

# Large-Scale Shell Model Investigation of Even-Even $^{66-76}\text{Ni}$ Isotopes

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**Abstract:** The structure of neutron-rich even-even  $^{66-76}\text{Ni}$  have been investigated by means of large-scale shell-model calculations. The energy levels for positive and negative parity states and the reduced transition probabilities  $B(E2;0 \rightarrow 2)$  are calculated by using the shell model code Nushellx by employing the effective interactions jun45 and jj44b. The results for excitation energies and reduced transition probabilities are compared with the recent available experimental data. Reasonable agreement is obtained for all isotopes under study.

**Keywords:** Shell model, energy levels, transition probabilities, Nushellx

## 1. Introduction

The nuclear shell model has been very successful in our understanding of nuclear structure: once a suitable effective interaction is found, the shell model can predict various observables accurately and systematically [1]. For light nuclei, there are several "standard" effective interactions such as the Cohen-Kurath [2]. The nickel isotopes ( $Z = 28$ ) cover three doubly-closed shells with number  $N = 38$ , to  $N = 48$  have been described by state-of-the-art shell model calculations with two recently available interactions using  $^{56}\text{Ni}$  as a core in the  $f_{5/2}p_{g_{9/2}}$  model space and therefore a unique testing ground to investigate the evolution of shell structure.

The  $^{66}\text{Ni}$  and its neighboring attracted the interest of recent research to answer the magicity versus superfluidity question related to doubly magic character of this nuclei [3,4,5,6,7]. Srivastava [8], performed shell model calculations for Ni, Cu and Zn isotopes by modifying the fpg interaction by modifying 28 two-body matrix element of the earlier interaction, the new interaction codenamed fpg9a is tuned for Cu isotopes and tested for Ni and Zn isotopes. Very recently F. Recchia et al. [9] investigated the level structure of  $^{68}\text{Ni}$  by two-neutron knockout and multi-nucleon-transfer reaction and they compare their experimental finding with the shell model calculations using several modern effective interactions. Y. Tsunoda [10], studied the shapes of neutron-rich exotic Ni isotopes by performing large-scale shell model calculations by the advanced Monte Carlo shell model (MCSM) in which the experimental energy levels are well described by using a single fixed Hamiltonian.

In this research we report the shell model calculations in the fpg-shell region for the even-even  $^{66-76}\text{Ni}$  isotopes by employing the modern jun45 [11] and jj44b [12] effective interactions, to test the ability of the present effective interactions to reproduce the experiment in this mass region.

## 2. Shell Model Calculation

The independent-particle Hamiltonian of an A-particle system can be written in terms two-particle interactions as [11],

$$H = \sum_{k=1}^A T_k + \sum_{k=1}^A \sum_{l=k+1}^A W(\vec{r}_k, \vec{r}_l) \quad (1)$$

where  $W(\vec{r}_k, \vec{r}_l)$  is the two-body interaction between the  $k^{\text{th}}$  and  $l^{\text{th}}$  nucleons. Choosing an average potential  $U(r_k)$ , the Hamiltonian becomes [11],

$$H = \sum_{k=1}^A [T_k + U(r_k)] + \sum_{k=1}^A \sum_{l=k+1}^A W(\vec{r}_k, \vec{r}_l) - \sum_{k=1}^A U(r_k) \quad (2)$$

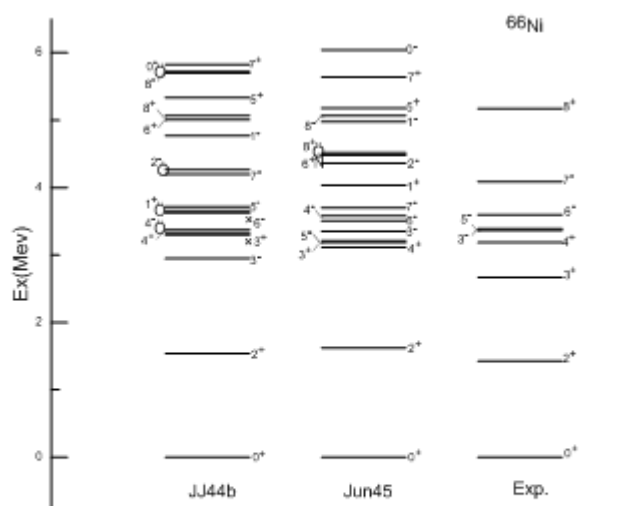


Fig. 1: Comparison of the excitation energies for  $^{66}\text{Ni}$  isotope with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

where the first term is identical to the independent-particle Hamiltonian, and the second and third account for the deviation from independent particle motion, known as the residual interaction. Separating the summations into core and valence contributions, eqn.(2) can be re-written [11],

