Design Method for Nearly Linear-Phase IIR Filters Using Constrained Optimization

Vamsee Krishna .A[1], A. K. Jagadish Kumar[2]

[1]Vignans Institute Of Information And Technology, M. Tech,Asst. Professor, Duvvada, Visakhapatnam, Andhra Pradesh, India

[2]Vignans Institute Of Information And Technology, M. Tech Student, Duvvada, Visakhapatnam, Andhra Pradesh, India

Abstract: ***The group delay deviation is minimized under the constraint that passband ripple and stopband are within the prescribed specifications and an applicable group delay can be achieved. By representing the filter in terms of a cascade of second-order sections, a non-restrictive stability constraint characterized by a set of linear inequality constraints can be incorporated in the optimization algorithm. Experimental results show that filters designed using the proposed method have much lower group-delay deviation for the same passband ripple and stopband attenuation when compared with corresponding filters designed with several state-of-the-art competing methods.***

Keywords: Delay equalization of filters, design of filters by optimization, IIR filter design, nearly linear-phase filters.

1. Introduction

Digital filters are integral parts of many digital signal processing systems, including control systems, systems for audio and video processing, communication systems and systems for medical applications. Due to the increasing number of applications involving digital signal processing and digital filtering the variety of requirements that have to be met by digital filters has increased as well Consequently there is a need for flexible techniques that can design digital filters satisfying sophisticated specifications. Compared with FIR digital filter design, the major difficulties for designing an IIR digital filter are its nonlinearity and stability problems. Many algorithms have been developed to implement stable IIR digital filters. Some approaches implement filters in an indirect way, that is, an FIR digital filter satisfying the filter specifications are designed first, and then model reduction techniques are applied to approximate the FIR digital filter by a reduced-order IIR digital filter. In such cases, IIR filters are more attractive than FIR filters for two main reasons. Firstly, they can satisfy the given filter specifications with a much lower filter order thereby reducing the computational requirement and/or the complexity of hardware and, secondly, they have a much smaller group delay. In such indirect designs, approximation procedures can substantially guarantee the stability of designed IIR digital filters, which facilitates the design procedures. However, it is difficult to design filters with accurate cut off frequencies using this design strategy. As well as the presence of the denominator polynomial in IIR filters makes their design more challenging than that of FIR filters because it results in a highly nonlinear objective function that requires highly sophisticated optimization methods. As IIR filters lack the inherent stability of FIR filters, stability constraints must be incorporated in the design process to ensure that the filter is stable, which means constraining the poles to lie within the unit circle of the plane.

The presence of the denominator polynomial in IIR filters makes their design more challenging than that of FIR filters because it results in a highly nonlinear objective function that requires highly sophisticated optimization methods. As IIR filters lack the inherent stability of FIR filters, stability constraints must be incorporated in the design process to ensure that the filter is stable, which means constraining the poles to lie within the unit circle of the z plane.

In applications where the phase response is not important, a fairly large choice of methods is available to the filter designer ranging from closed-form methods based on classical analog filter approximations to numerous optimization methods. In [1] unconstrained algorithms of the quasi-Newton family are used in a least-P^{th} formulation. Filter stability is achieved by means of a well-known stabilization technique whereby poles outside the unit circle are replaced by their reciprocals. MATLA function *irrlpnorm* in [2] implements an unconstrained least-P^{th} quasi-Newton algorithm of the type found in [3]. On the other hand, MATLAB™ function *irrlpnormc* in [2] implements a least-P^{th} Newton method that uses barrier constraints to assure the stability of the resulting filter and also provides for a specified stability margin.

Nearly linear-phase IIR filters can be designed by using the classical *equalizer approach* whereby an IIR filter satisfying prescribed amplitude-response specifications is designed and is then cascaded with an IIR delay equalizer to approximately linearize the phase response [4].More recent methods typically involve designing IIR filters that simultaneously satisfy both the amplitude- and phase-response constraints, as it results in filters of lower order.

