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Two Fluid Cosmological Model in A Kantowski-Sachs Space-Time in General Relativity

V. G. Mete[1], V. M. Umarkar[2], V. S. Bawane[3]

[1]Department of Mathematics, R.D.I.K. & K.D. College, Badnera– Amravati (M.S.), India

[2]Department of Mathematics, H.V.P.Mandal's College of Engineering, Amravati, India

[3]Department of Mathematics, Mahatma Fule Arts, Sitaramji Choudhary Science College, Warud, Dist. Amravati (M.S.), India

Abstract : ***A class of solutions of Einstein's field equations describing two fluid models of the universe in Kantowski-Sachs Space-time has been presented, in which the source of gravitational field consist of two commoving perfect fluids. One fluid is a radiation field, modeling the cosmic microwave background (CMB), while other is a matter field, modeling the material content of the universe. It is found that the both fluids are comoving in Kantowski –Sachs Space-time. The behavior of the radiation density, matter density, the ratio of the matter density to the radiation density and the pressure has been discussed. A subclass of solutions is found to describe models of a spatially homogeneous and partially isotropic evolving from a radiation dominated era to a pressure free matter dominated era. Further a table summarizes the asymptotic properties of all physically relevant variables.***

Keywords: Kantowski-Sachs.two fluid.general relativity

1. Introduction

The isotropic and homogeneous space-time due to Friedmann, Robertson and walker are simple models of the present stage of the expanding universe. Several researchers have studied the evolution of isotropic cosmological models filled with perfect fluids as well as viscous fluids. The discovery of 2.73*k* isotropic cosmic microwave background radiation (CMBR) motivated many researchers to investiga FRW with a two- fluid source [1-3]. In the two-fluid model, one fluid represents the matter content of the universe and another fluid is the radiation field corresponding to the observed cosmic microwave background (CMB) radiation [3-4]. The large scale matter distribution in the observable universe, largely manifested in the form of discrete structures, does not exhibit homogeneity of a high order. In contrast, the cosmic background radiation, which is significant in the microwave region is extremely homogeneous, however, recent space investigations detect anisotropy in the cosmic microwave background (CMB). In different angular scales, the observations from cosmic background explorer's differential microwave radiometer (COBE-DMR) have been detected and measured CMB an isotropies. These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination epoch and are being considered as indicative of the geometry and the content of the universe.

Universe is cooled sufficiently to form atomic hydrogen resulting into the release of photons in the recombination epoch. These have photons traveled freely through the universe forming the presently observed CMB. In 1992, COBE discovered temperature variations in the CMB level 1 part in 100,000. These small anisotropies are believed to have the information about the geometry and the content of the early universe. More about CMBR anisotropy is expected to be uncovered by the investigation of Microwave Anisotropy Probe (MAP) and COBRAS-SAMBA (Planck Surveyor) satellites. The observed anisotropies of CMB at the various angular scales make point of fresh look in the investigation of two-fluid models.

Bianchi type space-times exhibit spatial homogeniety and anisotropy. It is found that Bianchi Space-times of type I, II and VI_0 are asymptotically self similar into the past and the future [5]. Several cosmologists have constructed two-fluid cosmological models in general relativity.Coley and Dunn [6] investigated two-fluid Bianchi type VI_0 space- time.

The Kantowski–Sachs cosmological models containing perfect fluid with a zero cosmological constant was analyzed by Collins [7], he has carried out the qualitative study of the evolution of the Kantowski-Sachs model. Weber [8] applied Collin's method to a qualitative study of a Kantowski-Sachs models, in the presence of a nonzero cosmological constant $\wedge$.

Recently, Pant and Oli [9] constructed two-fluid Bianch type-II cosmological models. Oli [10, 11] have studied two –fluid Bianchi type-I cosmological models with and withoutvariable G and $\wedge$. Adhav et.al. [12-14] have studied anisotropic homogeneoue two-fluid cosmological models in Bianchi type-III and V space-times and also presented kantowski-Sachs model in presence of perfect fluid coupled with massless scalar field in general relativity.Katore et.al.[15] constructed Plane symmetric cosmological models with perfect fluid and dark energy. Two fluid cosmological models in higher dimensions have been presented by Mete et.al [16-18].

The main purpose of this paper is to construct two fluid Kantowski-Sachs space-time in the frame work of general relativity. In this paper, we have investigated physically sound co-moving two-fluid models in Kantowski-Sachs space-time.