$H = H_{core} + H_1 + H_2 + V(\vec{r}_1, \vec{r}_2)$  (3) In the above equation,  $H_{core}$  contains all of the interactions of nucleons making up the core,  $H_1$  and  $H_2$  are the single-particle contributions from particles 1 and 2, and  $V(\vec{r}_1, \vec{r}_2)$  is the residual interaction describing all interactions between particles 1 and 2 as well as any interaction with core nucleons. Inserting this form of the Hamiltonian into the Schrödinger equation yields an analogous expression for the energy [11],

$$E = E_{core} + E_1 + E_2 + \langle \Phi_{J,\tau} | V(\vec{r}_1, \vec{r}_2) | \Phi_{J,\tau} \rangle \quad (4)$$

Here,  $E_{core}$  is the binding energy of the core nucleus,  $E_1$  and  $E_2$  are defined as the single-particle energies of orbitals outside the core, and  $\langle \Phi_{J,\tau} | V(\vec{r}_1, \vec{r}_2) | \Phi_{J,\tau} \rangle$  is the residual interaction which needs to be defined by theory. It is important to note that the energy given by eqn. (4) is for pure configurations only. In principle, any close-lying state with the same total angular momentum J and total isospin will mix. The mixed eigen states are given by linear combinations of the unperturbed wave functions [11],

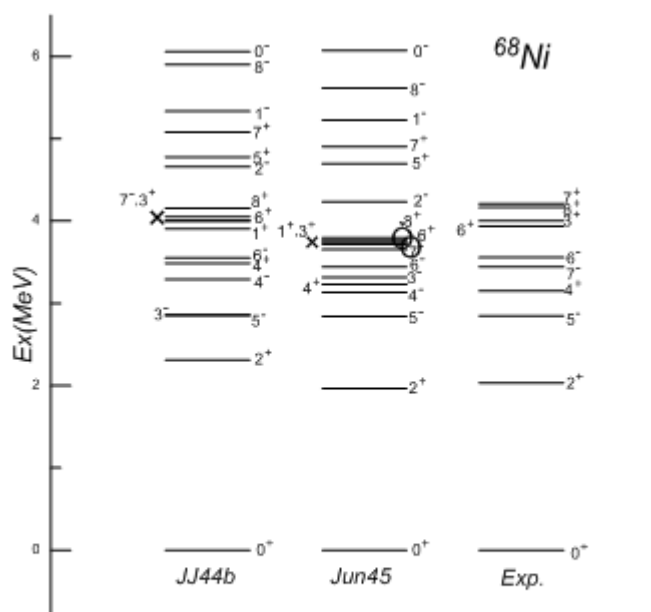


Fig. 2: Comparison of the excitation energies for  $^{68}\text{Ni}$  with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

$$(\Psi_{J,\tau})_p = \sum_{k=1}^g a_{kp} (\Phi_{J,\tau})_p \quad (5)$$

where g is the number of configurations that mix and the label p = 1, 2, ..., g. The coefficients  $a_{kp}$  fulfill the condition [4],

$$\sum_{k=1}^g |a_{kp}|^2 = 1 \quad (6)$$

Inserting eqn. (5) into the Schrödinger equation gives,

$$H(\Psi_{J,\tau})_p = E_p (\Psi_{J,\tau})_p \quad (7)$$

which leads to a system of linear equations.

### 3. Results and Discussions

#### 3.1 Excitation Energies

The core is taken at  $^{56}\text{Ni}$  for all isotopes under study with valence nucleons distributed  $p_{3/2}$ ,  $f_{5/2}$ ,  $p_{1/2}$  and  $g_{9/2}$  valence space by employing jun45 and jj44b effective interactions using the shell model code Nushellx. The comparison of the calculated energy levels for even-even  $^{66-76}\text{Ni}$  isotopes are presented in figures (1-6) employing jun45 and jj44b effective interactions with the recent available experimental data. Figure 1 presents the comparison of our calculations using the mentioned effective interactions for positive and

negative parity states, jj44b are in better agreement with the experiment than jun45 and both interactions are in reasonable agreement with the experimental data up to  $J^\pi = 8^+$ .

The calculations of the excitation energies for positive and negative parity states are shown in Fig.2 in which our theoretical calculations are in reasonable agreement with the experimental data, but none of the effective interactions are able to predict the correct energy level sequence for  $J^\pi > 4^+$ .

Figure 3 presents the comparison of the calculated energy levels for  $^{70}\text{Ni}$  nucleus with jun45 and jj44b effective residual interactions. The calculations using both effective interactions are in reasonable agreement the effective interaction jun45 are in better agreement than jj44b effective interaction with the experiment.