A methodology for the design of recursive digital filters having nearly linear phase response is in [5]. The underlying design method is of the direct type whereby the filter is designed as a single unit. The design problem is formulated as a cascade of filter sections where each section is represented by a bi-quadratic transfer function either in the conventional polynomial form or in the polar form. The design problem is then solved using a constrained Newton's method whereby constraints are used to assure the stability of the filter, to control the step size in order to achieve fast convergence, and to eliminate a real-axis pole-migration problem that often interferes with the design process.

2. The Optimization Problem

In this section, we frame the design problem at hand as a constrained optimization problem. To this end, we derive formulations for the stability constraints, group-delay deviation, pass-band ripple, stop-band attenuation, and transition-band gain constraints. Then, we incorporate the formulations within the framework of a constrained optimization problem.

We assume that the filter comprises a cascade of second-order sections (SOSs), which can be represented by a product of bi-quadratic transfer functions of the form

$$H(c,z) = H_0 \prod_{m=1}^{J} \frac{a_{0m} + a_{1m}z + z^2}{b_{0m} + b_{1m}z + z^2}$$

$$= H_0 \prod_{m=1}^{J} \frac{N_m(z)}{D_m(z)} \quad (2.1)$$

Where

$$c = [a_{01}\, a_{11}\, b_{01}\, b_{11} \ldots b_{0J}\, b_{ij}\, H_0]^T \quad (2.2)$$

J is the number of filter sections, N=2J is the filter order, And H_0 is a positive multiplier constant. An odd-order transfer function can be readily obtained by setting coefficients a_{0m} and $b_{0m}$to zero in one SOS.

2.1.1 Group-Delay Deviation

The group delay corresponding to transfer function H(c,z) in (2.1) is given by [4]

$$\tau_h(c,z) = -\sum_{i=1}^{J} \frac{\alpha_n(c,z,i)}{\beta_n(c,z,i)} + \sum_{i=1}^{J} \frac{\alpha_d(c,z,i)}{\beta_d(c,z,i)} \quad (2.3)$$

Where

$$\alpha_n(c,z,i) = 1 - a_{0i}^2 + a_{1i}(1-a_{0i})cosw \quad (2.4)$$

$$\beta_n(c,z,i) = a_{0i}^2 + a_{1i}^2 + 1 + 2a_{0i}(2cos^2 w - 1) + 2a_{1i}(a_{0i}+1)cosw \quad (2.5)$$

$$\alpha_d(c,z,i) = 1 - b_{0i}^2 + b_{1i}(1-b_{0i})cosw \quad (2.6)$$

$$\beta_d(c,z,i) = b_{0i}^2 + b_{1i}^2 + 1 + 2b_{0i}(2cos^2 w - 1) + 2b_{1i}(b_{0i}+1)cosw \quad (2.7)$$

The group-delay deviation at frequency w is given by

$$e_g(X, e^{jw}) = \tau_h(c, e^{jw}) - \tau \quad (2.8)$$

Where

$$X = [c^T \tau]^T \quad (2.9)$$

And τ is the group delay, which may be an optimization variable. If X_k is the value of X at the kth start of the iteration andδ isthe update to X_k , the updated value of the group-delay deviation can be estimated by using the linear approximation

$$e_g(X_k+\delta, e^{jw}) \approx e_g(X_k, e^{jw}) + \nabla e_g(X_k, e^{jw})^T \delta \quad (2.10)$$

Which becomes more accurate as $\|\delta\|_2$ gets smaller.

If w_{pl} and w_{ph} are the lower and upper edges of the passband, the L_p -norm of the passband group-delay deviation for the Kth iteration is given by

$$E_p^{(gd)}(k) = \left[\int_{w_{pl}}^{w_{ph}} |e_g(X_{k+1}, e^{jw})|^p dw\right]^{\frac{1}{p}} \approx$$

$$k_g \left[\sum_{i=1}^{N} |e_g(X_{k+1}, e^{jw_i})|^p\right]^{\frac{1}{p}} w_i \in \psi_p \quad (2.11)$$

$$\approx \left[\sum_{i=1}^{N} |k_g e_g(X_k, e^{jw_i}) + k_g \nabla e_g(X_k, e^{jw})^T \delta|^p\right]^{\frac{1}{p}}$$

