







$$\text{var} \left( \hat{R}_p \right) = \prod_{i=1}^m \left[ q_i^2 + \text{var} \left( \hat{q}_i \right) \right] - \prod_{i=1}^m q_i^2 \quad (10)$$

The biased estimate for  $\text{var} \left( \hat{R}_p \right)$

$$\hat{\text{var}} \left( \hat{R}_p \right)_b = \prod_{i=1}^m \left[ \left( 1 - \hat{r}_i \right)^2 + \hat{\text{var}} \left( \hat{r}_i \right) \right] - \prod_{i=1}^m \left( 1 - \hat{r}_i \right)^2 \quad (11)$$

The unbiased estimate for  $\text{var} \left( \hat{R}_p \right)$  is

$$\left[ \hat{\text{var}} \left( \hat{R}_p \right) \right]_{ub} = \prod_{i=1}^m \left( 1 - \hat{r}_i \right)^2 - \prod_{i=1}^m \left[ \left( 1 - \hat{r}_i \right)^2 - \hat{\text{var}} \left( \hat{r}_i \right) \right] \quad (12)$$

Since  $E \left( \hat{\text{var}} \left( \hat{R}_p \right) \right) = \text{var} \left( \hat{R}_p \right)$

### Mixed Configuration Models (2-step branching)

#### i) Series - Parallel model

Consider a system consisting of  $i = 1, 2, \dots, m$  subunits connected in series. Each of the subunits has  $j = 1, 2, \dots, n$  subunits connected in parallel, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace  $\hat{r}_i$  by  $1 - \prod_{j=1}^n \hat{q}_{ij}$  in equation (7), we arrive

$$\left[ \hat{\text{var}} \left( \hat{R}_{sp} \right) \right]_b = \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right)^2 + \hat{\text{var}} \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right) \right] - \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right)^2 \quad (15)$$

$$\left[ \hat{\text{var}} \left( \hat{R}_{sp} \right) \right]_b = \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right)^2 + \hat{\text{var}} \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right) \right] - \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right)^2 \quad (13)$$

Similarly the unbiased variance estimate for a series-parallel System Reliability estimate we replace  $\hat{r}_i$  by  $1 - \prod_{j=1}^n \hat{q}_{ij}$  in Equation (8)

$$\left[ \hat{\text{var}} \left( \hat{R}_{sp} \right) \right]_{ub} = \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right)^2 - \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right)^2 - \hat{\text{var}} \left( 1 - \prod_{j=1}^n \hat{q}_{ij} \right) \right] \quad (14)$$

### Mixed Configuration Models (3-step branching)

#### i) Series - Parallel – Series model

Consider a system consisting of  $i = 1, 2, \dots, m$  units connected in series. Each of the unit has  $j = 1, 2, \dots, n$  subunits connected in parallel and each of the subunits have  $k = 1, 2, \dots, r$  subunits connected in series, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace  $\hat{r}_{ij}$  by  $\prod_{k=1}^r \hat{r}_{ijk}$  in equation (13), we arrive

Similarly the unbiased variance estimate for a Series-Parallel-Series system reliability estimate we replace  $\hat{r}_{ij}$  by

$\prod_{k=1}^r \hat{r}_{ijk}$  in equation (14) and we arrive

$$\left[ \hat{\text{var}} \left( \hat{R}_{sp} \right) \right]_{ub} = \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right)^2 - \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right)^2 - \hat{\text{var}} \left( 1 - \prod_{j=1}^n \left( 1 - \prod_{k=1}^r \hat{r}_{ijk} \right) \right) \right] \quad (16)$$

#### ii) Series - Parallel – Parallel model

Consider a system consisting of  $i = 1, 2, \dots, m$  units connected in Series. Each of the unit has  $j = 1, 2, \dots, n$  subunits connected in parallel and each of the subunits have  $k = 1, 2, \dots, r$  subunits connected in Parallel, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace  $\hat{r}_{ij}$  by  $\left( 1 - \prod_{k=1}^r \left( 1 - \hat{r}_{ijk} \right) \right)$  in equation (13), we arrive

$$\left[ \hat{\text{var}} \left( \hat{R}_{sp} \right) \right]_b = \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r \left( 1 - \hat{r}_{ijk} \right) \right) \right) \right)^2 + \hat{\text{var}} \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r \left( 1 - \hat{r}_{ijk} \right) \right) \right) \right) \right] - \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r \left( 1 - \hat{r}_{ijk} \right) \right) \right) \right)^2 \quad (17)$$

Similarly the Unbiased Variance estimate for a Series-Parallel-Parallel System Reliability estimate we replace

$$\hat{r}_{ij} \text{ by } \left( 1 - \prod_{k=1}^r (1 - \hat{r}_{ijk}) \right) \text{ in Equation (14)}$$

$$\left[ \text{var} \left( \hat{R}_{ppp} \right) \right]_{ub} = \prod_{i=1}^m \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r (1 - \hat{r}_{ijk}) \right) \right) \right)^2$$

