A Comparison of Potential Infinity and Actual Infinity

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Abstract: ***Infinity is a useful concept of a process with no end. Sometimes its presence is explicit, sometimes it's not explicit. Mathematicians consider two types of infinities, potential and actual infinity. The main purpose of this paper is to compare actual infinity and potential infinity.***

Keywords: Actual infinity, potential infinity, infinitesimal, bigbang theory, steady state theory.

"Mathematical infinity is taken from reality, although uncon sciously and therefore it can only be explained from reality a nd not from itself or from mathematical abstraction."[1]

The concept of infinity appears in almost every field of mathematics. Sometimes its presence is explicit, sometimes it's not explicit. Arithmetic and classical algebra deal with numbers. All the numbers taken together form infinite collections. In geometry, we encounter points on a line, lines on a plane, planes in space. These are not infinite in number, but are also in extent. The core concepts in Mathematical analysis are limits, continuity, sequences, series, differentiation, integration etc. All these deals with infinitely large quantities and infinitely small quantities. The infinitesimal have been used to express the idea of objects so small that there is no way to see them or measure them.

Mathematicians consider two types of infinities, potential and actual infinity. Potential infinity was accepted as a legitimate mathematical object by mathematicians and logicians. But actual infinity is not accepted until George cantor defined the concept. In schools, teachers try to convince students $\frac{1}{0}$ is infinity(strictly for mathematicians $\frac{1}{0}$ is undefined. Algebraists dealing with only finite operations say that 0 has no multiplicative inverse, analysts say that the limit of $\frac{1}{x}$ as x approaches zero doesnot exist, fluctuates between plus infinity and minus infinity) by showing them that $\frac{1}{0.1}=10, \frac{1}{0.01}=100, \frac{1}{0.001}=1000, \ldots$ Thus as denominator diminishes, the quotient grows so that when denominator becomes zero, the quotient is infinite. This is potential infinity. It is obvious that the sum 1 + 2 + 3+…+ n, whatever the natural number 'n' may be the sum without bounds and so does the sum of the first 'n' natural numbers though at any given stage it is finite. Therefore, it is reasonable to assume that the sum of all natural numbers is infinite and that is potential infinity. But the number of terms are there in the sum of all natural numbers and totality of all natural numbers treated as actual infinity.

There are several theories of the origin of universe that we live in. According to big bang theory, the universe started with a big explosion and it continues to expand in all directions. If we accept this theory, the life of universe is infinite. It is potentially infinite. At any given instant its age will be finite . There is another theory called the steady state theory, the universe has no beginning and it has no end. If we subscribe this theory, then again the life of universe is infinite, but it will be actual infinity.

Aristotle (384-322 B.C.E.) made a positive step toward clarification by distinguishing two different concepts of infinity, *potential infinity* and *actual infinity*. The latter is also called *complete infinity* and *completed infinity*. The actual infinite is not a process in time; it is an infinity that exists wholly at one time. By contrast, Aristotle spoke of the potentially infinite as a never-ending process over time. The word "potential" is being used in a technical sense. A potential swimmer can learn to become an actual swimmer, but a potential infinity cannot become an actual infinity. Aristotle argued that all the problems involving reasoning with infinity are really problems of improperly applying the incoherent concept of actual infinity instead of the coherent concept of potential infinity.

Even though Aristotle promoted the belief that "the idea of the actual infinite–of that whose infinitude presents itself all at once–was close to a contradiction in terms…," [2] During those two thousand years others did not treat it as a contradiction in terms. Archimedes, Duns Scotus, William of Ockham, Gregory of Rimini, and Leibniz made use of it. Archimedes used it, but had doubts about its legitimacy. Leibniz used it but had doubts about whether it was needed.

Here is an example of how Gregory of Rimini argued in the fourteenth century for the coherence of the concept of actual infinity:
If God can endlessly add a cubic foot to a stone–which He can–then He can create an infinitely big stone. For He need only add one cubic foot at some time, another half an hour later, another a quarter of an hour later than that, and so on ad infinitum. He would then have before Him an infinite stone at the end of the hour.[2]

According to Indian mythology, Lord Krishna once gave a magical kaleidoscope to Yudhishtir to divert his attention from his brothers. When Yudhishtir looked into kaleidoscope, he found another Yudishtir inside, who was looking through a kaleidoscope. The original Yudhishtir start counting his replicas and lost him self in counting. Even the fastest computer in this world cannot count the number of replicas of Yudhishtir in the magical kaleidoscope. Infinity is not a number which is difficult to count, it is something which is impossible to count. The difficulty doesn't lie in the individual's ability, it lies in the very nature of the concept.

Leibniz envisioned the world as being an actual infinity of mind-like monads, and in (Leibniz 1702) he freely used the concept of being infinitesimally small in his development of the calculus in mathematics. Euler started denoting infinity by symbol∞ which is now universally used. Renowned astrophysicst, Sir Arthur Eddington cautioned his fellow physicist that∞ was a love knot with which no physicist should get entangled. The term "infinity" that is used in contemporary mathematics and science is based on a technical development of this earlier, informal concept of actual infinity. This technical concept was not created until late in the 19thcentury.

The material basis of mathematical infinite can be understood only when it is considered in dialectical harmony with the finite. Every mathematical theory is bound by a compulsory requirement for internal formal consistency. Thus the problem arises, how to unite this requirement with essentially contradictory character of the reality of infinity. The solution to this problem consists of the following: when, in the limit theory consider infinite limits or in the theory of sets consider infinite powers, this does not lead to internal formal inconsistencies in the indicated theories only because these distinct special forms of mathematical infinity are extremely simplified schematized forms of the different aspects of infinity in the real world.

References

[1] F. Engels, *Anti-Duhring*, 1966,
[2] Moore 2001