2. Metric and Field Equations

We consider the Kantowski-Sachs space-time is given by

$$ds^2 = dt^2 - R^2dr^2 - S^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

where R and S are functions of time t only.
The field equations are

$$G_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \quad ,$$

where $G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ is the Einstein's tensor. $T_{ij}^{(m)}$ is the energy momentum tensor for matter field described by a perfect fluid with density ρ_m, pressure p_m and four velocity

$$u_i^{(m)} = (1,0,0,0) \quad ,$$

where

$$g^{ij}u_i^{(m)}u_j^{(m)} = 1$$

$T_{ij}^{(r)}$,denotes the energy momentum tensor for the radiation field with density ρ_r, pressure $p_r = \frac{1}{3}\rho_r$ and four velocity $u_i^{(r)} = (1,0,0,0)$,
where

$$g^{ij}u_i^{(r)}u_j^{(r)} = 1$$

Thus,

$$T_{ij}^{(m)} = \left(\rho_m + p_m\right)u_i^m\, u_j^m - p_m g_{ij}$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^r\, u_j^r - \frac{1}{3}\rho_r g_{ij}$$

The off diagonal equations of (2) together with energy conditions

$\rho_m + \rho_r > 0,\ \rho_r > 0,$ lead to

$$u_1^{(m)} = u_2^{(m)} = u_3^{(m)} = u_1^{(r)} = u_2^{(r)} = u_3^{(r)} = 0,$$

From (3), (4) and (7),

$$u_0^{(m)} = u_0^{(r)} = 1, \qquad (8)$$

In our model, matter field and radiation field both are comoving.
From (5), (6), (7) and (8) the surviving field equations (2) are

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{S}^2}{S^2} + \frac{1}{S^2} = 8\pi\left(\rho_m + \rho_r\right)$$

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{1}{S^2} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right)$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right),$$

where overhead dot (.) denotes differentiation with respect to cosmic time t.

Out of these three equations in five unknowns R, S, ρ_m, p_m, and ρ_r only three are independent which may be written as follows:

Substracting equation (10) from equation (11), we get

$$\frac{\ddot{R}}{R} - \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} - \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} = 0$$

From equations (9), (10) and (11), we get

(1)

$$\rho_m = \frac{3}{2(4-3\gamma)}\left[\frac{7}{3}\frac{\dot{R}\dot{S}}{RS} + \frac{5}{3}\frac{\dot{S}^2}{S^2} + \frac{5}{3}\frac{1}{S^2} + \frac{3\ddot{S}}{S} + \frac{\ddot{R}}{R}\right]$$

(2)

$$\rho_r = \frac{3}{2(4-3\gamma)}\left[\frac{(3-4\gamma)\dot{R}\dot{S}}{RS} + \frac{(1-2\gamma)\dot{S}^2}{S^2} + \frac{(1-2\gamma)}{S^2} - \frac{3\ddot{S}}{S} - \frac{\ddot{R}}{R}\right].$$

Here there are five unknowns viz. R, S, ρ_m, p_m, and ρ_r .Thus to get a solution we require two additional relations among the field variables R, S and fluid variable ρ_m, p_m, and ρ_r. One of the additional relations will invariably be the equation of state of the matter field.
We assume a γ - law for the equation of state :

$$p_m = (\gamma - 1)\rho_m, \quad 1 \le \gamma \le 2 \qquad (3)$$

The field equations are highly non-linear .Therefore we require another relation between R and S to solve the field equation (12) . In the following sections we shall explore various possibilities in this respect so as to obtain physically meaningful models.

3. Power-Law Solution

Here we consider power law form for R and S : (4)

$$R(t) = At^m, \quad S(t) = Bt^n \ ,$$

where A, B, m and n are constants. Equation (12) yield (5)

$$n = 1, \quad B = \frac{1}{\sqrt{m^2 - 1}}, \quad 0 < 1 < m \qquad (6)$$

The resulting solution is

$$R = At^m$$

$$S = \frac{1}{\sqrt{m^2-1}}t \qquad (7)$$

$$\rho_m = \frac{1}{(4-3\gamma)}\left[2m(2m+1)\right]t^{-2}$$

$$\rho_r = \frac{3}{(4-3\gamma)}\left[-\gamma m^2 + 2m(1-\gamma)\right]t^{-2}$$

The solution is physically insignificant as either of ρ_m, ρ_r becomes positive in the range $0 < 1 < m$.