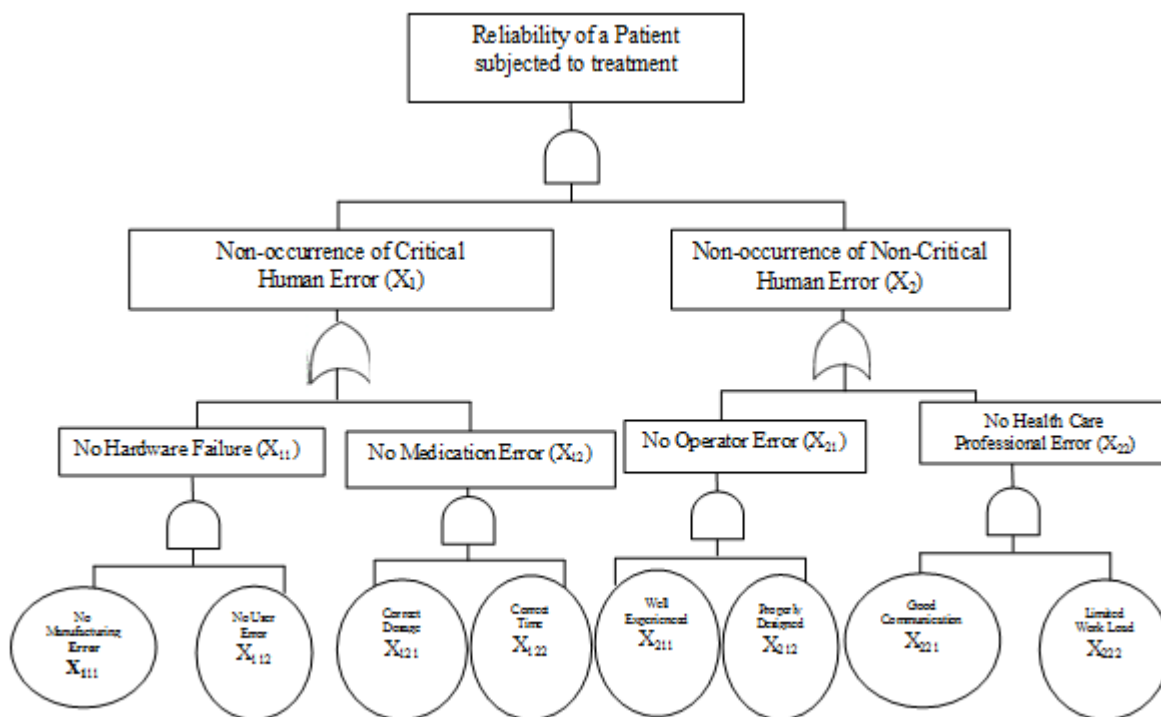
$$- \prod_{i=1}^m \left[ \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r (1 - \hat{r}_{ijk}) \right) \right) \right)^2 - \text{var} \left( 1 - \prod_{j=1}^n \left( 1 - \left( 1 - \prod_{k=1}^r (1 - \hat{r}_{ijk}) \right) \right) \right) \right] \quad (18)$$

**5. Illustrative Examples**

**SPS Model**

The FTA diagram given below deals with the Reliability of a patient in a hospital subjected to treatment, which depends on both non-occurrence of critical and non-critical human error. For the Non-occurrence of critical human error, there should be no hardware or medication error. Similarly for the non-occurrence of non-critical human

error, there should be no operator error or healthcare professional error. If there is no manufacturing error and user error that can avoid hardware failure. Correct dosage of medicine during appropriate time avoids medication error. If the medical device is well designed and the operator is experienced, operator error is not possible. Health care Professional error is not possible if there is good communication and he is restricted to limited work schedule.



**Figure 7: SPS Model**

**SPP Model:**

Consider a Medical Operator performing two tasks A & B independently. To finish Task A & B he depends either on subtasks 1 or 2 and subtasks 3 or 4 respectively. In order that all the subtasks to be completed each one of the subtasks can be finished by following any one of the two given steps. Thus the Medical Operator's Reliability is

analyzed using the Fault tree analysis approach and the variance estimates are evaluated for the given reliabilities of the sub-basic events where the Reliability of the sub-basic event  $X_{ijk}$  is  $\hat{r}_{ijk}$  and the FTA diagram is given below

