International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Unbiased Variance Estimators in System Reliability of Mixed Configuration Models

M. Reni Sagayaraj[1], A. Merceline Anita[2], A. Chandra Babu[3]

[1]Department of Mathematics, Sacred Heart College, Vellore, India
[2]Department of Mathematics, Sacred Heart College, Vellore, India
[3]Department of Mathematics, Noorul Islam University, Kanyakumari, India

Abstract: ***Unbiasedness is a highly desirable property of an Estimator if testing samples are small. We discuss the role of Unbiased Variance Estimator in System Reliability that can be applicable for any system decomposed into series-parallel connections as long as components Reliability Estimates are mutually independent. We present two different models in this paper and comparative study has been made based on the biased and unbiased nature of the system and components. Certain Illustrations related to these models are provided with Fault-Tree Analysis Approach.***

Keywords: System Reliability, Biased and Unbiased Estimators, Variance, Fault-Tree Analysis Approach, Series, Parallel Systems

Subject Classification: 62N02, 62N05, 65C20

Notations:

r_i = Reliability of i^{th} component, an unknown parameter

q_i = Unreliability of i^{th} component, $q_i = 1 - r_i$.

$\hat{r}_i$ = Reliability estimate of the i^{th} component, $\hat{q}_i$ = Unreliability estimate of the i^{th} component

var($\hat{r}_i$) = variance of Reliability estimate of the i^{th} component

var($\hat{q}_i$) = variance of Unreliability estimate of the i^{th} component

$\hat{\text{var}}(\hat{r}_i)$ = variance estimate of Reliability estimate of the i^{th} component

$\hat{\text{var}}(\hat{q}_i)$ = variance estimate of Unreliability estimate of the i^{th} component

$\hat{\text{var}}(\hat{r}_{ij})$ = variance estimate of Reliability estimate of the ij^{th} component

$\hat{\text{var}}(\hat{q}_{ij})$ = variance estimate of Unreliability estimate of the ij^{th} component

$\hat{\text{var}}(\hat{r}_{ijk})$ = variance estimate of Reliability estimate of the ijk^{th} component

n_i = sample size in life testing for component i,

x_i = no. of survivals for i^{th} component life test

y_i = no. of failures for i^{th} component in life test, t = test duration for components

$\hat{R}_s$ = Reliability estimate of series system, $\hat{R}_p$ = Reliability estimate of parallel system

$\hat{\text{var}}\left(\hat{R}_{sp}\right)$ = Variance estimate of Reliability estimate of series-parallel system

$\hat{\text{var}}\left(\hat{R}_{sps}\right)$ = Variance estimate of Reliability estimate of series-parallel-series system

$\hat{\text{var}}\left(\hat{R}_{spp}\right)$ = Variance estimate of Reliability estimate of series-parallel-parallel system

1. Introduction

System Reliability Analysis has been recognized as an essential need in Military and Defence System [1,2]. However, during last few decades, competition in the global market has forced System Reliability Analysis into almost every field of Science and Technology. System Reliability is calculated by modeling the system as an interconnection of parts in Series and Parallel where we adopt the following criteria to decide if components should be placed in Series or Parallel. If the failure of a part leads to the combination becoming inoperable the two parts are operating in Series. If the failure of a part leads to the other part taking over the operation of the failed parts, the two parts are considered to be operating in parallel. System reliability provides important knowledge for evaluating the new design and it is estimated based on individual component reliability [10].

The Quantitative study of uncertainty forms the starting point for a quantitative analysis of Reliability. The variance is a statistical metric which can be used to quantify the estimation uncertainty of the Reliability estimate. Estimating minimization of uncertainty are of particular importance for many decision makers to achieve higher Reliability and lower Reliability uncertainty because most decision makers prefer to obtain a risk-averse design.