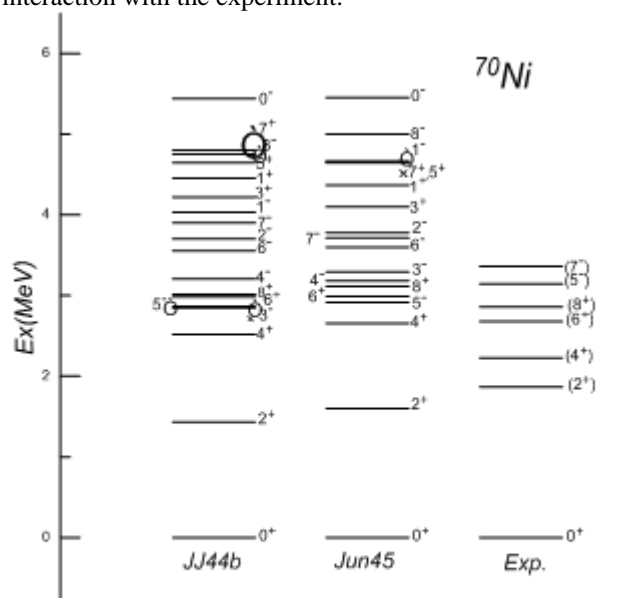


Fig. 3: Comparison of the excitation energies for  $^{70}\text{Ni}$  isotope with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

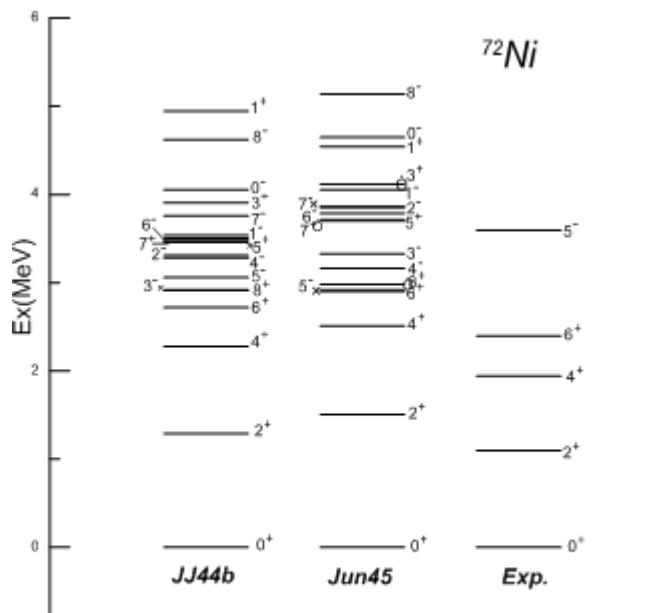


Fig. 4: Comparison of the excitation energies for  $^{72}\text{Ni}$  isotope with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

Figures 4, 5, and 6 shows same comparison for our theoretical energy levels using jun45 and jj44b residual effective interactions by considering the core at  $^{56}\text{Ni}$ . The comparison shows reasonable agreement with the experimental data and the best results achieved by using jj44b effective interaction for the isotopes  $^{74,74,76}\text{Ni}$ .

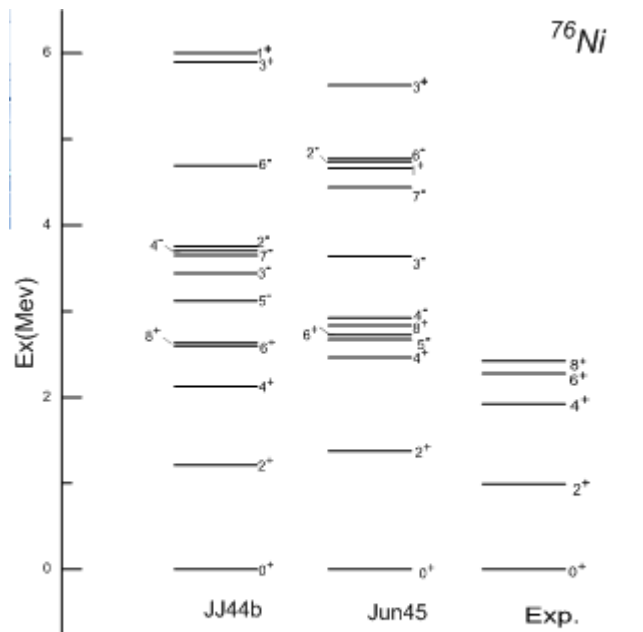


Fig. 6: Comparison of the excitation energies for  $^{76}\text{Ni}$  isotope with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

