stabilize the entire family, it is necessary and sufficient to stabilize the set of extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator coefficients with extreme values of the plant denominator coefficients.

If the plant numerator has degree m and the plant denominator is monic with degree n, the number of extreme plants can be as high as Next=2m+n+1 In [13], Barmish proved that, it is necessary and sufficient to stabilize only sixteen of the extreme plants. A complete survey of these extreme points is given in [14]. In [10], a necessary condition and sufficient condition for interval polynomials is proposed using the results of Nie [15] for fixed polynomials.

In the present paper, a method using interval analysis approach is proposed for the synthesis of a robustly stabilizing controller of a jet engine interval plant having parametric uncertainties using the robust stability conditions in [10]. The method is simple compared to approaches in [16, 17]. Basic definitions and important properties related to interval analysis are discussed in [18].

The present paper is organized as follows. In section 2 we give necessary condition and sufficient condition for robust stability of interval polynomial. Section 3 deals with design procedure for robust stability of interval plants. Section 4,Propose a particle swarm optimization to find optima control parameters. Section 5, the design of a robust controller is carried out using proposed technique for acceleration control loop with the manipulated variable as main burner fuel flow and controlled variable as the compressor speed acceleration. Conclusions are drawn in section 6.

2. Conditions for Robust Stability of Interval Polynomial

Consider the set of real polynomials of degree nof the form

$$\phi(s) = \phi_0 + \phi_1 s + \phi_2 s^2 + \phi_3 s^3 + \ldots + \phi_n s^n \ldots (1)$$

Where the coefficients lie within given ranges,

$$\phi_0 \in [x_0, y_0], \phi_1 \in [x_1, y_1], \ldots, \phi_n \in [x_n, y_n]$$

We assume that the degree remains invariant over the family, so that $0 \notin [x_n, y_n]$

Such a set of polynomial called a real interval family and is referred as an interval polynomial. The set of polynomials given by (1) is stable if and only if each and every element of the set is a Hurwitz polynomial. In [10] a necessary condition and sufficient condition for the robust stability of interval polynomial (1) is proposed using the algebraic stability criterion for fixed polynomials due to Nie [15] which are stated in the following lemmas.

Lemma 2.1 The interval polynomial $\phi(s)$

Defined in (1) is Hurwitz for all where i=0, 1, 2… n if the following necessary conditions are satisfied

$$\phi_i \in [x_i, y_i]$$
$$y_i \geq x_i > 0, i = 0,1,2,\ldots, n$$
$$x_i x_{i+1} > y_{i-1} y_{i+2}, i = 0,1,2,\ldots, n-2 \quad (2)$$

Proof: See [10].

Lemma 2.2 The interval polynomial $\phi(s)$

Defined in (1) is Hurwitz for all where i=0, 1, 2,…,n if the following necessary conditions are satisfied $\phi_i \in [x_i, y_i]$

$$y_i \geq x_i > 0, i = 0,1,2,\ldots, n$$
$$0.4655 \; x_i x_{i+1} > y_{i-1} y_{i+2}, i = 0,1,2,\ldots n-2 \quad (3)$$

Proof: See [10].

3. Design Steps for Robust Stabilization of Interval Plants

Consider a interval plant consisting of all plants of the form,

$$G(s, p, q) = \frac{N(s, p)}{D(s, q)} \quad (4)$$

where the numerator and denominator polynomials are of the form

$$N(s, p) = p_0 + p_1 s + p_2 s^2 + \ldots + p_{m-1} s^{m-1} + p_m s^m$$
$$D(s, q) = q_0 + q_1 s + q_2 s^2 + \ldots + q_{n-1} s^{n-1} + q_n s^n \quad (5)$$

where vectors p and q lie in given rectangles P and Q, respectively, i.e.,

$$p \in P = \{p : p_i^- \leq p_i \leq p_i^+\} \text{ for } i = 0,1,\ldots, m$$
$$q \in Q = \{q : q_i^- \leq q_i \leq q_i^+\} \text{ for } i = 0,1,\ldots, n$$

