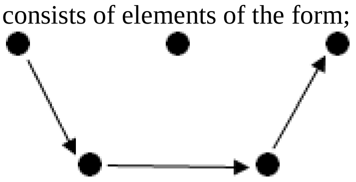
The conjugacy class, C_{12} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&2&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&3&4\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&2&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&3&5&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&4&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&4&5&-\end{pmatrix},$$
$$\begin{pmatrix}1&2&3&4&5\\2&3&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&-&4&5&-\end{pmatrix}\}$$

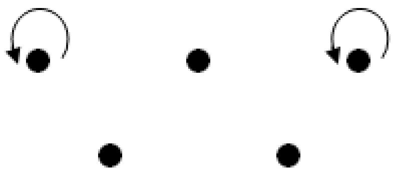
consists of elements of the form;



The conjugacy class, C_{13} =

$$\{\begin{pmatrix}1&2&3&4&5\\1&2&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&5\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&2&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&5\end{pmatrix},$$
$$\begin{pmatrix}1&2&3&4&5\\-&2&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&4&5\end{pmatrix}\}$$

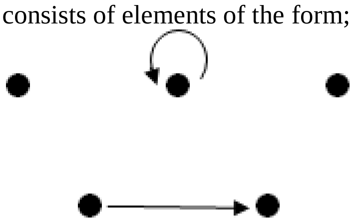
consists of elements of the form;



The conjugacy class, C_{14} =

$$\{\begin{pmatrix}1&2&3&4&5\\1&-&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&3&-&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\1&-&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&4&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&4&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&1&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&5&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&5&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&1&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&3&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&2&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&4\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\2&-&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&5&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&5\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\3&-&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&-&-&-&5\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\4&-&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&4&-&-&5\end{pmatrix},$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&2&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&3&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\1&-&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&4&-&5\end{pmatrix}\}$$

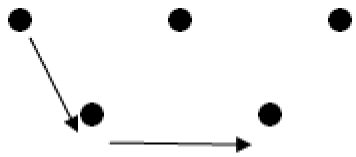
consists of elements of the form;



The conjugacy class, C_{15} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&3&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&-&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&3&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&4&-&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&5&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&-&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&3&4\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\3&-&5&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&5&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\4&-&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&4&5&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&4&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&4&5&-\end{pmatrix}\},$$

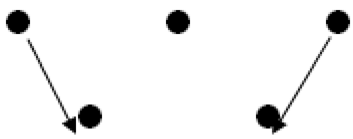
consists of elements of the form;



The conjugacy class, C_{16} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&-&1&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&1&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&3&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&-&1&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&4\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&1&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&-&-&3\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\2&-&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&5&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&2&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&4&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&-&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&-&-&4\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&1&5&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\2&-&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&5&-&-&-\end{pmatrix}$$
$$\begin{pmatrix}1&2&3&4&5\\-&1&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\3&-&-&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\4&5&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\4&-&5&-&-\end{pmatrix}\},$$

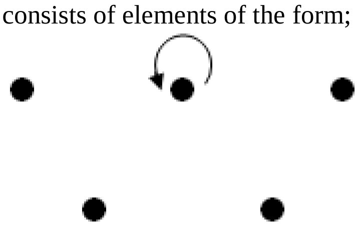
consists of elements of the form;



The conjugacy class, C_{17} =

$$\{\begin{pmatrix}1&2&3&4&5\\1&-&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&4&-\end{pmatrix}\}$$
$$\begin{pmatrix}1&2&3&4&5\\-&-&-&-&5\end{pmatrix}$$

consists of elements of the form;



The conjugacy class, C_{18} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&1\end{pmatrix}$$

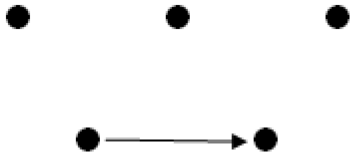
$$\begin{pmatrix}1&2&3&4&5\\2&-&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&2\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\4&-&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&4&-&-&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\5&-&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&5&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&5&-&-\end{pmatrix}$$

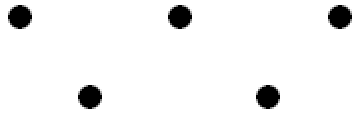
$$\begin{pmatrix}1&2&3&4&5\\3&-&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&3&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&4&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&5&-\end{pmatrix}\},$$

consists of elements of the form;



The conjugacy class, $C_{19} = \{\begin{pmatrix}1&2&3&4&5\\-&-&-&-&-\end{pmatrix}\},$

consists of elements of the form;



Thus, we see that IO_5 has nineteen conjugacy classes.

4. Result

The summary of the sequence on the number of conjugacy classes of Order - preserving partial one - -one transformation semigroup is 2,4, 7, 12,19, . . . A000070 of the Online Encyclopedia of Integer Sequences (OEIS) [7].

5. Conclusion

It was discovered that the transformation semigroup can be understood better in nature and properties with the aids of graphs and the unlabelled graph can be effectively use to determine the conjugacy class of Order - preserving partial one - -one transformation semigroup.

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