Where $\psi_p \epsilon [w_{pl}, w_{ph}]$is the set of passband frequency sample points and k_g is a constant. Expressing (2.11) in matrix form, we get

$$E_p^{(gd)}(k) \approx \|C_k\delta + d_k\|_p \quad (2.12)$$

$$\text{Where } C_k = \begin{bmatrix} K_g \nabla e_g(X_k, e^{jw_1})^T \\ \vdots \\ K_g \nabla e_g(X_k, e^{jw_{N_p}})^T \end{bmatrix} \quad (2.13)$$

$$d_k = [d_1 d_2 \ldots d_{N_p}]^T \quad (2.14)$$

$$d_i = k_g e_g(X_k, e^{iw_i}), w_i \in \psi_p. \quad (2.15)$$

The right-hand side of (2.12) is the L_p -norm of an affine function of $\delta$and, therefore, it is convex with respect to δ[1]. The quality of the group-delay characteristic of the filter can be measured by using the normalized maximum variation of the filter group delay,τ_h , over the passband as a percentage, i.e., [3]

$$Q_\tau = \frac{100(\tau_{max} - \tau_{min})}{2\tau_{avg}} \quad (2.16)$$

Where

$$\tau_{avg} = \frac{\tau_{max} + \tau_{min}}{2} \quad (2.17)$$

$$\tau_{max} = \max_{w \epsilon \psi_p} \tau_h(e^{jw}) \quad (2.18)$$

$$\tau_{min} = \min_{w \epsilon \psi_p} \tau_h(e^{jw}). \quad (2.19)$$

Hence,

$$Q_T = \frac{100(\tau_{max} - \tau_{min})}{(\tau_{max} + \tau_{min})} \quad (2.20)$$

Q_τ will be referred to as the maximum group-delay deviation hereafter.

2.1.2 Pass-band Error

If $H_d(w)$is the desired frequency response of the filter and C_k is the value of vector at the start of the Kth iteration, a passband error function at frequency w can be defined as

$$e_h(c_k, e^{jw}) = |H(c_k, e^{jw})|^2 - |H_d(w)|^2. \quad (2.21)$$

Without loss of generality, we can assume that the desired amplitude response is unity in the pass-band. Therefore, the pass-band error function becomes

$$e_h^{(pb)}(c_k, e^{jw}) = |H(c_k, e^{jw})|^2 - 1, w\epsilon\psi_p. \quad (2.22)$$

Using the same approach as in above(group delay deviation), theL_p -norm of the pass-band error function,$E_p^{(pb)}(k)$, in matrix form can be approximated as

$$E_p^{(pb)}(k) \approx \|D_k^{(pb)}\delta + f_k^{(pb)}\|_p \quad (2.23)$$

Where

$$D_k^{(pb)} = \begin{bmatrix} k_{pb}\nabla e_h^{(pb)}(c_k, e^{jw_1})^T & 0 \\ \vdots & \\ k_{pb}\nabla e_h^{(pb)}(c_k, e^{jw_{N_p}})^T & 0 \end{bmatrix}, w_i \in \psi_p \quad (2.24)$$

$$f_k^{(pb)} = [f_1^{(pb)} f_2^{(pb)} \dots f_{N_p}^{(pb)}]^T, \quad (2.25)$$

$$f_i^{(pb)} = k_{pb}\, e_h^{(pb)}(c_k, e^{jw_i}), \quad (2.26)$$

$$\delta = [\delta_c^T \delta_\tau]^T. \quad (2.27)$$

In the above equations, δ_c is the vector update for C_k, δ_τ is the scalar update for τ, k_{pb} and is a constant. The elements of the last column of $D_k^{(pb)}$ in (2.24) are all zeros since (2.23) are independent.

3. Design Approach

Two general strategies for the design of digital filters have been developed to deal with design problems where the group delay is not specified or with problems where a prescribed group delay is required. In the former case, the group delay can be used as an additional independent variable that can be optimized in order to bring about additional improvements to the filter being designed.