(9)

4. A Class of Physically Meaningful Solutions

(10)

By the transformation

$$R = \lambda(t)S$$

Equation (12) reduces to (11)

$$\frac{d}{dt}\left(S^2\right) + 2\left(\frac{\ddot{\lambda}}{3\lambda} + \frac{\dot{\lambda}}{3\lambda}\right)S^2 = \frac{2}{3\lambda^3\dot{\lambda}}, \quad \text{provided } \dot{\lambda} \neq 0$$

Here we have followed the method of Pant and Oli [9].
Equation (19) a is first order linear differential equation in S^2 and can be solved easily by a proper choice of $\lambda(t)$ corresponding to the other required additional relation. Here we assume a power law form for λ :

$$\lambda = t^{-n},$$

where n is a non-zero arbitrary constant.

The resulting solution is given by

$$R=\sqrt{\frac{C}{nV}}\; t^{\frac{1-2n}{3}} \qquad (21a)$$

$$S=\sqrt{\frac{C}{nV}}\; t^{\frac{n+1}{3}} \qquad (21b)$$

$$\rho_m=\frac{1}{9(4-3\gamma)}\left[6n^2+30V(n-1)-6\right]t^{-2}, \qquad (21c)$$

$$\rho_r=\frac{1}{3(4-3\gamma)}\left[(4-3\gamma)V^2+\{n(2-9\gamma)+2(1+3\gamma)\}V+3(\gamma-2)(n-1)\right]t^{-2}, \qquad (21d)$$

where

$$V=C(n-2)\left[C+t^{\frac{-2(n-2)}{3}}\right]^{-1},$$

where $n\neq 0,2$ and C is an arbitrary constant.The above is a class of solutions with n as parameter.

From (21a, b) and (22), we have the following realistic conditions on C, n and $V(t)$:

$C > 0,\quad n > 0,\quad V(t) > 0$ (23 a)

$C > 0,\quad 2 < n < 0,\quad V(t) < 0$ (23 b)

$C > 0,\quad (n\text{-}2) < 0,\quad V(t) < 0$ (23 c)

The set of conditions (23a) limits the validity of the solution (21) to a finite interval of time :

$$0\le t<t_0=C^{\frac{3}{-2(n-2)}}, \qquad (24\ a)$$

On the other hand, the set of conditions (23 b, c) put a lower bound on t :

$$t>t_0=C^{\frac{3}{-2(n-2)}}. \qquad (24\ b)$$

Moreover,we note that since (15) is the equation of state for the matter distribution which is distinct from the radiation field, γ is restricted to the following range in view of (21 c, 21d) :

$$1\le\gamma\le\frac{4}{3} \qquad (25)$$

However, the solution (21) can also be used to construct another class of physically meaningful models for

$$\frac{4}{3}\le\gamma\le 2. \qquad (26)$$

Our aim is to find suitable intervals of time, from the ranges provided by (24 a), (24 b), in which ρ_m , ρ_r , p_m and $\frac{\rho_m}{\rho_r}$ behave reasonably.For a physically meaningful model $\rho_m\ge 0$, $\rho_r\ge 0$, $p_m\ge 0$. Also, for the evolution of the universe from a radiation dominated stage to a matter dominated stage, ρ_r must decrease faster than ρ_m. Such property can be ascertained by the study of the function $\frac{\rho_m}{\rho_r}$ which, for a reasonable model of universe, must be a monotonically increasing function of time.

Further,from equation (21), we observe that ρ_r is quadratic function of V (t), so that vanishing of each density will correspond to two values of V (t) for given n and γ.

$$\rho_m=0\Rightarrow V(t)=A,\,V(t)=B,\ A<B$$

$$\rho_r=0\Rightarrow V(t)=a,\,V(t)=b,\ a<b.$$

Now it is not difficult to explore the nature of ρ_m in $A < V(t) < B$ and that of ρ_r in $a < V(t) < b$. Having done so, we shall seek an interval for $V(t)$ or in other words an interval for t where ρ_m and ρ_r are non-negative functions :

$$V(t_1)\le V(t)\le V(t_2)$$

for

$$t_1\le t\le t_2\ ,$$

where

$$t_1=\left[C\left(\frac{(n-2)}{V(t_1)}-1\right)\right]^{\frac{3}{-2(n-2)}} \qquad (22)$$

$$t_2=\left[C\left(\frac{(n-2)}{V(t_2)}-1\right)\right]^{\frac{3}{-2(n-2)}}.$$

Here the constant C provides a scaling factor for time. By proper choice of C, the interval of time (t_1,t_2) is taken large enough. It is found that the solution (21) describes a class of reasonable models of the universe in a finite interval of time (30) for $n < -2.5$, provided γ is restricted by (25). In case γ obeys (26), one gets reasonable models for $n > 2.5$.