This paper develops Unbiased Variance Estimates for a few mixed configuration Systems involving Parallel and

Series combinations. These types of system occur in almost all fields. The Unbiased Variance Estimates are based on the information of the component Reliability estimate. In this paper, we present a method to estimate variance of System Reliability estimate for configurations of the type Series-Parallel-Series (SPS) and Series-Parallel-Parallel (SPP) Models. No parametric assumptions are required for component time to failure. The component Reliability and variance are estimated based on the binomial testing data. For many system design problems, there are limited failure and survival data based on the component level. Thus one must consider the uncertainty (measured by variance) of the reliability estimate at the system level where the uncertainty of the system reliability estimate is associated with the propagation of uncertainty at the component level [10].

The rest of the paper is organized as follows. Section 2 Reviews some basic concepts such as Series and Parallel Systems, Unbiased Estimate and Fault-Tree Analysis Approach. Section 3 proposes biased and unbiased variance estimates for mixed configuration systems like SPS and SPP models involving Series and Parallel Configurations. In Section 4, a comparative study of the four types for two different models has been made with illustrative examples. Section 5 draws the conclusion.

2. Basic Definitions and Related Concepts

Series System

In a Series system, all components in the system should be operating to maintain the required operation of the system. Thus the failure of any one component of the system will cause failure of the whole system.

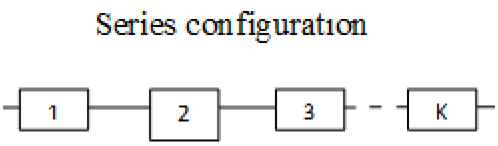


Figure 1: Block diagram of K Units Series System

The System Reliability of the Series system is given by

$$R_s = r_1 r_2 \ldots r_m = \prod_{i=1}^{m} r_i$$

3. Parallel System

In a Parallel system, the system operates if one or more components operate, and the system fails if all components fail. The parallel n-components are represented by the following block diagram.

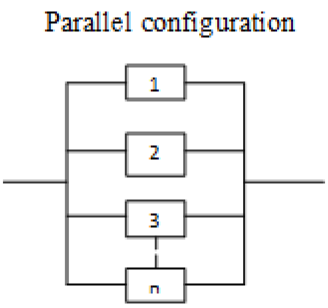


Figure 2: n Units Parallel System

$$R_p(t) = 1-(1-r_1(t))(1-r_2(t))\ldots(1-r_n(t))$$
$$= 1-\prod_{i=1}^{n}(1-r_i(t))$$

where $R_p(t)$ is the parallel system reliability at time t.

$r_i(t)$ is the reliability of the i^{th} component

.

Unbiased Estimate

An estimate $\hat{\theta}$ of parameter θ is said to be Unbiased if its Bias is equal to zero for all values of parameter θ. The Bias of estimator is defined to be

$$\text{Bias}\left(\hat{\theta}\right) = \text{E}\left(\hat{\theta}\right) - \theta.$$

Fault Tree Analysis (FTA) Approach

Fault Tree Analysis Approach is a top-down approach of a system analysis that is used to determine the possible occurrence of undesirable events or failures. Over the years, the method has gained favor over other reliability analysis approaches because of its versatility in degree of detail of complex systems. There are many symbols used to construct fault trees. The basic four symbols are given below.

The fig 3 denotes a fault event that occurs from the logical combination of fault events through the input of logic gates such as OR and AND.

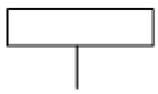


Figure 3: Fault Event

The Figure 4 denotes a basic fault event.

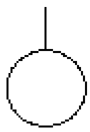


Figure 4: Basic Fault Event

The Figure 5 denotes the OR gate. It denotes the output fault event if one or more of input fault event occur.

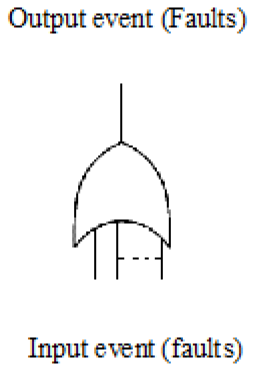


Figure 5: OR Gate

The Figure 6 denotes the AND gate. It denotes that an output fault tree event occurs if all the input fault events occur