3.1 Optimized Group Delay

When the group delay is assumed to be an independent variable, it is important that the initialization filter be chosen to be close to the desired optimal filter in order to assure fast convergence. To this end, a good first step would be to design the lowest-order IIR filter that would satisfy only the amplitude-response specifications. An *elliptic* filter would be the most suitable choice since it gives the lowest-order IIR filter for any given amplitude-response specifications because of the optimality of the elliptic approximation. Such a filter can be obtained by using the design method described in Chap. 12 of [3].

To reduce the group-delay deviation of the filter in the pass-band, a number of additional general bi-quadratic SOSs are included depending on the degree of linearity required in the phase response. To achieve fast convergence, the additional SOSs are initialized as all pass sections. The poles and zeros of the additional SOSs are initially distributed in the pass-band sector of the z plane, namely, the sector bounded by the pass-band edge frequencies. Under these circumstances, the transfer function assumes the form

$$H_{init}(z) = H_{ellip}(z)$$
$$G_0 \prod_{k=1}^{M} \frac{(z-r_k^{-1}e^{jw_k})(z-r_k^{-1}e^{-jw_k})}{(z-r_k e^{jw_k})(z-r_k e^{-jw_k})}, \quad (3.1) \quad w_k \epsilon \Psi_p$$

Where $H_{ellip}(z)$ is the transfer function of the elliptic filter, G_0 is a normalizing gain factor, M is the number of additional allpass SOSs, and $0 < r_k < 1$. An initialization group delay that was found to work well is the average of the pass-band filter-equalizer combination, which can be estimated as

$$\tau_{init} = \frac{-1}{w_{ph}-w_{pl}} \int_{w_{pl}}^{w_{ph}} \frac{d}{dw}\{\arg[H_{init}(e^{jw})]\}dw. \quad (3.2)$$

The required filter can be designed by using the following algorithm:

Step 1: Obtain the transfer function of the required elliptic filter, $H_{ellip}(z)$, that satisfies the required amplitude response specifications; e.g., by using the D-Filter software package [20].

Step 2: Set the number of additional general bi quadratic SOSs to M and select r_k and w_k to construct the transfer function $H_{init}(z)$, as in (3.1); from $H_{ellip}(z)$, compute the initialization group delay τ_{init}, using (3.2). This can be easily done by using D-Filter [20].

Step 3a: Using $H_{init}(z)$, and τ_{init} for initialization, solve the optimization problem.

Step 3b (optional): Solve the optimization problem using $\{H_{init}(z), \tau_{init}^{(max)}\}$ and $\{H_{init}(z), \tau_{init}^{(min)}\}$ for initialization and then select the solution that has the smallest value of Q_τ in *Steps 3a* and *3b*.

Step 4: Using (2.20), compute the maximum group delay deviation of the filter, Q_τ, obtained in *Step 3*. If Q_τ is less than the prescribed value, the filter specifications are satisfied and the algorithm is terminated; otherwise, set M=M+1 and go to *Step 2*.

The optional step, *Step 3b*, can be carried out if the amount of computation required is not a critical factor, in order to increase the possibility for obtaining a better solution.

3.2 Prescribed Group Delay

When a prescribed group delay is required, the initialization procedure described in 3.1 is not appropriate. Amore appropriate initialization scheme would be to use the balance model truncation (BMT) method described in [21], [22].The main steps of the BMT involve converting a high-order FIR filter into a state-space balanced model, then reducing the model order, and finally converting the lower-order model to a reduced-order IIR filter.

To ensure that the IIR filter obtained with the BMT method as a group delay that is close to the prescribed value, the initialization linear-phase FIR filter is designed to have a group delay, τ_{pr}, that is close to the prescribed value. This can be done by selecting the filter length

$$L_{fir} = 2\lceil \tau_{pr} \rceil + 1 \quad (3.3)$$

Where [.] is the ceiling operator. The transfer function of the IIR filter can be expressed as

$$H_{init}(z) = \widehat{G_0} \frac{\prod_i (z-\check{z}_i)}{D_{bmt}(z)} = \frac{\sum_i \check{b}_i z^{m-i}}{D_{bmt}(z)} \quad (3.4)$$

Sometimes, the filter obtained with the BMT method may have one or more zeros that are located far away from the origin. Such zeros can slow down the optimization algorithm. Experimental results have shown that faster convergence can be achieved by moving any zeros with a radius greater than a prescribed maximum $r_{max}^{(z)}$ to the origin of the z plane by letting

$$\check{z}_i = \begin{cases} 0 \ if \ |\bar{z}_i| > r_{max}^{(z)} \\ \bar{z}_i \ otherwise. \end{cases} \quad (3.5)$$

Where the normalizing gain factor $\widehat{G_0}$ is chosen to ensure that the average pass-band gain of the filter is unity.

