

# Design of Robust PI/PID Controller for a Parametric Uncertain Jet Engine Using PSO

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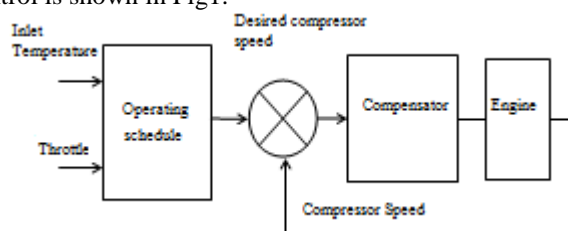
**Abstract:** The modern jet engine consists of various control loops and subjected to several disturbances. The design of controller under the presence of parametric uncertainties is a difficult task. In this paper, an interval analysis is proposed for design of a robust PI/PID controller for jet engine. The proposed method applied to acceleration control loop of a jet engine in the presence of parametric uncertainties. The proposed method uses necessary and sufficient conditions for stability of interval polynomials. These conditions are used to derive a set of inequalities in terms of the controller parameters which are solved and find the optimal control parameters with the help of particle swarm optimization. The results show the efficiency of the proposed topology.

**Keywords:** Robust controller, parametric uncertain system, interval systems, PI/PID controller

## 1. Introduction

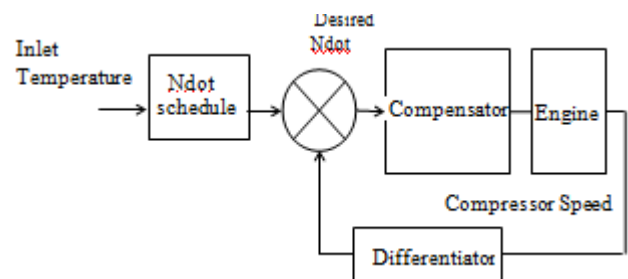
The performance requirements of modern, high technology aircraft have placed severe demands to engine control capability. Control requirements applied to gas turbine engines consist of ensuring safe, stable engine operation. Specific engine performance rating points are generally defined as basic steady state design goals for the control. A review of the basic theory of aero gas turbine engine operation and on Control Design which is currently in commercial and military use is dealt by Spang and Brown [1].

The control logic of the modern Full Authority Digital Engine Control (FADEC) having many control loops, each has a specific function. Control loops include a high or low rotor speed governor, an acceleration and deceleration loop, and various limiting loops for temperature, speed and fuel flow. With electronic controls, more accurate control of engine thrust can be obtained through control of compressor speed [2]. A block diagram of a typical compressor speed control is shown in Fig1.



**Figure 1:** Block diagram of Compressor Speed control Loop of Jet Engine

A compressor speed demand schedule establishes the desired compressor speed as a function of inlet temperature and throttle position. A variant of proportional control uses the derivative of rotor speed (N dot) to control engine acceleration and deceleration as a function of inlet temperature. A block diagram of aNdot control is shown in Fig2.



**Figure 2:** Jet Engine's Ndot controller Block diagram

Direct control of acceleration, rather than speed, allows tighter control of engine acceleration thereby improving transient response and reducing mechanical stress.

All existing systems are subject to various disturbances and uncertainties. Robust stability analysis with uncertain parameters has been very important research topic. Since control systems operate under large uncertainties it is important to study stability robustness in the presence of uncertainty. The uncertainty in the control system causes decrease of system performance and destabilization. An important approach to this subject is via expressing the characteristic polynomial by an interval polynomial, i.e. a polynomial whose coefficient each varies independently in a prescribed interval. The stability analysis of polynomials subjected to parameter uncertainty have received considerable attention after the celebrated theorem of Kharitonov [3], which assures robust stability under the condition that four specially constructed "extreme polynomials", called Kharitonov polynomials are Hurwitz. The problem of robust stability of interval polynomial is also dealt in [4, 5, 6, 7, 8, 9, 10]. Several results have appeared in the literature which aims at reducing test of Hurwitz stability of entire family to a small subset of entire family. In this regard few extreme point results are available in the literature. These includes work due to Ghosh [11], where he has shown that a pure gain controller  $C(s)=k$  stabilizes entire interval plant family if and only if it stabilizes a distinguished set of eight of the extreme plants. Holot and Fang [12] considered the same setup as Ghosh but allow the controller to be first order. They prove that to robustly

stabilize the entire family, it is necessary and sufficient to stabilize the set of extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator coefficients with extreme values of the plant denominator coefficients.