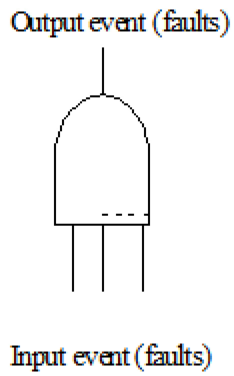


Figure 6: AND Gate

4. Variance Estimation

Component reliability estimates and the variance associated with these estimates can be derived from various types of data including Binomial testing data. For the ith type of component $(i=1,2...m)$ used in the system, let n_i be the number of units on test for t hours and let x_i be the number of units survived during the testing period. Component survival number, a random variable, can be modeled by a binomial distribution of $B(n_i, r_i)$. Then the unbiased estimate for r_i and the exact variance of the estimate $\hat{r}_i$ are given by

$$\hat{r}_i = \frac{x_i}{n_i} \tag{1}$$

$$\operatorname{var}\left(\hat{r}_i\right) = \frac{r_i(1-r_i)}{n_i} \tag{2}$$

The actual r_i is not known in (2) and if the estimate $\hat{r}_i$ is used to replace r_i, then an approximation of the variance is given as

$$\hat{\operatorname{var}}\left(\hat{r}_i\right) = \frac{\hat{r}_i\left(1-\hat{r}_i\right)}{n_i} \tag{3}$$

Equation (3) is a biased estimate for $\operatorname{var}\left(\hat{r}_i\right)$.

An unbiased estimate of $\operatorname{var}\left(\hat{r}_i\right)$ is

$$\hat{\operatorname{var}}\left(\hat{r}_i\right) = \frac{\hat{r}_i\left(1-\hat{r}_i\right)}{n_i - 1} \tag{4}$$

Series System

The variance of the reliability estimate for the series system is given by

$$\operatorname{var}\left(\hat{R}_s\right) = \operatorname{var}\left(\prod_{i=1}^{m} \hat{r}_i\right) = \operatorname{var}\left(\prod_{i=1}^{m} \frac{X_i}{n_i}\right) \tag{5}$$

where X_i follows a binomial distribution $B(n_i, r_i)$ for component type $i = 1, 2, ...m$

which gives

$$\operatorname{var}\left(\hat{R}_s\right) = \prod_{i=1}^{m}\left[r_i^2 + \operatorname{var}\left(\hat{r}_i\right)\right] - \prod_{i=1}^{m} r_i^2 \tag{6}$$

replace r_i by $\hat{r}_i$ and $\operatorname{var} r_i$ by $\hat{\operatorname{var}}\left(\hat{r}_i\right)$ we get the biased estimate which is given as follows

$$\left[\hat{\operatorname{var}}\left(\hat{R}_s\right)\right]_b = \prod_{i=1}^{m}\left[\hat{r}_i^{\,2} + \hat{\operatorname{var}}\left(\hat{r}_i\right)\right] - \prod_{i=1}^{m} \hat{r}_i^{\,2} \tag{7}$$

An unbiased estimate for var ($\hat{R}_s$) is given by

$$\left[\hat{\operatorname{var}}\left(\hat{R}_s\right)\right]_{ub} = \prod_{i=1}^{m} \hat{r}_i^{\,2} - \prod_{i=1}^{m}\left(\hat{r}_i^{\,2} - \hat{\operatorname{var}}\left(\hat{r}_i\right)\right) \tag{8}$$

Since $E\left(\hat{\operatorname{var}}\left(\hat{R}_s\right)\right) = \operatorname{var}\left(\hat{R}_s\right)$

Parallel System

Reliability estimate for a parallel system is given by

$$\hat{R}_P = 1 - \hat{Q}_P \tag{9}$$

where $\hat{Q}_p = \prod_{i=1}^{m} \hat{q}_i = \prod_{i=1}^{m}\left(\frac{Y_i}{n_i}\right)$

for component i, assuming n_i units are tested for time t and Y_i units failed

$$\operatorname{var}\left(\hat{R}_P\right) = \operatorname{var}\left[1 - \left(\prod_{i=1}^{m} \frac{Y_i}{n_i}\right)\right] = \operatorname{var}\left(\prod_{i=1}^{m} \frac{Y_i}{n_i}\right)$$

$$\operatorname{var}\left(\hat{R}_p\right)=\prod_{i=1}^{m}\left[q_i^2+\operatorname{var}\left(\hat{q}_i\right)\right]-\prod_{i=1}^{m}q_i^2 \quad (10)$$