The required filter can then be designed by using the following procedure:

Step 1: Design a linear-phase FIR filter of length given by (3.4) with the prescribed pass-band and stop-band edge frequencies, by using D-Filter or the MATLAB™ function *fir1* or some other way.

Step 2: If the total number of SOSs is M_{tot} , the IIR filter order is $2M_{tot}$. Using the BMT method, transform the FIR filter obtained in *Step 1* to an IIR filter of order $2M_{tot}$.

Step 3: Form the transfer function in (3.5).

Step 4: Using$H_{init}\,(z)$, in (3.5) for initialization, solve the optimization problem for the prescribed group delay ofτ_{pr} .

Step5: Using (2.20) compute the group-delay deviation,Q_τ,of the filter obtained in *Step 4*. If $Q_\tau$is less than the maximum prescribed value, the filter specifications are satisfied and the algorithm is terminated; otherwise, set$M_{tot} = M_{tot} + 1$ and go to *Step 2*.

4. Simulation Results

This chapter illustrates the complete details about the performance evaluation of proposed approach. In order to compare the proposed method with other state-of-the-art competing methods, we have designed and tested many nearly linear-phase IIR filters satisfying a diverse range of specifications.

Example1

Low pass Digital Filter

The following table illustrates the parameters required for the design of low pass filter.

Parameters	Values
Maximum pass band ripple, dB	0.2
Minimum stop band attenuation, dB	50
Pass band edge, rad/sec	0.36 π
Stop band edge ,rad/sec	0.44π
Minimum pole radius	0.98

And their corresponding waveforms are as follows

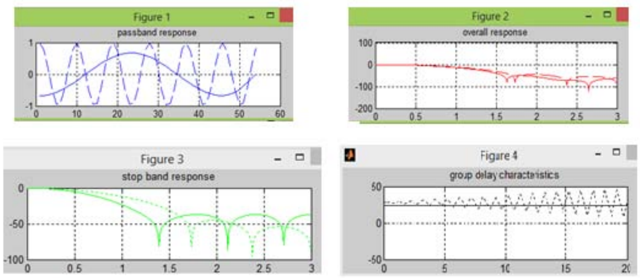

Figure 4.1:.Passband, overall and stop band amplitude responses and group-delay characteristic for Design of the proposed method (solid curves) and the method in [4] (dashed curves) for Example 1.

Example 2

High pass Digital filter

The following table illustrates the parameters required for the design of High pass filter.

Parameters	Values
Maximum pass band ripple, dB	0.1
Minimum stop band attenuation, dB	73
Pass band edge, rad/sec	0.6 π
Stop band edge ,rad/sec	0.4π
Minimum pole radius	0.98

And there corresponding waveforms are as follows

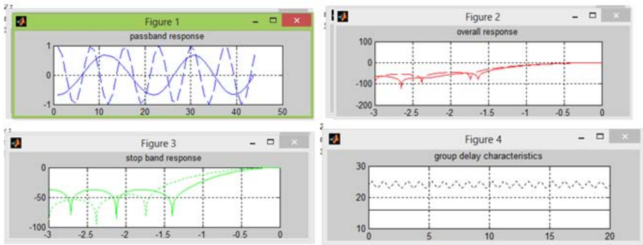

Figure 4.2: Passband, overall and stop band amplitude responses and group-delay characteristic for Design of the proposed method (solid curves) and the method in [4] (dashed curves) for Example 2.

Example3

Band pass Digital Filter

The following table illustrates the parameters required for the design of Band pass Digital filter.