If the plant numerator has degree  $m$  and the plant denominator is monic with degree  $n$ , the number of extreme plants can be as high as  $N_{\text{ext}}=2m+n+1$ . In [13], Barmish proved that, it is necessary and sufficient to stabilize only sixteen of the extreme plants. A complete survey of these extreme points is given in [14]. In [10], a necessary condition and sufficient condition for interval polynomials is proposed using the results of Nie [15] for fixed polynomials.

In the present paper, a method using interval analysis approach is proposed for the synthesis of a robustly stabilizing controller of a jet engine interval plant having parametric uncertainties using the robust stability conditions in [10]. The method is simple compared to approaches in [16, 17]. Basic definitions and important properties related to interval analysis are discussed in [18].

The present paper is organized as follows. In section 2 we give necessary condition and sufficient condition for robust stability of interval polynomial. Section 3 deals with design procedure for robust stability of interval plants. Section 4, Propose a particle swarm optimization to find optima control parameters. Section 5, the design of a robust controller is carried out using proposed technique for acceleration control loop with the manipulated variable as main burner fuel flow and controlled variable as the compressor speed acceleration. Conclusions are drawn in section 6.

## 2. Conditions for Robust Stability of Interval Polynomial

Consider the set of real polynomials of degree  $n$  of the form

$$\phi(s) = \phi_0 + \phi_1 s + \phi_2 s^2 + \phi_3 s^3 + \dots + \phi_n s^n \quad (1)$$

Where the coefficients lie within given ranges,

$$\phi_0 \in [x_0, y_0], \phi_1 \in [x_1, y_1], \dots, \phi_n \in [x_n, y_n]$$

We assume that the degree remains invariant over the family, so that  $0 \in [x_n, y_n]$

Such a set of polynomial called a real interval family and is referred as an interval polynomial. The set of polynomials given by (1) is stable if and only if each and every element of the set is a Hurwitz polynomial. In [10] a necessary condition and sufficient condition for the robust stability of interval polynomial (1) is proposed using the algebraic stability criterion for fixed polynomials due to Nie [15] which are stated in the following lemmas.

### Lemma 2.1 The interval polynomial $\phi(s)$

Defined in (1) is Hurwitz for all where  $i=0, 1, 2, \dots, n$  if the following necessary conditions are satisfied

$$\begin{aligned} & \phi_i \in [x_i, y_i] \\ & y_i \geq x_i > 0, i = 0, 1, 2, \dots, n \\ & x_i x_{i+1} > y_{i-1} y_{i+2}, i = 0, 1, 2, \dots, n-2 \end{aligned} \quad (2)$$

**Proof:** See [10].

### Lemma 2.2 The interval polynomial $\phi(s)$

Defined in (1) is Hurwitz for all where  $i=0, 1, 2, \dots, n$  if the following necessary conditions are

$$\text{satisfied } \phi_i \in [x_i, y_i]$$

$$y_i \geq x_i > 0, i = 0, 1, 2, \dots, n$$

$$0.4655 \ x_i x_{i+1} > y_{i-1} y_{i+2}, i = 0, 1, 2, \dots, n-2 \quad (3)$$

**Proof:** See [10].

## 3. Design Steps for Robust Stabilization of Interval Plants

Consider a interval plant consisting of all plants of the form,

$$G(s, p, q) = \frac{N(s, p)}{D(s, q)} \quad (4)$$

where the numerator and denominator polynomials are of the form

$$N(s, p) = p_0 + p_1 s + p_2 s^2 + \dots + p_{m-1} s^{m-1} + p_m s^m$$

$$D(s, q) = q_0 + q_1 s + q_2 s^2 + \dots + q_{n-1} s^{n-1} + q_n s^n \quad (5)$$

where vectors  $p$  and  $q$  lie in given rectangles  $P$  and  $Q$ , respectively, i.e.,

$$p \in P = \{p : p_i^- \leq p_i \leq p_i^+ \} \text{ for } i = 0, 1, \dots, m$$

$$q \in Q = \{q : q_i^- \leq q_i \leq q_i^+ \} \text{ for } i = 0, 1, \dots, n$$

Where  $q_i \in [1, 1]$  and the bound on  $p_i^-, p_i^+, q_i^-, q_i^+$  are

specified a priori. To stabilize the interval plant family we consider a proper PI or PID controller and its transfer function is given by

$$C_{PI}(s) = k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)} \quad (\text{for PI})$$

$$C_{PID}(s) = k_1 + \frac{k_2}{s} + k_3 s = \frac{N_c(s)}{D_c(s)} \quad (\text{for PID})$$