The biased estimate for $\operatorname{var}\left(\hat{R}_p\right)$

$$\hat{\operatorname{var}}\left(\hat{R}_p\right)_b=\prod_{i=1}^{m}\left[\left(1-\hat{r}_i\right)^2+\hat{\operatorname{var}}\left(\hat{r}_i\right)\right]-\prod_{i=1}^{m}\left(1-\hat{r}_i\right)^2 \quad (11)$$

The unbiased estimate for $\operatorname{var}\left(\hat{R}_p\right)$ is

$$\left[\hat{\operatorname{var}}\left(\hat{R}_p\right)\right]_{ub}=\prod_{i=1}^{m}\left(1-\hat{r}_i\right)^2-\prod_{i=1}^{m}\left[\left(1-\hat{r}_i\right)^2-\hat{\operatorname{var}}\left(\hat{r}_i\right)\right] \quad (12)$$

Since $E\left(\hat{\operatorname{var}}\left(\hat{R}_p\right)\right)=\operatorname{var}\left(\hat{R}_p\right)$

Mixed Configuration Models (2-step branching)

i) Series - Parallel model

Consider a system consisting of $i=1,2...m$ subunits connected in series. Each of the subunits has $j=1,2...n$ subunits connected in parallel, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace $\hat{r}_i$ by $1-\prod_{j=1}^{n}\hat{q}_{ij}$ in equation (7), we arrive

$$\left[\hat{\operatorname{var}}\left(\hat{R}_{sp}\right)\right]_b=\prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)^2+\hat{\operatorname{var}}\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)\right]-\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)^2 \quad (13)$$

Similarly the unbiased variance estimate for a series-parallel System Reliability estimate we replace $\hat{r}_i$ by $1-\prod_{j=1}^{n}\hat{q}_{ij}$ in Equation (8)

$$\left[\hat{\operatorname{var}}\left(\hat{R}_{sp}\right)\right]_{ub}=$$

$$\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)^2-\prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)^2-\hat{\operatorname{var}}\left(1-\prod_{j=1}^{n}\hat{q}_{ij}\right)\right] \quad (14)$$

Mixed Configuration Models (3-step branching)

i) Series - Parallel – Series model

Consider a system consisting of $i=1,2...m$ units connected in series. Each of the unit has $j=1,2...n$ subunits connected in parallel and each of the subunits have k = 1,2…r subunits connected in series, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace $\hat{r}_{ij}$ by $\prod_{k=1}^{r}\hat{r}_{ijk}$ in equation (13), we arrive

$$\left[\hat{\operatorname{var}}\left(\hat{R}_{sps}\right)\right]_b=\prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}(1-\prod_{k=1}^{r}\hat{r}_{ijk}\right)^2+\hat{\operatorname{var}}\left(1-\prod_{j=1}^{n}\left(1-\prod_{k=1}^{r}\hat{r}_{ijk}\right)\right)\right]-\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}\left(1-\prod_{k=1}^{r}\hat{r}_{ijk}\right)\right)^2 \quad (15)$$

Similarly the unbiased variance estimate for a Series-Parallel-Series system reliability estimate we replace $\hat{r}_{ij}$ by $\prod_{k=1}^{r}\hat{r}_{ijk}$ in equation (14) and we arrive

$$\left[\hat{\operatorname{var}}\left(\hat{R}_{sps}\right)\right]_{ub}=\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}(1-\prod_{k=1}^{r}\hat{r}_{ijk})\right)^2-\prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}(1-\prod_{k=1}^{r}\hat{r}_{ijk}\right)^2-\hat{\operatorname{var}}\left(1-\prod_{j=1}^{n}(1-\prod_{k=1}^{r}\hat{r}_{ijk}\right)\right] \quad (16)$$

ii) Series - Parallel – Parallel model

Consider a system consisting of $i=1,2...m$ units connected in Series. Each of the unit has $j=1,2...n$ subunits connected in parallel and each of the subunits have k = 1,2…r subunits connected in Parallel, provided all the subunits are independent.

