to Riemannian connection ∇ is defined by

$$\begin{aligned}&Z(X,Y)U = \\ &R(X,Y)U-\frac{r}{n(n-1)}[g(Y,U)X-g(X,U)Y]\end{aligned} \tag{4.7}$$

Definition.4.4: Let M^n be an n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. The concircular curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

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$$\bar{Z}(X,Y)U = \bar{R}(X,Y)U - \frac{\bar{r}}{n(n-1)}[g(Y,U)X - g(X,U)Y] \quad (4.8)$$

From (2.16), (2.18) and (4.8), we get

$$\begin{aligned}\bar{Z}(X,Y)U &= Z(X,Y)U + 2\beta\eta(U)g(\phi X,Y)\xi \\ &+ \alpha[\eta(X)Y - \eta(Y)X]\eta(U) + \\ &\alpha[g(\phi Y,U)\phi X - g(\phi X,U)\phi Y] \\ &+ \beta[g(X,U)\phi Y - g(Y,U)\phi X]\end{aligned} \quad (4.9)$$

If $\alpha = 0$ and $\beta = 0$, then (4.9) give the following theorem:

Colloary5: The concircular curvature tensor of cosympletic manifold with respect to Riemannian connection is equal to the concircular curvature tensor of cosympletic manifold admitting the quarter-symmetric non metric connection.

$$Z(X,Y)U = \bar{Z}(X,Y)U \quad (4.10)$$

Definition.4.5: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D, then conharmonic curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

$$\bar{V}(X,Y,Z,U) = \bar{K}(X,Y,Z,U) - \frac{1}{(n-2)}\begin{bmatrix}\bar{S}(Y,Z)g(X,U) - \bar{S}(X,Z)g(Y,U) + \\ \bar{S}(X,U)g(Y,Z) - \bar{S}(Y,U)g(X,Z)\end{bmatrix} \quad (4.11)$$

If $\bar{S} = 0$, (4.11) gives

$$\bar{V}(X,Y,Z,U) = \bar{K}(X,Y,Z,U) \quad (4.12)$$

Hence we can state the following theorem:

Theorem6: If in an odd n- dimensional Trans-Sasakian manifold the ricci tensor of a quarter-symmetric non metric connection D vanishes, then the curvature tensor of M^n with respect to quarter-symmetric non metric connection D is equal to the con-harmonic curvature tensor of quarter-symmetric non metric manifold.

Definition.4.6: Let M^n be an n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. The quasi conformal curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

$$\begin{aligned}\bar{\hat{C}}(X,Y,Z,U) &= ag(\bar{R}(X,Y)Z,U) + \\ &b\begin{bmatrix}\bar{S}(Y,Z)g(X,U) - \bar{S}(X,Z)g(Y,U) \\ + g(Y,Z)S(X,U) - g(X,Z)S(Y,U)\end{bmatrix} - \\ &\frac{r}{n}\left[\frac{a}{(n-1)} + 2b\right][g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]\end{aligned} \quad (4.13)$$

Where a and b are constant such that $a \neq 0, b \neq 0$.

If $a = 1, b = \frac{1}{(n-1)}$ then quasi conformal curvature tensor reduces to conformal curvature tensor.

Now using (2.16), (2.17), (2.18) and (4.13) we get

$$\begin{aligned}\bar{\hat{C}}(X,Y,Z,U) &= a\begin{bmatrix}K(X,Y,Z,U) + 2\beta\eta(Z)\eta(U)g(\phi X,Y) \\ + \alpha\{\eta(X)g(Y,U) - \eta(Y)g(X,U)\}\eta(Z) \\ + \alpha\{g(\phi Y,Z)g(\phi X,U) - g(\phi X,Z)g(\phi Y,U)\} \\ + \beta\{g(X,Z)g(\phi Y,U) - g(Y,Z)g(\phi X,U)\}\end{bmatrix} \\ &+ b\begin{bmatrix}S(Y,Z)g(X,U) - (n-1)\alpha\eta(Y)\eta(Z)g(X,U) \\ + \alpha g(\phi Y,\phi Z)g(X,U) + \beta g(\phi Y,Z)g(X,U) \\ - S(X,Z)g(Y,U) + (n-1)\alpha\eta(X)\eta(Z)g(Y,U) \\ - \alpha g(\phi X,\phi Z)g(Y,U) - \beta g(\phi X,Z)g(Y,U) \\ + g(Y,Z)S(X,U) - (n-1)\alpha\eta(X)\eta(U)g(Y,Z) \\ + \alpha g(\phi Y,\phi U)g(Y,Z) + \beta g(\phi X,U)g(Y,Z) \\ - S(Y,U)g(X,Z) + (n-1)\alpha\eta(Y)\eta(U)g(X,Z) \\ - \alpha g(\phi Y,\phi U)g(X,Z) - \beta g(\phi Y,U)g(X,Z)\end{bmatrix} \\ &- \frac{r}{n}\left[\frac{a}{(n-1)} + 2b\right][g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]\end{aligned} \quad (4.14)$$

