Comparative Study of High Density Salt and Pepper Noise Removal (Spatial Domain Methods used in Image Processing)

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Abstract: This paper deals with the study of Salt & Pepper noise and different de-noising methods to remove them. Noise densities varies from 10 to 98% in an image. Various filters such as Mean Filters, Order-Statistic Filters and Adaptive filter are used for image de-noising. From Mean Filter Arithmetic Mean Filter, Max and Min Filter from Order-Statistic Filters and Adaptive mean Filter from Adaptive Filters and Median filters respectively. Comparison of different noise removal technique is based on three parameters Mean Absolute Error (MAE), Mean Square Error (MSE), and Peak Signal Noise Ratio (PSNR). The mean absolute error is a quantity used to measure how close forecast or predictions are to the eventual outcomes. The mean square error of an estimator is one of many ways to quantify the difference between an estimator and the true value of quantity being estimated. MSE is a risk function, corresponding to the excepted value of the squared error loss or quadratic loss. MSE measures the average of the square of the “error”. The error is the amount by which the estimator differs from the quantity to be estimated. The phrase Peak Signal to Noise Ratio, often known as PSNR, is an engineering term for the ratio between maximum possible power of a signal and the corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. Our quantitative results with help of MATLAB simulations show that for lower noise densities to medium-high densities (10 to 98%), Max filter gives the optimum performance followed by adaptive filter, Median filters. For noise densities from 60% to 80%, Min Filter showed best results, followed by Max and Adaptive median filter respectively. For higher noise values (>90%) the Min Filter showed the best results followed by Adaptive mean filter and median filter.

Keywords: Salt and pepper noise, Image de-noising, MAE, MSE and PSNR.

1. Introduction

Noise is any undesired Information that contaminates an image. Noise appears in an image from a variety of sources. The salt & pepper type noise is typically caused by malfunctioning of the pixel elements in the camera sensors, faulty image location or timing errors in digitization process. For the images corrupted by salt & pepper noise, the noise pixel can take maximum for salt and minimum for pepper [1]. To de-noise an image several filters are used, depending on noise density.

2. Classification of Noise

Basically, there are three standard noise models that affect an image. They are additive noise, multiplicative noise, and impulse noise [5].

2.1 Additive Noise

Additive noise is independent of the pixel values in the original image. Additive noise is represented by:

\[ f'[m,n] = f[m,n] + \eta[m,n] \]

where \( f[m,n] \) is original image, \( f'[m,n] \) be the noise digitized version and \( \eta[m,n] \) is the noise function which returns values coming from an arbitrary distribution. Additive noise is good model for the thermal noise within photo-electronic sensors.

2.2 Multiplicative Noise

Multiplicative noise or speckle noise is a signal-dependent from of noise whose magnitude is related to the value of the original pixel. The simple mathematical expression for a multiplicative noise model is following:-

\[ f'[m,n] = f[m,n] + \eta[m,n] f[m,n] \]

2.3 Impulse Noise

Impulse noise has the property of either leaving a pixel unmodified with probability 1-p, or replacing altogether with a probability p. Restricting \( \eta[m,n] \) to reduce only the extreme intensities 0 or 1 results in salt-and-pepper noise. The source of impulse noise is usually the result of an error in transmission or an atmospheric or man-made disturbance. Gaussian and Salt-and –Pepper noise are called Impulsive noise [3].

A. Gaussian Noise

Gaussian noise can be expressed by following mathematical equation:-

\[ p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-z^-)^2}{2\sigma^2}} \]

Here z represents intensity and \( z^- \) is mean value of z, \( \sigma \) is its standard deviation. \( \sigma^2 \) is called variance of z.
B. Salt-and-Pepper

The salt-and-pepper noise is given by

\[ p(z) = \begin{cases} \frac{Pa}{m n} & \text{for } z = a \\ \frac{Pb}{m n} & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \]

If \( b > a \), intensity \( b \) will appear as a light dot in image. Here \( Pa, Pb \) are the probabilities, \( P(x) \) is the probabilities density of Salt and Pepper noise in image and \( A \times B \) is the array size of image[2].

3. Filters

Filters are used for removing effect of noise from an image. Basic Filters for Image De-noising are:

Mean Filters, Order-Statistic Filters, Adaptive Filters.

I. Mean Filters

Mean Filters are following types: Arithmetic mean Filter, Geometric mean filter, Harmonic mean filter, ContraHarmonic Filter.