To find the biased variance estimate of the Reliability estimate, we replace $\hat{r}_{ij}$ by $\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)$ in equation (13), we arrive

$$\left[\hat{\operatorname{var}}\left(\hat{R}_{spp}\right)\right]_b=\prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)^2+\hat{\operatorname{var}}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)\right]$$

$$-\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)^2 \quad (17)$$

Similarly the Unbiased Variance estimate for a Series-Parallel-Parallel System Reliability estimate we replace

$$\hat{r}_{ij} \text{ by } \left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right) \text{ in Equation (14)}$$

$$\left[\widehat{\mathrm{var}}\left(\hat{R}_{spp}\right)\right]_{ub} = \prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)^2 - \prod_{i=1}^{m}\left[\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)^2 - \widehat{\mathrm{var}}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\prod_{k=1}^{r}\left(1-\hat{r}_{ijk}\right)\right)\right)\right)\right] \tag{18}$$

5. Illustrative Examples

SPS Model

The FTA diagram given below deals with the Reliability of a patient in a hospital subjected to treatment, which depends on both non-occurrence of critical and non-critical human error. For the Non-occurrence of critical human error, there should be no hardware or medication error. Similarly for the non-occurrence of non-critical human error, there should be no operator error or healthcare professional error. If there is no manufacturing error and user error that can avoid hardware failure. Correct dosage of medicine during appropriate time avoids medication error. If the medical device is well designed and the operator is experienced, operator error is not possible. Health care Professional error is not possible if there is good communication and he is restricted to limited work schedule.

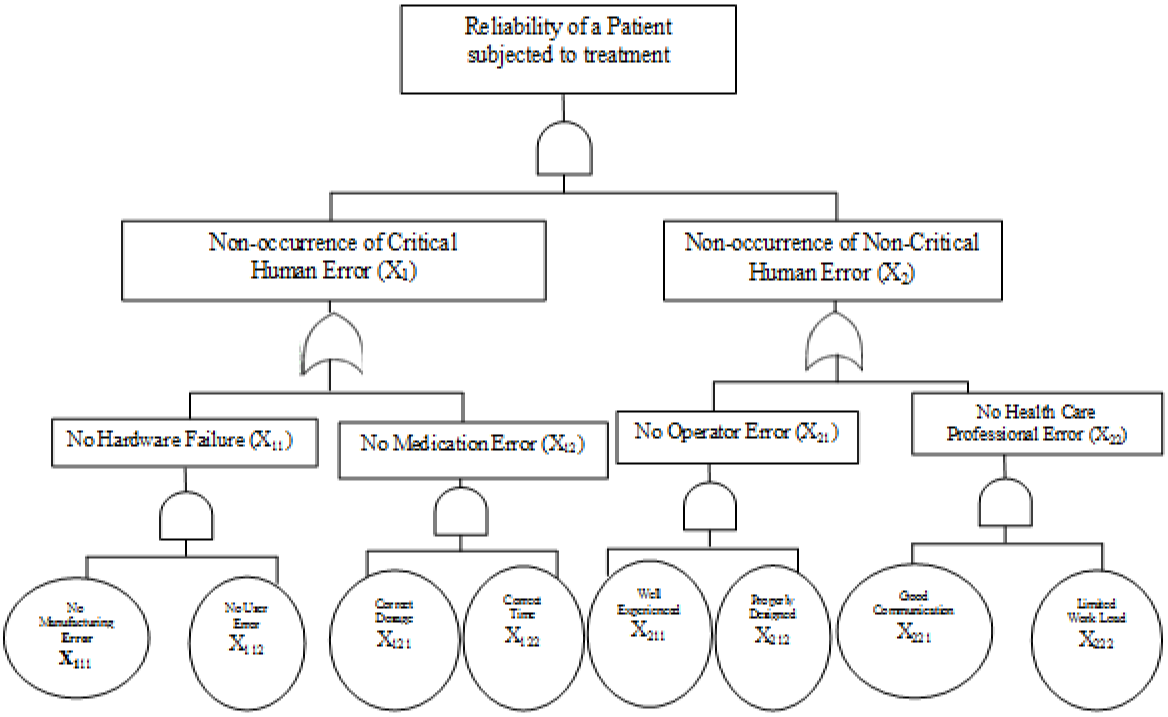


Figure 7: SPS Model

SPP Model:

Consider a Medical Operator performing two tasks A & B independently. To finish Task A & B he depends either on subtasks 1 or 2 and subtasks 3 or 4 respectively. In order that all the subtasks to be completed each one of the subtasks can be finished by following any one of the two given steps. Thus the Medical Operator's Reliability is analyzed using the Fault tree analysis approach and the variance estimates are evaluated for the given reliabilities of the sub-basic events where the Reliability of the sub-basic event X_{ijk} is $\hat{r}_{ijk}$ and the FTA diagram is given below

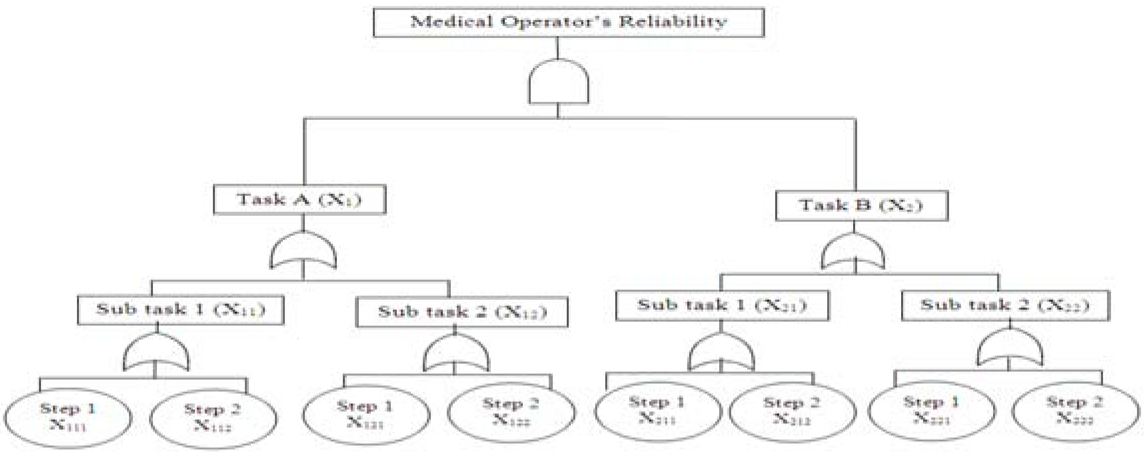


Figure 8: SPP Model

For both the examples, all the events are independent and the base event probabilities are calculated for a small sample of size 10.

Let us consider the values of the sub-basic events as

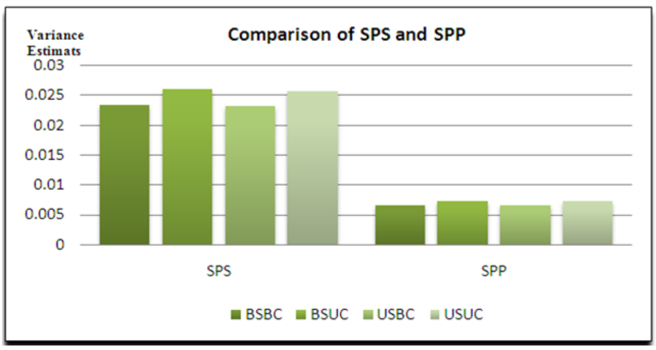
$$\hat{r}_{111} = 0.1,\ \hat{r}_{112} = 0.3,\ \hat{r}_{121} = 0.5,\ \hat{r}_{122} = 0.9,\ \hat{r}_{211} = 0.8,\ \hat{r}_{212} = 0.9,\ \hat{r}_{221} = 1,\ \hat{r}_{222} = 0.8$$