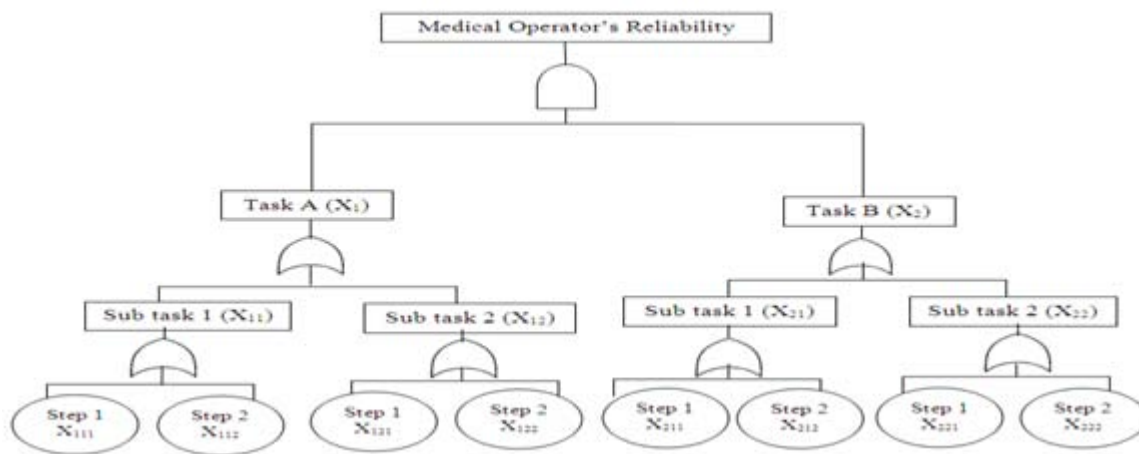


Figure 8: SPP Model

For both the examples, all the events are independent and the base event probabilities are calculated for a small sample of size 10.

Let us consider the values of the sub-basic events as

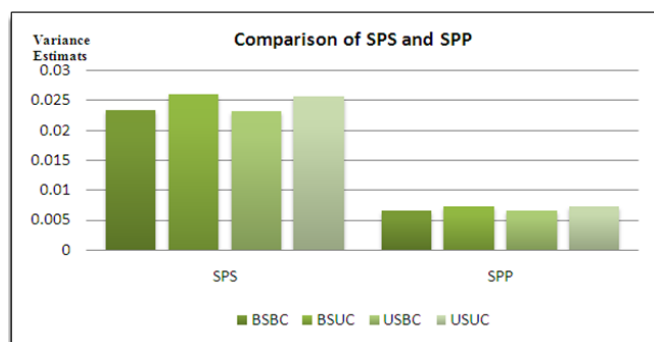
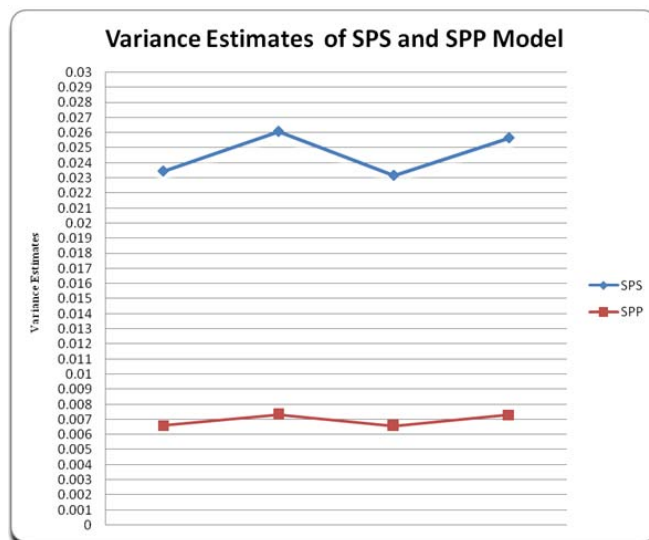
$$\hat{r}_{111} = 0.1, \hat{r}_{112} = 0.3, \hat{r}_{121} = 0.5, \hat{r}_{122} = 0.9, \hat{r}_{211} = 0.8, \hat{r}_{212} = 0.9, \hat{r}_{221} = 1, \hat{r}_{222} = 0.8$$

We consider the four types of study for the above examples such as:

- Biased System Biased Components (BSBC)
- Unbiased System Unbiased Components (USUC)
- Biased System Unbiased Components (BSUC)
- Unbiased System Biased Components (USBC)

Comparison Table

Types	Models	
	SPS	SPP
BSBC	$2.346 \times 10^{-2}$	$6.592 \times 10^{-3}$
BSUC	$2.608 \times 10^{-2}$	$7.326 \times 10^{-3}$
USBC	$2.3197 \times 10^{-2}$	$6.568 \times 10^{-3}$
USUC	$2.567 \times 10^{-2}$	$7.296 \times 10^{-3}$



## 6. Conclusion

In this paper, we derive the formulae to evaluate the biased and unbiased variance estimates of System Reliability for the mixed models namely SPS and SPP models which involve Series and Parallel configurations. Comparative study has been made based on the biased and unbiased nature of the System and Components. The Variance Estimates of the System Reliability Estimates are very less in SPP than SPS model. This clearly reveals that the System Reliability is higher when the System model involves two Parallel configurations than one Parallel configuration.

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