We say that this controller  $C(s)$  robustly stabilizes the interval plant family if  $p \in P$ , for all and all,

the  $q \in Q$  resulting closed loop polynomial

$$\Delta(s, p, q) = N_c(s)N(s, p) + D_c(s)D(s, q) \quad (6)$$

has all its roots in the strict left half plane; that is

$\Delta(s, p, q)$  is Hurwitz. This is being the case,  $C(s)$  is said to be a robust stabilizer and the closed loop system is said to be robustly stable. Let the closed loop interval polynomial be in the form

$$\Delta(s, p, q) = [x_0, y_0] + [x_1, y_1] s + \dots + [x_n, y_n] s^n + [1, 1] s^{n+1} \quad (7)$$

The stability conditions in (2) and (3) can be applied to closed loop characteristic polynomial in (7), which leads to inequalities in terms of controller parameters. These inequalities can be solved to obtain controller parameters. Even though the method in [13] provides a necessary and sufficient condition for robust stabilization using only sixteen extreme plants and which is used in [16] to obtain a robust controller for jet engine, the method still involves much computational complexity since it is required to construct sixteen Routh table and solve the constraints (obtained by enforcing positivity in the first column of the

Routh table) for stability. Although the method in [10] is based on necessary Condition and sufficient condition it involves less Computational complexity and provides a simple Way to obtain a robust controller as can be seen from the application of the method for robust control of the Jet engine.

#### 4. Particle Swarm Optimization Algorithm Operation [19]:

Particle Swarm Optimization optimizes an objective function using population based search. The population consists a group of solutions; named particles are randomly initialized and freely fly across the multi-dimensional search space. During flight, each particle updates its own velocity and position based on the best experience of its own and the entire population. The various steps involved in Particle Swarm Optimization algorithm are as follows:

**Step 1:** The velocity and position of all particles are randomly set to within pre-defined ranges.

**Step 2:** Velocity updating at each iteration, the velocities of all particles are updated according to,

$$v_i = w \cdot v_i + c_1 R_1 (p_{i,best} - p_i) + c_2 R_2 (g_{i,best} - g_i) \quad (8)$$

where  $p_i$  and  $v_i$  are the position and velocity of particle  $i$ , respectively;  $p_{best}$  and  $g_{best}$  is the position with the 'best' objective value found so far by particle  $i$  and the entire population respectively;  $w$  is a parameter controlling the dynamics of flying;  $R_1$  and  $R_2$  are random variables in the range  $[0,1]$ ;  $c_1$  and  $c_2$  are factors controlling the related weighting of corresponding terms. The random variables help the PSO with the ability of stochastic searching.

**Step 3:** Position updating- The positions of all particles are updated according to,

$$p_i = p_i + v_i \quad (9)$$

after updating,  $p_i$  should be checked and limited to the allowed range.

**Step 4:** Memory updating – Update  $p_{i,best}$  and  $g_{i,best}$  when condition is met,

$$\begin{aligned} p_{i,best} &= p_i & \text{if } f(p_i) < f(p_{i,best}) \\ g_{i,best} &= g_i & \text{if } f(g_i) < f(g_{i,best}) \end{aligned} \quad (10)$$

Where:  $f(x)$  is the fitness function to be optimized.

#### Fitness Function:

The fitness function to be minimized is the ISE performance criterion. The integral square error (ISE) criterion is defined as

$$ISE = \int_0^t [r(t) - y(t)]^2 dt \quad (11)$$

Where:  $r(t)$ =reference signal  
 $y(t)$ =output signal measured

**Step 5:** Stopping Condition – The algorithm repeats steps 2 to 4 until certain stopping conditions are met, such as a pre-defined number of iterations. Once stopped, the algorithm

reports the values of  $g_{best}$  and  $f(g_{best})$  as its solution.

PSO utilizes several searching points and the searching points gradually get close to the global optimal point using its  $p_{best}$  and  $g_{best}$ . Initial positions of  $p_{best}$  and  $g_{best}$  are different. However, using different direction of  $p_{best}$  and  $g_{best}$ , all agents gradually get close to the global optimum. The above steps are resolved in to a flowchart as shown in Fig3.

