Holographic Dark Energy in Higher-Dimensions

Gitumani Sarma

Research Scholar, Department of Mathematics, Gauhati University

Abstract: ***In this work It is discussed the behaviour of holographic dark energy density in higher dimension. Exact solutions of Einstein's field equation have been obtained by assuming a particular relation between two metric coefficients. It is also observed that for a certain condition Universe is accelerating and holographic dark energy density decreases during evolution of the Universe. Some physical parameters are also discussed.***

1. Introduction

A recent cosmological observation indicates that the present universe is undergoing an accelerated expansion. This acceleration of the universe a well established fact that is confirmed by various independent observational data including SNIa, CMB radiation etc. However this discovery can be maintained in general relativity by introducing a mysterious kind of energy source called the dark energy that can generate repulsive gravity [1, 2]. It is well known that perfect fluid with a constant equation of state (EOS) parameter lower than $-1/3$. For solution of the Einstein's field equation one can seek by introducing by some kinematical ansatz that are consistent with the observation kinematics of the universe and may investigate the dynamics of the fluid as a possible candidate of the DE. Astronomical observations indicate that our universe currently consists of approximately 70% dark energy, 25% dark matter and 5% baryonic matter and radiation.

Holographic dark energy is the nature of DE can also be studied according to some basic quantum. Gravitational principle. According to this principle [3], the degrees of freedom in a bounded system should be finite and does not scale by it volume but with its boundary era. Here ρ_Λ is the vacuum energy density. Using this idea in cosmology we take ρ_Λ as DE density. The holographic principle is considered as another alternative to the solution of DE problem. This principle was first considered by G. 't Hooft [4] in the context of black hole physics. In the context of dark energy problem though the holographic principle proposes a relation between the holographic dark energy density ρ_Λ and the Hubble parameter H as $\rho_\Lambda = H^2$, it does not contribute to the present accelerated expansion of the universe. In [5], Granda and Olivers proposed a holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$, where H is the Hubble parameter and α, β are constants which must satisfy the conditions imposed by the current observational data.

2. Metric and Field Equation

We consider the spatially flat, homogeneous and anisotropic universe in five-dimensional FRW metric

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] + b^2(t)dy^2 \tag{1}$$

Where $a(t)$ and $b(t)$ represent scale factor of four-dimensional space time and extra dimension respectively. r is the radial component, (θ, ϕ) are the two angular component.

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + \overline{T}_{ij}) \tag{2}$$

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{3}$$

And,

$$\overline{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j - g_{ij}\rho_\Lambda \tag{4}$$

Where ρ_m, ρ_Λ are energy densities of matter and holographic dark energy and p_Λ is the pressure of holographic dark energy.

Field equations are

$$3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{b}}{ab} = \rho_m + \rho_\Lambda \tag{5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -p_\Lambda \tag{6}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p_\Lambda \tag{7}$$

For the metric (1) we have the following form

$$V = A^2 B = R^3 \tag{8}$$

$$\theta = V^\mu;_\mu = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \tag{9}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{2\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2}\right) - \frac{1}{6}\theta^2 \tag{10}$$

$$H = \frac{\dot{R}}{R} \tag{11}$$

The holographic dark energy density are given by

$$\rho_\Lambda = \frac{2}{\alpha - \beta}\left(\dot{H} + \frac{3\alpha}{2}H^2\right) \tag{12}$$

The continuity equation can be obtained as

$$\dot{\rho}_m + \dot{\rho}_\Lambda + \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)(\rho_m + \rho_\Lambda + p_\Lambda) = 0 \qquad (13)$$

The continuity equation of the matter is

$$\dot{\rho}_m + \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)\rho_m = 0 \qquad (14)$$

The continuity equation of the holographic dark energy is

$$\dot{\rho}_\Lambda + \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)(\rho_\Lambda + p_\Lambda) = 0 \qquad (15)$$

The barotropic equation of state

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \qquad (16)$$

3. Solution of Field Equation:

There are four equation (5)-(7) and (16) with five unknown $a, b, \rho_m, \rho_\Lambda$, and p_Λ. Therefore to solve the field equation we need one condition. Let us assume a relation between two metric coefficients,

$$b = Ka^n \qquad (17)$$

Where K and n are constant.
Using (17) in (5)-(7), we get

$$3(1+n)\frac{\dot{a}^2}{a^2} = \rho_m + \rho_\Lambda \qquad (18)$$

$$(2+n)\frac{\ddot{a}}{a} + (n^2+n+1)\frac{\dot{a}^2}{a^2} = -p_\Lambda \qquad (19)$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p_\Lambda \qquad (20)$$

From equation (19) and (20) we get

$$a = \left[(n+3)(K_1 t + K_2)\right]^{\frac{1}{n+3}} \qquad (21)$$

Where K_1 and K_2 are integration constant.
Using (17) and (21)

$$b = K\left[(n+3)(K_1 t + K_2)\right]^{\frac{n}{n+3}} \qquad (22)$$

Using (21) and (22) in (8)

$$R = \left(a^3 b\right)^{1/4} = K_\circ (K_1 t + K_2)^{1/4} \qquad (23)$$

Where $K_\circ = K^4 (n+3)^{1/4}$
Using (23) in (11)

$$H = \frac{\dot{R}}{R} = \frac{K_1}{4(K_1 t + K_2)} \qquad (24)$$

Using (12), (17) and (16) in (15), we obtain

$$\omega_\Lambda = -1 - \frac{\left(\ddot{H} + 3\alpha H \dot{H}\right)}{4H\left(\dot{H} + \frac{3\alpha}{2}H^2\right)} \qquad (25)$$