A. Arithmetic mean filter

This is the simple of the mean filters. Let \( S_{xy} \) represents the set of co-ordinates in a rectangular sub image of window of size \( m \times n \), centered at point \((x, y)\). The arithmetic mean filtering process computes the average value of the corrupted image \( g(x, y) \) in the area defined by \( S_{xy} \). The value of the restored image \( f(x, y) \) at any point \((x, y)\) is simply the Arithmetic mean which is computed using the pixels in the neighborhood defined by \( S_{xy} \). In other words,

\[ f(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t) \]

This operation can be implemented using a spatial filter of size \( m \times n \) in which all coefficients have value \( 1/mn \). A mean filter smooth local variations in an image, and image is reduced as a result of blurring [3].

B. Geometric mean filter

An image restored using a geometric mean filter is given by the expression

\[ f(x, y) = \left[ \prod_{(s,t) \in S_{xy}} g(s, t) \right]^{1/mn} \]

Here, each restored pixel is given by the product of the pixels in the sub-image window, raised to the power \( 1/mn \).[6]

C. Harmonic mean filter

The harmonic mean filtering operation is given by the expression

\[ f(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s, t)}} \]

D. Contra harmonic mean filter

The contra harmonic mean filter yields a restored image based on the expression

\[ f(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q} \]

where \( Q \) is called the order of the filter and this filter is well suited for reducing or virtually eliminating the effect of salt/pepper noise. For positive values of \( Q \), the filter eliminates pepper noise. For negative values of \( Q \) it eliminates salt noise. It cannot do both simultaneously. Note that the contra harmonic mean filter reduces to the arithmetic mean filter if \( Q = 0 \), and to the harmonic Mean filter if \( Q = -1 \) [3].

II. Order-Statistic Filters

Order-statistic filters are spatial filters whose response is based on ordering the value of the pixel Contained in the image area encompassed by filter. The ranking result determines the response of the filter. Its types are:

III. Median filter, Max and min filters.

A. Median filter

The best-known order-statistic filter is median filter, which as its name implies, replaces the value of a pixel by the median of the intensity levels in the neighborhood of the pixel. Median filters are particularly effective in the presence of both bipolar and unipolar impulse noise [3].

B. Max and Min filters

Although the median filter is by far the order-statistic filter most used in image processing, it is by no means the only one. The median represents the 50th percentile of a ranked set of number, but you will recall from basic statistic that ranking lends itself to many other possibilities. For example, using the 100th percentile results in the so-called max filter [3], given by

\[ f(x, y) = \max_{(s,t) \in S_{xy}} g(s, t) \]

This 0th percentile filter is the min filter given by

\[ f(x, y) = \min_{(s,t) \in S_{xy}} g(s, t) \]

IV. Adaptive median filter

The median filter performs well as long as the spatial density of the impulse noise is not large. Adaptive median filtering can handle impulse noise with probabilities even larger than these. An additional benefit of the adaptive median filter is that it seeks to preserve detail while smoothing non-impulse noise, something that the ‘traditional’ median filter does not do[2].
4. Experiment Verification

The filters were implemented using (MATLAB 7.1) on PC equipped with 2.0 GHz and 4GB of (RAM). The following given image is tested with several noise models and denoised with filters.

![Figure 1](image1)

Each of the given filter is tested using the image (deer.jpg) of 8 bits/pixel image. The image corrupted by impulse noise at various noise densities changed the pixels value. The filters were applied to restore the image and restoration performance was measured using the following parameters: Peak Signal-to-Noise Ratio (PSNR), Mean Square Error (MSE), Mean Absolute Error (MAE). These are defined by following formulae:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i,j) - K(i,j)]^2$$

The PSNR is defined as:

$$PSNR = 10 \cdot \log_{10} \left( \frac{MAX^2}{MSE} \right)$$

Here, $MAX_t$ is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, this is 255. More generally, when samples are represented using linear PCM with $B$ bits per sample, $MAX_t$ is $2^B - 1$. For color image with three RGB values per pixel, the definition of PSNR is the same except the $MSE$ is the sum over all squared value differences divided by image size and by three. The mean absolute error (MAE) is given by

$$MAE = \frac{1}{n} \sum_{i=0}^{n} |f_i - y_i| = \frac{1}{n} \sum_{i=0}^{n} |e_i|$$

As the name suggests, the mean absolute error is an average of the absolute errors $e_i = f_i - y_i$

Where $f_i$ is the prediction and $y_i$ is the true value. Note that alternative formulations may include relative frequencies as weight factors.