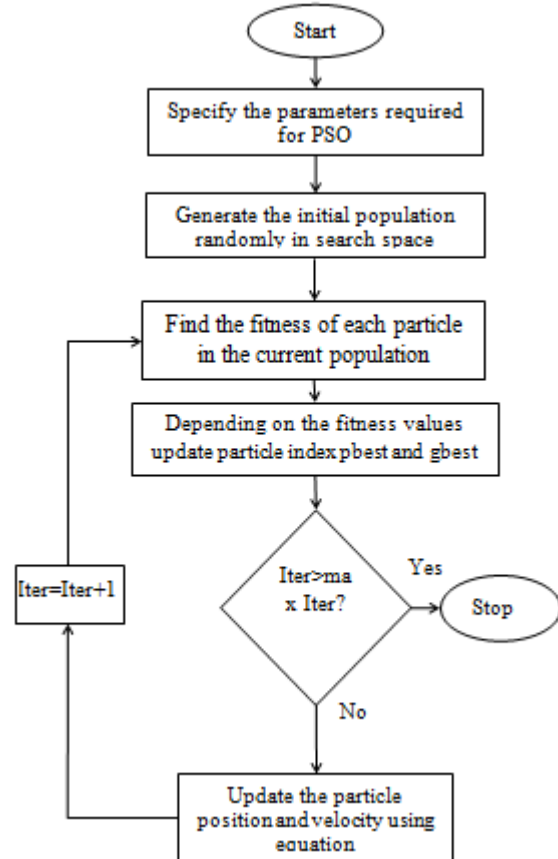


Figure 3: Flowchart of particle swarm optimization (psa)

#### 5. Jet Engine Application

Consider the SISO Jet engine interval plant with input as fuel flow and output as acceleration of compressor speed,  $Ndot$  (refer fig.2)

$$G(s, p, q) = \frac{N(s, p)}{D(s, p)} = \frac{q_0}{s^3 + r_2 s^2 + r_1 s}$$

Let uncertainty bounds are

$$q_0 \in [940, 980], r_1 \in [97, 107], r_2 \in [215, 230]$$

#### Design of Robust PI controller:

Let us synthesize a controller of the form

$$C(s) = k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)}$$

to stabilize the interval model of jet engine. The  $C(s)$  will stabilize the given model of jet engine if the closed loop interval polynomial in (7) is stable. The closed loop interval polynomial in (7) becomes

$$s^4 + [215, 230]s^3 + [97, 98]s^2 + [940k_1, 980k_1]s + [940k_2, 980k_2] \quad (12)$$

On applying the necessary and sufficient conditions given in section2 to this polynomial the following inequality

constraints are obtained.

$$\begin{aligned} &-91180 \quad k_1 + 484216.526 \quad k_2 < 0 \\ &-91180 \quad k_1 + 225400 \quad k_2 < 0 \\ &-20855 \quad + 2105.26 \quad k_1 < 0 \\ &-20855 \quad + 980 \quad k_1 < 0 \\ &-940 \quad k_1 \leq 0 \\ &-980 \quad k_2 \leq 0 \end{aligned}$$

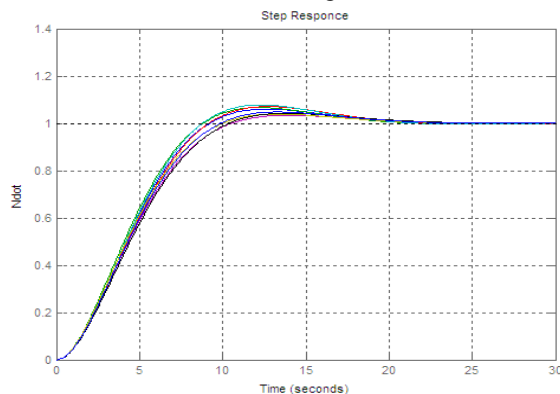
By solving these above constraints to find the range of  $k_1$ ,  $k_2$  and are optimized with the help of proposed PSO algorithm in section 4 to obtain optimal  $k_1$ ,  $k_2$  values for a minimized ISE.

The control parameters are  $k_1=0.0258$ ,  $k_2=0.0000154$  and  $ISE=1.69 \times 10^{-16}$

Then the set Kharitonov polynomials are:

$$\begin{aligned} \Delta_1(s) &= s^4 + 215 s^3 + 107 s^2 + 25.284 s + 0.011446 \\ \Delta_2(s) &= s^4 + 230 s^3 + 107 s^2 + 24.252 s + 0.011446 \\ \Delta_3(s) &= s^4 + 215 s^3 + 97 s^2 + 25.284 s + 0.015092 \\ \Delta_4(s) &= s^4 + 230 s^3 + 97 s^2 + 24.252 s + 0.015092 \end{aligned}$$

All the four Kharitonov polynomials are Hurwitz stable. Hence the designed PI controller stabilizes the jet engine. The closed-loop step response for  $k_1=0.0258$  and  $k_2=0.0000154$  are shown in Fig 4.