Parameters	Values
Maximum pass band ripple, dB	1.0
Minimum stop band attenuation, dB	41
Low stop band edge, rad/sec	0.2 π
Low pas band edge ,rad/sec	0.3π
High pass band edge, rad/sec	0.5 π
High stop band edge, rad/sec	0.7 π
Minimum pole radius	0.98

And there corresponding waveforms are as follows

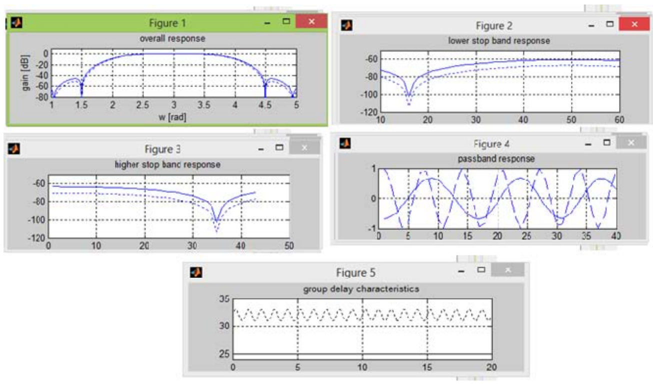

Figure 4.3: Overall, stop band, and pass band amplitude responses and group-delay characteristic for Design of the proposed method (solid curves) and the method in [3] (dashed curves) for Example 3.

5. Conclusion

A method for the design of nearly linear-phase IIR digital filters that satisfy prescribed specifications has been described. In the proposed method, the group-delay deviation is minimized under the constraints that the pass-band ripple and minimum stop-band attenuation meet the specifications and either a prescribed or an optimized group delay can be achieved. By designing the filter as a cascade of second-order sections, a non-restrictive stability constraint characterized by a set of linear inequality constraints can be incorporated in the optimization algorithm. An additional feature of the method, which is very useful in certain applications, is the inherent capability of constraining the maximum gain in transition bands to be below a prescribed value. This facilitates the elimination of transition-band anomalies which sometimes occur in filters designed by optimization.

Experimental results have shown that the nearly linear-phase IIR filters designed using the proposed method have a much lower maximum group-delay deviation for the same pass-band ripple and minimum stop-band attenuation when compared with several filters designed with state-of-the-art competing methods. It has also been demonstrated that nearly linear-phase IIR filters offer some substantial advantages when compared with their exact linear-phase FIR counterparts such as lower group delay and filter complexity without compromising the required amplitude-response specifications.