Where $q_i \in [1, 1]$ and the bound on $p_i^-, p_i^+, q_i^-, q_i^+$ are specified a prior. To stabilize the interval plant family we consider a proper PI or PID controller and its transfer function is given by

$$C_{PI}(s) = k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)} \quad \text{(for PI)}$$

$$C_{PID}(s) = k_1 + \frac{k_2}{s} + k_3 s = \frac{N_c(s)}{D_c(s)} \quad \text{(for PID)}$$

We say that this controller $C(s)$ robustly stabilizes the interval plant family if $p \in P$, for all and all, the $q \in Q$ resulting closed loop polynomial

$$\Delta(s, p, q) = N_c(s) N(s, p) + D_c(s) D(s, p) \quad (6)$$

has all its roots in the strict left half plane; that is

$\Delta(s, p, q)$ is Hurwitz. This is being the case, C(s) is said to be a robust stabilizer and the closed loop system is said to be robustly stable. Let the closed loop interval polynomial be in the form

$$\Delta(s,p,q) = [x_0, y_0] + [x_1, y_1]\, s + \ldots + [x_n, y_n]\, s^n + [1,1]\, s^{n+1} \quad (7)$$

The stability conditions in (2) and (3) can be applied to closed loop characteristic polynomial in (7), which leads to inequalities in terms of controller parameters. These inequalities can be solved to obtain controller parameters. Even though the method in [13] provides a necessary and sufficient condition for robust stabilization using only sixteen extreme plants and which is used in [16] to obtain a robust controller for jet engine, the method still involves much computational complexity since it is required to construct sixteen Routh table and solve the constraints (obtained by enforcing positivity in the first column of the

Routh table) for stability. Although the method in [10] is based or necessary Condition and sufficient condition it involves less Computational complexity and provides a simple Way to obtain a robust controller as can be seen from the application of the method for robust control of the Jet engine.

4. Particle Swarm Optimization Algorithm Operation [19]:

Particle Swarm Optimization optimizes an objective function using population based search. The population consists a group of solutions; named particles are randomly initialized and freely fly across the multi-dimensional search space. During flight, each particle updates its own velocity and position based on the best experience of its own and the entire population. The various steps involved in Particle Swarm Optimization algorithm are as follows:

Step 1: The velocity and position of all particles are randomly set to within pre-defined ranges.

Step 2: Velocity updating at each iteration, the velocities of all particles are updated according to,

$$v_i = w^* v_i + c_1 R_1 (p_{i,best} - p_i) + c_2 R_2 (g_{i,best} - g_i) \quad (8)$$

where pi and vi are the position and velocity of particle i, respectively; pbest and gbest is the position with the 'best' objective value found so far by particle i and the entire population respectively; w is a parameter controlling the dynamics of flying; R1 and R2 are random variables in the range [0,1]; c1 and c2 are factors controlling the related weighting of corresponding terms. The random variables help the PSO with the ability of stochastic searching.

Step 3: Position updating- The positions of all particles are updated according to,

$$p_i = p_i + v_i \quad (9)$$

after updating, pi should be checked and limited to the allowed range.

Step 4: Memory updating – Update pi,best and gi,best when condition is met,

$$p_{i,best} = p_i \qquad \text{if } f(p_i) > (p_{i,best})$$

$$g_{i,best} = g_i \qquad \text{if } f(g_i) > (g_{i,best}) \quad (10)$$

Where: f(x) is the fitness function to be optimized.

Fitness Function:

The fitness function to be minimized is the ISE performance criterion. The integral square error (ISE) criterion is defined as

$$ISE = \int_0^t [r(t) - y(t)]^2 dt \quad (11)$$

Where: r(t)=reference signal
y(t)=output signal measured

Step 5: Stopping Condition – The algorithm repeats steps 2 to 4 until certain stopping conditions are met, such as a pre-defined number of iterations. Once stopped, the algorithm reports the values of gbest and f(gbest) as its solution.

PSO utilizes several searching points and the searching points gradually get close to the global optimal point using its pbest and gbest. Initial positions of pbest and gbest are different. However, using different direction of p_{best} and g_{best}, all agents gradually get close to the global optimum. The above steps are resolved in to a flowchart as shown in Fig3.