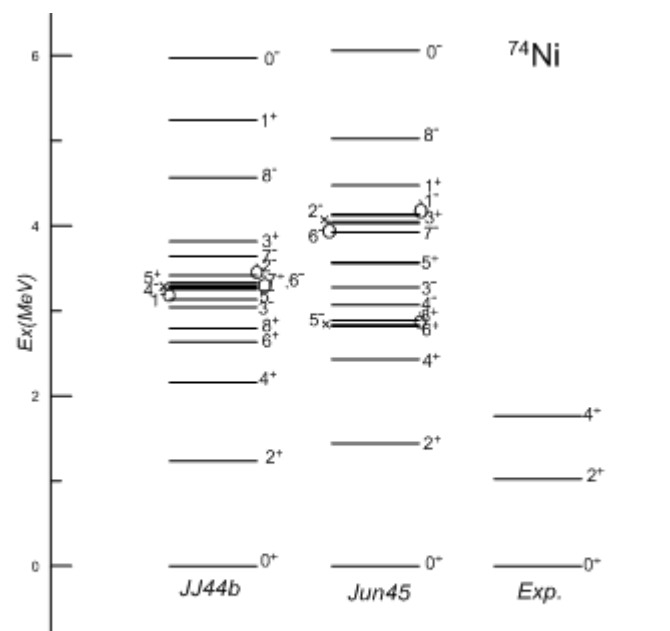


Fig. 5: Comparison of the excitation energies for  $^{74}\text{Ni}$  isotope with their corresponding experimental values using jun45 and jj44b effective interactions. Experimental data taken from ref. [14].

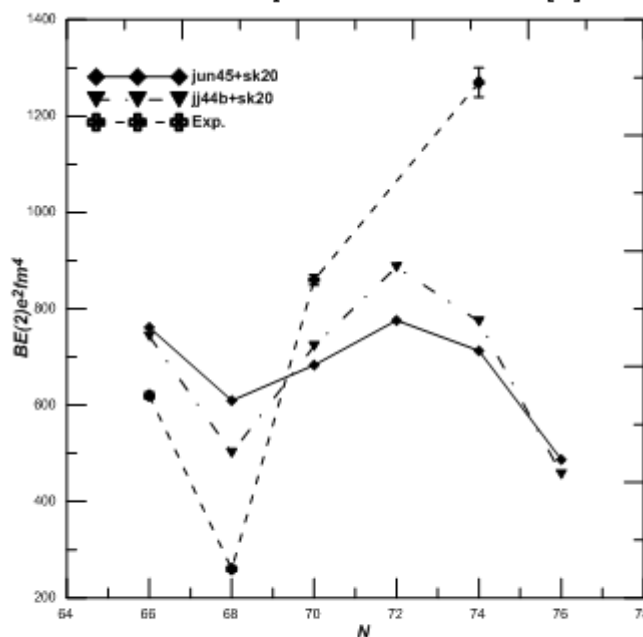


Fig.7: Comparison of the experimental transition  $B(E2; 0 \rightarrow 2)$  taken from Ref. [15] with the present theoretical work using jun45 and jj44b effective interactions. Experiment data taken from ref. [15].

### 3.2 Reduced Transition Probabilities

Since the transition rates represent a sensitive test for the most modern effective interactions that have been developed to describe fp-g9/2-shell nuclei. The transition strengths calculated in this work performed using the Skyrme potential (sk20) for each in-band transition by assuming pure E2 transition. Core polarization effect were included by choosing the effective charges for proton  $e_p^{eff} = 1.0 e$  and for neutron  $e_n^{eff} = 1.56e$ . Our theoretical results and experimental values [15] are tabulated in Table 1 and plotted as shown in Fig7..

**Table 1:** theoretical and experimental result transition  $B(E2; 0 \rightarrow 2)$   $^{66-76}\text{Ni}$  isotopes in units of  $e^2\text{fm}^4$ .

$J_i^\pi \rightarrow J_f^\pi$	Ni isotopes	EXP	jun45	jj44b
0 $\rightarrow$ 2	$^{66}\text{Ni}$	611	761.2	743.1
0 $\rightarrow$ 2	$^{68}\text{Ni}$	260	609.6	500.9
0 $\rightarrow$ 2	$^{70}\text{Ni}$	860	683.2	722.8
0 $\rightarrow$ 2	$^{72}\text{Ni}$	-	775.4	886.8
0 $\rightarrow$ 2	$^{74}\text{Ni}$	1270	712.9	773.8
0 $\rightarrow$ 2	$^{76}\text{Ni}$	-	487	456.9

#### 4. Conclusions

In the present work large scale shell model calculations have been performed for neutron rich even-even  $^{66-76}\text{Ni}$  isotopes. The energy levels and transition strength are calculated by employing jun45 and jj44b effective interactions. The effects of core polarization are taken into consideration by using the effective charges with fixed values for the entire set of isotopes. The facts that core-polarization contribution is considered through the effective charges are not always an adequate choice for the calculation for the reduced transition probabilities and the core polarization should be included through a microscopic theory. The systematic study of the reduced transition probabilities proves limitation to reproduce the experiment and microscopic theory for considering the effect of core polarization might improve the situation.

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#### Author Profile



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