5. Investigation of Dust Models

Here we shall present a detailed study of the solution (21) with

$$\rho_m=0 \text{ or } \gamma=1 \qquad (31)$$

The solution for the exploration of physically meaningful dust models for different ranges of values of n , will be investigated as :

In view of (23), it is to be noted that for $V(t) > 0$, the possibility of the existence of a physically meaningful model lies only in the interval bounded by

$V(t) = b,\ V(t) = B$,

where as for $V(t) < 0$, for a meaningful investigation, our study will be limited to the interval bounded by

$V(t) = a,\ V(t) = A$

Table 1: Various parameter associated with the dust models corresponding different values of *n*

S.	n	A	B	a	b	$V(t_1)$	$V(t_2)$	$t_1 C^{\frac{3}{2(n-2)}}$	$t_2 C^{\frac{3}{2(n-2)}}$	$\rho_r(t_1) C^{-\frac{3}{(n-2)}}$	$\rho_m(t_2) C^{-\frac{3}{(n-2)}}$
1	2.5	-2.26974	0.5	-2.28194	0.5	0.5	0.5	0	0	0	0
2	2.6	-2.49285	0.29245	-2.48877	0.30669	0.30669	0.29245	1.11808	0.88181	1.15384	6.94532
3	2.7	-2.51044	0.34407	-2.50151	0.34528	0.34528	0.34407	0.94383	0.93003	1.93089	7.10331
4	3	-2.55344	0.47905	-2.52874	0.48141	0.48141	0.47905	0.89440	0.88181	3.50669	10.9658
5	3.3	-2.88364	0.63511	-2.82343	0.63871	0.63871	0.63511	0.96070	0.94850	4.62243	12.7410
6	3.5	-2.91854	0.73469	-2.84403	0.75	0.75	0.73469	0.74157	0.95999	5.4375	14.7815
7	4	-3.39305	0.99783	-3.25423	1.00207	1.00207	0.99783	1.00310	0.99675	8.63271	20.1087
8	5	-3.56865	1.46938	-3.32442	1.66666	1.66666	1.46938	1.11802	0.97979	15.4074	37.0751
9	6	-4.41878	1.94338	-3.99448	1.96627	1.96627	1.94338	1.98742	0.99856	28.8826	56.7656
10	7	-4.57930	1.98947	-4.98761	2.38344	2.38344	1.98947	0.97239	0.85588	43.1739	94.6229
11	8	-5.03997	2.91544	-5.10664	2.92108	2.92108	2.91544	0.98692	0.98600	59.9003	113.173
12	9	-5.58786	3.48600	-5.44899	3.51951	3.51951	3.48600	1.00239	0.99828	80.0018	154.424
13	10	-6.05589	3.77921	-6.01983	3.86201	3.86201	3.77921	0.98714	0.99348	101.541	186.328
14	20	-11.4333	8.91848	-10.9957	10.5377	10.5377	8.91848	1.02917	0.99849	542.329	833.349
15	50	-29.2304	21.7476	-27.3036	23.6166	23.6166	21.7476	0.99900	0.99413	3286.24	5270.86
16	100	-52.6098	47.4405	-52.1817	47.9060	47.9060	47.4405	0.99931	0.99902	13446.5	22822.64
17	1000	-569.400	447.477	-448.775	567.780	567.780	447.477	1.00041	0.99968	1618862.5	1621615.05
18	10,000	-5021.40	4480.62	-4980.10	5685.09	5685.09	4480.62	1.000041	0.999968	162318775.8	162703324.8
19	10,0000	-50236.6	44812.06	-49794.6	56858.2	56858.2	44812.06	1.0000041	0.9999968	$1.62392589 \times 10^{10}$	$1.627575403 \times 10^{10}$

Table 2: Variation of radiation density matter density and ratio of matter density to radiation density in the dust model from the epoch $t = t_1$ to $t = t_2$ for $n = 3$