References

[1] A. Antoniou and W.-S. Lu, Practical Optimization: Algorithms and Engineering Applications. New York: Springer, 2007.
[2] D. J. Shpak, *MATLAB™ functions iirlpnorm and iirlpnormc*in the *MATLAB™ Filter Design Toolbox*. Natick, MA, USA: The Math-Works, 2002.
[3] A. Antoniou, *Digital Signal Processing: Signals, Systems, and Filters*.New York: McGraw-Hill, 2005.
[4] C. Charalambous and A. Antoniou, "Equalisation of recursive digital filters," *Proc. Inst. Elect. Eng.*, vol. 127, no. 5, pp. 219–225, Oct. 1980.
[5] D. Guindon, D. J. Shpak, and A. Antoniou, "Design methodology for nearly linear-phase recursive digital filters by constrained optimization,"*IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 57, no. 7, pp.1719–1731, Jul. 2010.
[6] W.-S. Lu, "Design of recursive digital filters with prescribed stability margin: A parameterization approach," *IEEE Trans. Circuits Syst. I:Fund. Theory Appl.*, vol. 45, pp. 1289–1298, Sep. 1998.
[7] W.-S. Lu, "An argument-principle based stability criterion and application to the design of IIR digital filters," in *Proc. IEEE Intern. Symp.Circuits Syst.*, 2006, pp. 4431–4434.
[8] J. L. Sullivan and J.W. Adams, "PCLS IIR digital filters with simultaneous frequency response magnitude and group delay specifications," *IEEE Trans. Signal Process*, vol. 46, no. 11, pp. 2853–2861, Nov. 1998.
[9] A. Jiang and H. K. Kwan, "IIR digital filter design with new stability constraint based on argument principle," *IEEE Trans. Circuits Syst. I,Reg. Papers*, vol. 56, pp. 583–593, Mar. 2009.
[10] X. Lai and Z. Lin, "Minimax design of IIR digital filters using a sequential constrained least-squares method," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3901–3906, Jul. 2010.
[11] X. Lai and Z. Lin, "Minimax phase error design of IIR digital filters with prescribed magnitude and phase responses," *IEEE Trans. SignalProcess.*, vol. 60, no. 2, pp. 980–986, Feb. 2012.
[12] A. Jiang and H. K. Kwan, "Minimax design of IIR digital filters using iterative SOCP," *IEEE Trans.Circuits Syst. I, Reg. Papers*, vol. 57, no.6, pp. 1326–1337, Jun. 2010.
[13] A. Jiang and H. K. Kwan, "Minimax design of IIR digital filters using SDP relaxation technique," *IEEE Trans. Circuits Syst. I, Reg. Papers*,vol. 57, pp. 378–390, Feb. 2010.
[14] X. Lai, Z. Lin, and H. K. Kwan, "A sequential minimization procedure for minimax design of IIR filters based on second-order factor updates," *IEEE Trans. Circuits Syst.-II: Exp. Briefs*, vol. 58, no. 1, pp.51–55, Jan. 2011.
[15] W.-S. Lu and A. Antoniou, "Design of digital filters and filter banks by optimization: A state of the art review," in *Proc. Eur. Signal Process. Conf. (EUSIPCO)*, Tampere, Finland, Sep. 2000, pp. 351–354.
[16] B. Dumitrescu and R. Niemist, "Multistage IIR filter design using convex stability domains defined by positive realness," *IEEE Trans.Signal Process.*, vol. 52, no. 4, pp. 962–974, Apr. 2004.
[17] M. C. Lang, "Least-squares design of IIR filters with prescribed magnitude and phase responses and a pole radius constraint," *IEEE Trans. Signal Process.*, vol. 48, no. 11, pp. 3109–3121, Nov. 2000.
[18] W.-S. Lu and T. Hinamoto,"Optimal design of IIR digital filters with robust stability using conic-quadratic-programming updates," *IEEETrans. Signal Process.*, vol. 51, no. 6, pp. 1581–1592, Jun. 2003.
[19] J. F. Sturm,"Using SeDuMi1.02, a MATLAB™toolbox for optimization over symmetric cones," *Optimiz. Methods Softw.*, vol. 11-12, pp.625–653, 1999.
[20] [Online]. Available: http://www.ece.uvic.ca/~dsp/ .
[21] I. Kale, J. Gryka, G. D. Cain, and B. Beliczynski, "FIR filter order reduction: Balancedmodel truncation andHankel-norm optimal approximation,"*IEEE Proc. Vis. Image Signal Process.*, vol. 141, pp. 168–174, Jun. 1994.
[22] L. Prenobo and L. M. Silverman, "Model reduction via balanced state space representation," *IEEE Trans. Autom. Control*, vol. AC-27, no. 4,pp. 382–387, Apr. 1982.
[23] A. Antoniou, "Improved minimax optimization algorithms and their application in the design of recursive digital filters," *IEE Proc.*, vol.138, no. 6, pp. 724–730, Dec. 1991.
[24] [Online].Available:http://www.ece.uvic.ca/~andreas/JournalPapers/Poles_and_Zeros_and_Additional_Results.pdf.

Author Profile

Vamsee Krishna .A obtained his B.Tech Degree from Thandra Paparaya Institute of Science and Technology, Affiliated to JNTUK, Andhra Pradesh, India. He obtained his M.Tech Degree from Gokul Institute of Technology And Sciences, Affiliated to JNTUK, Andhra Pradesh, India. He is presently working as Asst. Professor in the Department of ECE, VIIT, Visakhapatnam. His Interests are Image Processing.

A. K. Jagadish Kumar, obtained his B.Tech Degree from Visakha Institute Of Technology And Science, Affiliated to JNTUK, Andhra Pradesh, India. Now he is pursuing his M.Tech degree in Department Of Electronics and Communication Engineering at Vignans Institute of Information and Technology, Visakhapatnam. He is interested in the field of Robotics and Digital Electronics.