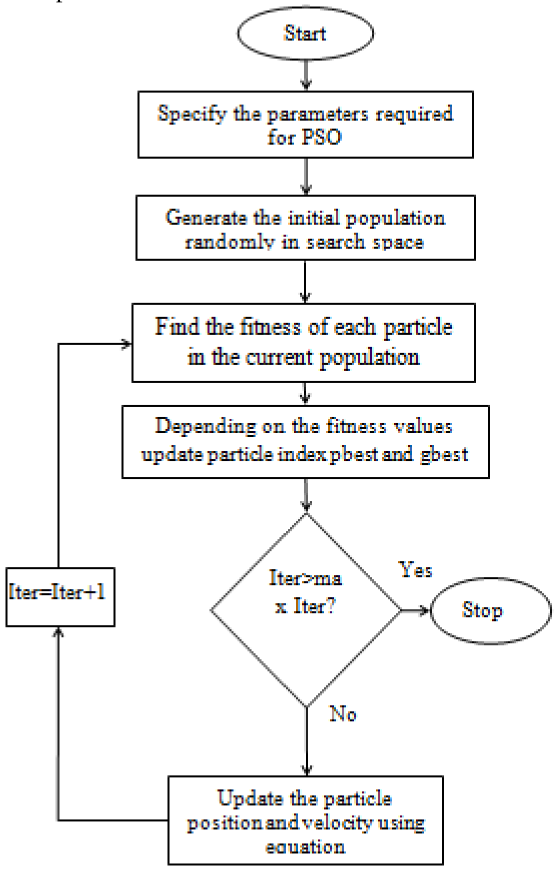

Figure 3: Flowchart of particle swarm optimization (pso)

5. Jet Engine Application

Consider the SISO Jet engine interval plant with input as fuel flow and output as acceleration of compressor speed, Ndot (refer fig.2)

$$G(s,p,q) = \frac{N(s,p)}{D(s,p)} = \frac{q_0}{s^3 + r_2 s^2 + r_1 s}$$

Let uncertainty bounds are

$$q_0 \in [940, 980], r_1 \in [97, 107], r_2 \in [215, 230]$$

Design of Robust PI controller:

Let us synthesize a controller of the form

$$C(s) = k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)}$$

to stabilize the interval model of jet engine. The C(s) will stabilize the given model of jet engine if the closed loop interval polynomial in (7) is stable. The closed loop interval polynomial in (7) becomes

$$s^4 + [215,230]s^3 + [97,980]s^2 + [940k_1, 980k_1]s + [940k_2, 980k_2] \quad (12)$$

On applying the necessary and sufficient conditions given in section2 to this polynomial the following inequality constraints are obtained.

$$-91180\ k_1 + 484216.526\ k_2 < 0$$
$$-91180\ k_1 + 225400\ k_2 < 0$$
$$-20855 + 2105.26\ k_1 < 0$$
$$-20855 + 980\ k_1 < 0$$
$$-940\ k_1 \le 0$$
$$-980\ k_2 \le 0$$

By solving these above constraints to find the range of k_1, k_2 and are optimized with the help of proposed PSO algorithm in section4 to obtain optimal k_1, k_2 values for a minimized ISE.

The control parameters are k_1=0.0258, k_2=0.0000154 and ISE=1.69×10^{-16}

Then the set Kharitonov polynomials are:

$$\Delta_1(s) = s^4 + 215\ s^3 + 107\ s^2 + 25.284\ s + 0.011446$$
$$\Delta_2(s) = s^4 + 230\ s^3 + 107\ s^2 + 24.252\ s + 0.011446$$
$$\Delta_3(s) = s^4 + 215\ s^3 + 97\ s^2 + 25.284\ s + 0.015092$$
$$\Delta_4(s) = s^4 + 230\ s^3 + 97\ s^2 + 24.252\ s + 0.015092$$

All the four Kharitonov polynomials are Hurwitz stable. Hence the designed PI controller stabilizes the jet engine. The closed-loop step response for k_1=0.0258 and k_2=0.0000154 are shown in Fig4.