Now interchanging X and Y in (4.14), we have

$$\bar{\hat{C}}(X,Y,Z,U) + \bar{\hat{C}}(Y,X,Z,U) = 0 \quad (4.15)$$

Hence we state:

Theorem7: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D, then the quasi conformal curvature tensor of M^n is skew-symmetric with respect to quarter-symmetric non metric connection D in X and Y.

Definition.4.7: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D, then pseudo projective curvature tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

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$$\tilde{\bar{P}}(X,Y)Z = a\bar{R}(X,Y)Z + b\left[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y\right] - \frac{\bar{r}}{n}\left\{\frac{a}{(n-1)} + b\right\}\left[g(Y,Z)X - g(X,Z)Y\right] \quad (4.16)$$

Where a and b are constant such that $a,b \neq 0$

By using (2.16), (2.17), (2.18) and (4.16), we get

$$\begin{aligned}\tilde{\bar{P}}(X,Y)Z &= \tilde{P}(X,Y)Z + 2a\beta\eta(Z)g(\phi X,Y)\xi \\ &+ a\alpha[\eta(X)Y - \eta(Y)X]\eta(Z) + \\ &+ a\alpha[g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y] \\ &+ b\beta[g(\phi Z,Y)X + g(\phi Z,X)Y] \\ &+ 2b\alpha(n-1)[\eta(X)Y - \eta(Y)X]\eta(Z) \\ &+ b\alpha g(Y,Z)X\end{aligned} \quad (4.17)$$

Now from (4.17), we have

$$\tilde{\bar{P}}(X,Y)Z + \tilde{\bar{P}}(Y,X)Z = 0 \quad (4.18)$$

Hence we can state:

Theorem8: The pseudo projective curvature is skew-symmetric for an odd n dimensional Trans-Sasakian manifold with the quarter symmetric non metric connection D.

5. Projective RICCI Tensor

Definition.5.1: Let M^n be an odd n dimensional Trans-Sasakian manifold with the quarter-symmetric non metric connection D. The projective ricci tensor tensor of M^n with respect to quarter-symmetric non metric connection D is defined by

$$\hat{\bar{P}}(X,Y) = \frac{n}{(n-1)}\bar{S}(X,Y) - \frac{\bar{r}}{(n-1)}g(X,Y) \quad (5.1)$$

From (2.17), (2.18) and (5.1), we have

$$\hat{\bar{P}}(X,Y) = \frac{n}{(n-1)}[\alpha g(X,Y) + \beta g(\phi Y,X) - n\alpha\eta(X)\eta(Y)] \quad (5.2)$$

From (5.2), we get

$$\hat{\bar{P}}(X,Y) + \hat{\bar{P}}(Y,X) = \frac{n\alpha}{n-1}[g(X,Y) - n\eta(X)\eta(Y)] \quad (5.3)$$

If $\alpha = 0$ in (5.3), we can state

Corollary9: Let M^n be β-Kenmostu manifold admitting the quarter-symmetric non metric connection D, then the projective ricci tensor of M^n is skew-symmetric.

Again from (5.2), we get

$$\hat{\bar{P}}(X,Y) - \hat{\bar{P}}(Y,X) = \frac{2n\beta}{(n-1)}g(\phi Y,X) \quad (5.4)$$

If $\beta = 0$, then we can get

Corollary10: Let M^n be α-Sasakian manifold admitting the quarter-symmetric non metric connection D, then the projective ricci tensor of M^n is symmetric.

Now from (5.1), we have

$$\hat{\bar{P}}(X,Y) + \hat{\bar{P}}(Y,X) = \frac{n}{(n-1)}\begin{bmatrix} S(X,Y) - 2n\alpha\eta(X)\eta(Y) + \\ S(Y,X) + 2\alpha g(X,Y) - \frac{2n}{n}g(X,Y)\end{bmatrix} \quad (5.5)$$

If $\hat{\bar{P}}(X,Y)$ is skew-symmetric then L.H.S of (5.5) is vanishes and

$$S(X,Y) = n\alpha\eta(X)\eta(Y) - \left[\alpha - \frac{r}{n}\right]g(X,Y) \quad (5.6)$$

Hence we can state:

Theorem11: An odd n dimensional Trans-Sasakian manifold admitting the quarter-symmetric non metric connection becomes the Einstein manifold if the projective ricci tensor is skew- symmetric.

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