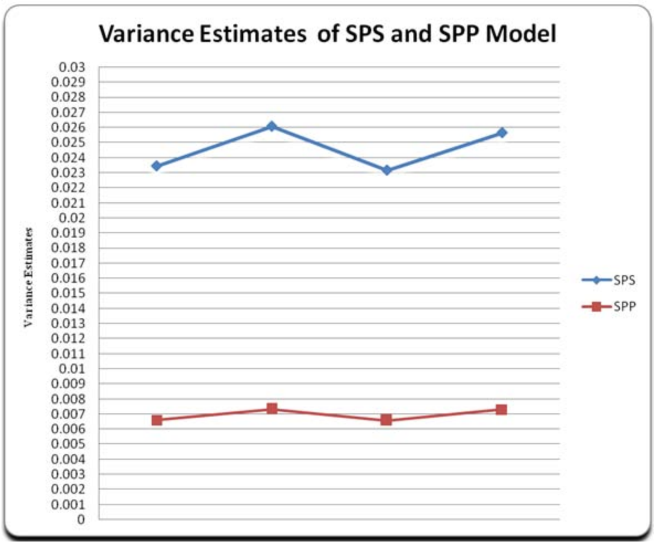
We consider the four types of study for the above examples such as:

- Biased System Biased Components (BSBC)
- Unbiased System Unbiased Components (USUC)
- Biased System Unbiased Components (BSUC)
- Unbiased System Biased Components (USBC)

Comparison Table

Types	*Models*	
	SPS	*SPP*
BSBC	2.346×10^{-2}	6.592×10^{-3}
BSUC	2.608×10^{-2}	7.326×10^{-3}
USBC	2.3197×10^{-2}	6.568×10^{-3}
USUC	2.567×10^{-2}	7.296×10^{-3}





6. Conclusion

In this paper, we derive the formulae to evaluate the biased and unbiased variance estimates of System Reliability for the mixed models namely SPS and SPP models which involve Series and Parallel configurations. Comparative study has been made based on the biased and unbiased nature of the System and Components. The Variance Estimates of the System Reliability Estimates are very less in SPP than SPS model. This clearly reveals that the System Reliability is higher when the System model involves two Parallel configurations than one Parallel configuration.

References

[1] British Ministry of Defense, Reliability and Maintainability Assurance Guides – Part 1: One shot Devices / Systems in Defense Standard, 1997
[2] Coit. D, and Jin T, Prioritizing System Reliability Prediction Improvements, IEEE, Transactions on Reliability, Vol. 50, No.1, p.17, 2001
[3] Coit .D, System Reliability Confidence – Intervals for Complex Systems with Estimated Component – Reliability, IEEE Transactions on Reliability, Vol. 46, No.4, p.487, 1997
[4] Dhillon B.S, Medical Device Reliability And Associated Areas, CRC press,. Washington, D.C., 2000
[5] Dhillon, B.S., Reliability Engineering in Systems Design and Operation, Van Nostrand Reinhold Company, New York, 1983.
[6] Jose Emmanuel Ramirez-Marquez and Wei Jiang – On Improve confidence Bounds for System Reliability, IEEE Transactions on Reliability, Vol. 55, No.1, March 2006.
[7] Marvin Rausand, Arnljot Hoyland, System Reliability Theory, Models, Statistical Methods and Applications, John wiley & Sons, Inc.
[8] Merceline Anita. A, Chandrababu. A, Reni Sagayaraj. M, System Reliability Approach to Problems in Medical Field, International Journal of Biometrics & Bio-informatics, Vol.3, No. 4, April 2011, pp. 207-214.
[9] Navidi, William (2006), Statistics for Engineers and Scientists, Mc-Graw Hill, 2006.
[10] Reni Sagayaraj. M, A. Merceline Anita. A, Chandra Babu. A, The Role of Unbiased Variance Estimators in System Reliability with Applications, International Journal of Mathematical Sciences and Engineering Applications, Vol.6, No. IV, July 2012, pp. 189 – 202, ISSN: 0973 – 9424
[11] Srinath L.S, Reliability Engineering **,** East west press, New Delhi.,2003
[12] Wright D and Bierbaum R, Nuclear Weapon Reliability Evaluation Methodology, SAND Report. SAND 2002 – 8133, 2002