Restoration results are shown in Figure 2 for deer.jpg image. The visual quality clearly show that the ‘Max’ filter outperform than the other filter in terms of noise removal.

Table I, II and III show the comparison of PSNR, MSE, and MAE of various filters for the deer.jpg image for different noise densities.

Figure 3,4,5 show the comparison graph of PSNR,MSE,MAE of various filters for the deer.jpg image for different noise densities.

![Figure 2 (deer.jpg) : (a)Original Image corrupted with 70% salt-&-pepper noise density. Restoration results of (b) AMF (c) AdMF (d)Median (e) Max (f)Min](image2)
Table 1: Comparison table of PSNR of different filters for image deer.jpg with Salt & Pepper Noise

<table>
<thead>
<tr>
<th>Noise Density (%)</th>
<th>AMF</th>
<th>AdMF</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>31.63</td>
<td>31.39</td>
<td>37.69</td>
<td>64.12</td>
<td>27.48</td>
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<tr>
<td>20</td>
<td>30.46</td>
<td>28.08</td>
<td>36.13</td>
<td>58.69</td>
<td>27.64</td>
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<tr>
<td>30</td>
<td>30.16</td>
<td>27.18</td>
<td>34.97</td>
<td>53.89</td>
<td>27.85</td>
</tr>
<tr>
<td>40</td>
<td>30.16</td>
<td>26.71</td>
<td>33.61</td>
<td>48.79</td>
<td>28.14</td>
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<tr>
<td>50</td>
<td>30.27</td>
<td>26.31</td>
<td>32.61</td>
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<td>60</td>
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<td>26.07</td>
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<tr>
<td>70</td>
<td>30.51</td>
<td>25.85</td>
<td>29.32</td>
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<td>29.88</td>
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<tr>
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<td>28.03</td>
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<td>98</td>
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Table 2: Comparison table of MSE of different filters for image deer.jpg with Salt & Pepper Noise

<table>
<thead>
<tr>
<th>Noise Density (%)</th>
<th>AMF</th>
<th>AdMF</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
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<td>90.3486</td>
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<td>11.7778</td>
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<tr>
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<td>48.7919</td>
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<tr>
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<td>42.1000</td>
<td>183.6855</td>
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<td>40.3516</td>
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Table 3: Comparison table of MAE of different filters for image deer.jpg with Salt & Pepper Noise

<table>
<thead>
<tr>
<th>Noise Density (%)</th>
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<th>AdMF</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
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<td>10.1466</td>
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<tr>
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</tr>
<tr>
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<tr>
<td>50</td>
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<tr>
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<td>70.0103</td>
<td>8.5543</td>
</tr>
</tbody>
</table>

Figure 3: Comparison Graph for PSNR of different filters for salt & pepper noise Corrupted deer.jpg image

Figure 4: Comparison Graph for MSE of different filters for salt & pepper noise Corrupted deer.jpg image

Figure 5: Comparison Graph for MSE of different filters for salt & pepper noise Corrupted deer.jpg image

5. Conclusion

Based upon our simulation work, we were able to classify the filter requirements depending upon the noise density of corrupting noise along with PSNR, MSE and MAE as deciding parameters for salt and pepper noise, we have the following results.

- The noise density of around 10% will give PSNR of around 64 which is provided by ‘Max’ filter. On increasing the noise density, the ‘Max’ filter gives around 58 PSNR as seen from table-1.
- The noise density of around 30% will give PSNR 53 which is provided by the ‘Max’ filter as seen from table-1 and second best filter was found to be Adaptive median filter. On increasing noise density to next 10%, we have ‘Max’ filter which gives higher values of PSNR than the median filter.
- When noise density is 60% the maximum PSNR values of the ‘Max’ filter is 39, when noise density increased by 30%, maximum value of PSNR is achieved by MAX filter followed by Adaptive median filter and median filters respectively. Both have same value of PSNR around 25.
- When noise density reaches to 80%, the Max filter is better than other filters as its PSNR values is around 30.5508 and then comes ‘Min’ filter.
- When the noise density is 90% then the highest PSNR value is 40.3516 which is given by ‘Min’ filter. All three Median, Max and Adaptive median have almost same value of PSNR.
- On increasing noise density above 90% then the highest PSNR value is 40.3516 of ‘Min’ filter, followed by
Adaptive median and Median filters which have almost the same values of PSNR.

- When values of PSNR are compared using the table-1, for different noise densities and given filters, it was found out that PSNR values for Max filter showed variation from 64 to 23. The least variation was the Arithmetic mean Filter (PSNR form 31 to 30).

References


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