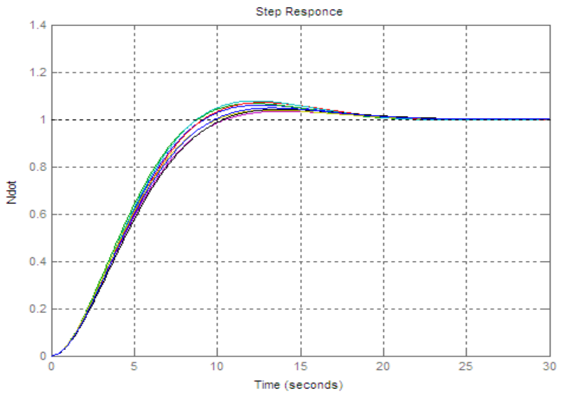

Figure 4: Closed loop step response with designed PI controller for all extreme plants

Design of PID controller:

Let us synthesize a controller of the for

$$C(s) = k_1 + \frac{k_2}{s} + k_3 s = \frac{N_c(s)}{D_c(s)}$$

to stabilize the interval model of jet engine. The C(s) will stabilize the given model of jet engine if the closed loop interval polynomial In (7) is stable. The closed loop interval polynomial in (7) becomes

$$s^4 + [215, 230]\, s^3 + [97 + 940\, k_3, 107 + 980\, k_3]\, s^2 + [940\, k_1, 980\, k_1]\, s + 940\, k_2, 980\, k_2] \quad (13)$$

On applying the necessary and sufficient conditions given in section2 to this polynomial the following inequality constraints are obtained.

$$-42444.29\ k_1 - 411315.8\ k_1 k_3 + 225400\ k_2 < 0$$
$$-91180\ k_1 - 883600\ k_1 k_3 + 225400\ k_2 < 0$$
$$-9708.0025 - 94077.55\ k_3 + 980\ k_1 < 0$$
$$-20855 - 202100\ k_3 + 980\ k_1 < 0$$
$$-97 - 940\ k_3 < 0$$
$$-940\ k_1 \le 0$$
$$-980\ k_2 \le 0$$

By solving these above constraints we get the controller parameters k_1=0.438, k_2=0.0003, k_3=0.49 and ISE=1.69×10^{-16}

Then the set Kharitonov polynomials are:

$$\Delta_1(s) = s^4 + 215\ s^3 + 587.2 s^2 + 372.4 s + 0.282$$
$$\Delta_2(s) = s^4 + 230\ s^3 + 587.2 s^2 + 357.2 s + 0.282$$
$$\Delta_3(s) = s^4 + 215\ s^3 + 557.6 s^2 + 372.4 s + 0.294$$
$$\Delta_4(s) = s^4 + 230\ s^3 + 557.6 s^2 + 357.2 s + 0.294$$

All the four Kharitonov polynomials are Hurwitz stable. Hence the designed PID controller stabilizes the jet engine. The closed-loop step response for k_1= 0.438, k_2=0.0003 and k_3=0.49 are shown in Fig5.

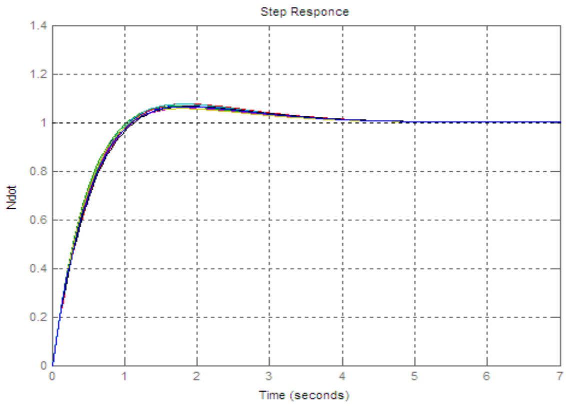

Figure 5: Closed loop step response with designed PID controller for all extreme plants

6. Conclusions

In this paper, a new approach using interval analysis is proposed for design of a robust controller to jet engine. The proposed method is applied to synthesize the acceleration control loop of a jet engine in the presence of parametric uncertainties. The proposed topology uses a necessary condition and sufficient condition for stability of interval polynomial. These conditions are used to derive a set of inequalities in terms of the controller parameters which can be solved to obtain a robust controller. Although the proposed method is based on a necessary condition and a sufficient condition it is simple, involves less computational complexity and provides an easy method to obtain a robust controller. The PSO Algorithm is used to determine the optimal control parameters for a minimized ISE which are obtained with the help of MATLAB [20]. The extension of this technique using Artificial intelligence to tune the PI/PID parameters is a part of further research work. The results show the efficacy of the proposed method.

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