S.No.	$C^{\frac{3}{2}} t$	$C^{-3}\rho_m$	$C^{-3}\rho_r$	$\frac{\rho_m}{\rho_r}$
1.	$C^{\frac{3}{2}} t_1 = 0.89440$	10.6788	3.50669	3.04526
2.	0.89356	10.6980	3.51223	3.04587
3.	0.89272	10.7173	3.51779	3.04660
4.	0.89188	10.7365	3.52336	3.04728
5.	0.89104	10.7559	3.52893	3.04792
6.	0.89020	10.7758	3.53452	3.04873
7.	0.88936	10.7946	3.54011	3.04922
8.	0.88852	10.8140	3.54571	3.04988
9.	0.88768	10.8335	3.55132	3.05055
10.	0.88684	10.8530	3.55693	3.05122
11.	0.88600	10.8725	3.56256	3.05187
12.	0.88516	10.8921	3.56820	3.05254
13.	0.88432	10.9117	3.57384	3.05321
14.	0.88348	10.9313	3.57949	3.05387
15.	0.88264	10.9510	3.58515	3.05454
16.	$C^{\frac{3}{2}} t_2 = 0.88181$	10.9658	3.58940	3.055050

Case (i) : $0 < n < -2.5$. In this case $\rho_m > 0$, $\rho_r > 0$ in the interval $a < V(t) < A$.

Case (ii) : $n = -2.5$. Here $a < A$. Thus no interval exists for a meaningful model.

Case (iii) : $n_1 < n < n_2$, $n \neq 2$, where $n_1 < -2.5$ and $n_2 < 0$ are the roots of equation.

$$0.66666n^2 + 1.0223n - 1.68896 = 0.$$

In this range of values of *n*; *A* and *B* becomes imaginary and ρ_r is positive for all *t*.

Case (iv) : $n = n_1 . n_2$. In these cases $B > A$ and $\rho_m \geq 0$,

Case (v) : $-2.5 < n < n_1$, $\rho_m > 0$, $\rho_r > 0$ in the interval $a < V(t) < A$.

Case (vi) : $n = 2.5$. Here $B = b$. Thus no interval exists for a meaningful model.

Case (vii) : $n > 2.5$. In this case $A < a < B < b$.

We find $\rho_m \geq 0$, $\rho_r \geq 0$ in the following interval for $V(t) > o$:

$$b = V(t_1) \leq V(t) \leq V(t_2) = B.$$

Since $\rho_r(t_1) = 0$, $\rho_m(t_2) = 0$ and $t_1 > t_2$, the solution describes a set of models where in the early stages matter dominates radiation and in later stages radiation dominates matter. This does not agree with the actual universe.

Case (viii) : $n < -2.5$. In this case $A < a < b < B$ and $\rho_m \geq 0$, $\rho_r \geq 0$, in the interval

$$A = V(t_1) \leq V(t) \leq V(t_2) = a .$$

Here $\rho_m(t_1) > 0$, $\rho_r(t_2) > 0$ and $t_1 < t_2$. This corresponds to a class of models where the matter density increases with time, whereas the radiation density which is increases. For $n > 2.5$, the solution (21) provides a class of dust models in which, for the time interval $\left(C^{\frac{3}{2(n-2)}}t_1, C^{\frac{3}{2(n-2)}}t_2\right)$. ρ_m and ρ_r are positive and the ratio ρ_m/ρ_r is monotonically increasing.

6. Conclusion

In this paper we present a class of solutions of Einstein's field equations describing two fluid models of the universe in Kantowski-Sachs Space-time. In these models one fluid is the radiation distribution which represents the cosmic microwave background and the other fluid is the perfect fluid representing the matter content of the universe. It is found that the both fluids are comoving in Kantowski – Sachs Space-time.

A numerical study of the model has been presented by calculating A, B, a, b, $V(t_1)$, $V(t_2)$, $C^{\frac{3}{2(n-2)}}t_1$, $C^{\frac{3}{2(n-2)}}t_2$, $\rho_m(t_2)$, $\rho_r(t_1)$ for some values of $n > 2.5$ (Table – I). As an illustration of the physical behaviour of the model, the variation of ρ_m, ρ_r, ρ_m/ρ_r in the interval $\left(C^{\frac{3}{2(n-2)}}t_1, C^{\frac{3}{2(n-2)}}t_2\right)$ is obtained for $n = 3$ (Table -II). A qualitative study of two fluid Kantowski-Sachs Space-time has been presented.

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