From equation (20) and (21)

$$p_\Lambda = \frac{3K_1^2(n+1)}{(n+3)^2 (K_1 t + K_2)^2} = \frac{48(n+1)}{(n+3)^2} H^2 \qquad (26)$$

From (12) and (24)

$$\rho_\Lambda = \frac{(3\alpha - 8)K_1^2}{16(\alpha - \beta)(K_1 t + K_2)^2} = \frac{(3\alpha - 8)}{(\alpha - \beta)} H^2 \qquad (27)$$

From equation (14)

$$\rho_m = K_3 R^{-4} \qquad (28)$$

4. Some Physical Parameter of The Model

The Physical and kinematical properties of the model have the following expressions:

$$V = R^4 = (a^3 b) = K_\circ^{\,4}(K_1 t + K_2) \qquad (29)$$

$$\theta = \frac{K_1}{(K_1 t + K_2)} \qquad (30)$$

$$\sigma^2 = \frac{nK_1^2 (n-3)}{3(n+3)^2 (K_1 t + K_2)^2} \qquad (31)$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_Λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} \qquad (32)$$

From equation (5)

$$\rho_m + \rho_\Lambda = \frac{K_1^2}{(n+3)(K_1 t + K_2)^2} \qquad (33)$$

$$\Omega_m + \Omega_\Lambda = \frac{\rho_m + \rho_\Lambda}{3H^2} = 1 - \frac{7-3n}{3(n+3)} \qquad (34)$$

Equation (34) shows that the sum of energy density parameter constant. I, e the present universe is isotropic. This is due to dark energy, which is found to be similar result by pradhan et al [37].
Equation (12) and (24) give

$$\rho_\Lambda = \frac{K_1^2 (3\alpha - 8)}{16(\alpha - \beta)(K_1 t + K_2)^2} \qquad (35)$$

Since recent observational data indicates that the universe is accelerating, So n must be lies between -3 and 0.

From equation (21), we have observed that $n > -3$. And (22) gives n must be less than zero. So we have a condition $0 > n > -3$ for accelerating universe.

5. Discussion

The following result are obtained-

(i) For $0 > n > -3$, the cosmological model represents an accelerating universe (i, e a increase and b decrease).
(ii) From equation (35), it is observed that the holographic dark energy density decreases with the evolution of the universe. And $\alpha \neq \beta$.
(iii) From equation (34) shows that present universe is isotropic.
(iv) Under certain condition the solutions describes the accelerated expansion of the universe.
(v) The EOS parameter of the holographic dark energy also behaves like quintessence EOS.

(vi) Observational data also suggest that the dark energy is responsible for gearing up the universe some five billion years ago.

(vii) From Fig-1 we observed that constant K_1 must be positive.

References

[1] Copeland, E. J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D 15, 1753-1935(2006).
[2] Li, M., et al.: Commun. Theor. Phys. 56 525-604(2011)
[3] Susskind, L.: J. Math. Phys. 36, 6377(1995)
[4] ‘t Hooft, G.: gr-qc/9310026
[5] Granda, L. N., Oliveros, A.: Phys. Lett. B 669, 275(2008)
[6] G. F. R. Ellis: Modern Cosmology, Institute of Physics Publishing, Bristol, pp138-139(2002)
[7] Singh, C. P., Kumar, S.: Int. J. Mod. Phys. D 15, 419-438(2006)
[8] Pradhan, A., Amirhaschi, H.: arxiv: 1110. 1019v1 [physics. gen-ph]. 5 oct. 2011
[9] Debnath, U.: Class. Quantum Gravity 25, 205019(2008)
[10] Huang, J. Z., et al.: Astrophysics. space sci. 315, 175(2008)
[11] Shao, Y., Gui, Y.: Phys. Lett. A 23, 65(2008)
[12] Pradhan, A., et al.: Int. J. Theor. Phys. (2011). doi: 10. 1007/s10773-0100793-z. arXiv: 1010. 1121[gr-qc]
[13] Perlmutter, S., et al.: Nature(London) 391, 51(1998)
[14] Riess, A. G., et al.: Astron. J. 116, 1009(1998)
[15] Sarkar, S., Mahanta, C. R.: Int. J. Theor. Phys. 52, i482 (2013)
[16] Samanta, G. C.: Int. J. Theor. Phys. 52; 4389(2013)
[17] Adhav, K. S.: Eur. Phys. J. Plus 126, 52(2011)
[18] Adhav, K. S.: Astrophys. Space sci. 335, 611(2011)
[19] B. Saha, Asrophys. Space sci. 302, 83(2006).
[20] S. D. Maharaj, R. Naidoo, Astrophys. space sci. 208, 261(1993)
[21] S. Kumar, Astrophys. space sci. 332, 449(2011)
[22] A. Pradhan, H. Amirhaschi, B. Saha, Int. J. Theor. Phys. 50, 2923(2011)
[23] P. S. Baghel, J. P. Singh, Research in Astron. Astrophys 11, 1457(2012)

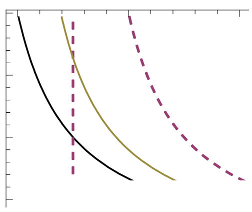

Figure 1: The plot of Hubble parameter and time.

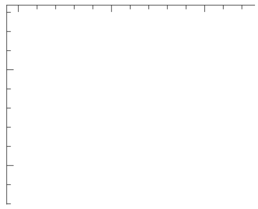

Figure 2: The plot of Hubble parameter and holographic dark energy density