**Figure 4:** Closed loop step response with designed PI controller for all extreme plants

#### Design of PID controller:

Let us synthesize a controller of the for

$$C(s) = k_1 + \frac{k_2}{s} + k_3 s = \frac{N_c(s)}{D_c(s)}$$

to stabilize the interval model of jet engine. The  $C(s)$  will stabilize the given model of jet engine if the closed loop interval polynomial in (7) is stable. The closed loop interval polynomial in (7) becomes

$$s^4 + [215, 230] s^3 + [97 + 940 k_3, 107 + 980 k_3] s^2 + [940 k_1, 980 k_1] s + 940 k_2, 980 k_2 \quad (13)$$

On applying the necessary and sufficient conditions given in section 2 to this polynomial the following inequality constraints are obtained.

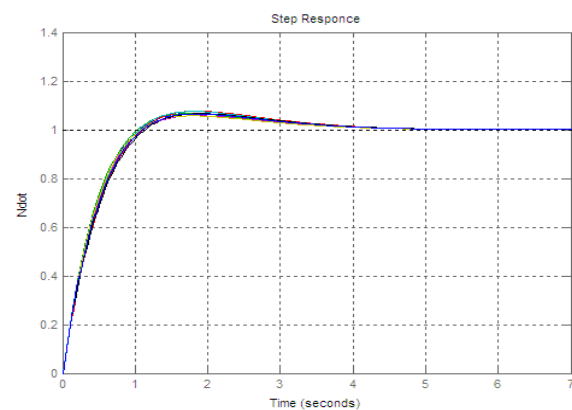
$$\begin{aligned} &-42444.29 \quad k_1 - 411315.8 \quad k_1 k_3 + 225400 \quad k_2 < 0 \\ &-91180 \quad k_1 - 883600 \quad k_1 k_3 + 225400 \quad k_2 < 0 \\ &-9708.0025 \quad -94077.55 \quad k_3 + 980 \quad k_1 < 0 \\ &-20855 \quad -202100 \quad k_3 + 980 \quad k_1 < 0 \\ &-97 \quad -940 \quad k_3 < 0 \\ &-940 \quad k_1 \leq 0 \\ &-980 \quad k_2 \leq 0 \end{aligned}$$

By solving these above constraints we get the controller parameters  $k_1=0.438$ ,  $k_2=0.0003$ ,  $k_3=0.49$  and  $ISE=1.69 \times 10^{-16}$

Then the set Kharitonov polynomials are:

$$\begin{aligned} \Delta_1(s) &= s^4 + 215 s^3 + 587.2 s^2 + 372.4 s + 0.282 \\ \Delta_2(s) &= s^4 + 230 s^3 + 587.2 s^2 + 357.2 s + 0.282 \\ \Delta_3(s) &= s^4 + 215 s^3 + 557.6 s^2 + 372.4 s + 0.294 \\ \Delta_4(s) &= s^4 + 230 s^3 + 557.6 s^2 + 357.2 s + 0.294 \end{aligned}$$

All the four Kharitonov polynomials are Hurwitz stable. Hence the designed PID controller stabilizes the jet engine. The closed-loop step response for  $k_1=0.438$ ,  $k_2=0.0003$  and  $k_3=0.49$  are shown in Fig 5.



**Figure 5:** Closed loop step response with designed PID controller for all extreme plants

## 6. Conclusions

In this paper, a new approach using interval analysis is proposed for design of a robust controller to jet engine. The proposed method is applied to synthesize the acceleration control loop of a jet engine in the presence of parametric uncertainties. The proposed topology uses a necessary condition and sufficient condition for stability of interval polynomial. These conditions are used to derive a set of inequalities in terms of the controller parameters which can be solved to obtain a robust controller. Although the proposed method is based on a necessary condition and a sufficient condition it is simple, involves less computational complexity and provides an easy method to obtain a robust controller. The PSO Algorithm is used to determine the optimal control parameters for a minimized ISE which are obtained with the help of MATLAB [20]. The extension of this technique using Artificial intelligence to tune the PI/PID parameters is a part of further research work. The results show the efficacy of the proposed method.



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## References



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