manner in which the state variable change as a function of time may be thought of as a trajectory in a dimensional space, called the state space two-dimensional state space is sometimes referred to as the phase-plane when one state is derivative of the other.

In these systems the differential equation of equation (4) the state of a system is described by a set of second-order differential equation in terms of the variable (θ) and input variables (v), then the matrixes of this system is.

A= $\begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$

B = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

C= $\begin{bmatrix} 0 & 1 \end{bmatrix}$

D= $[0]$

We converting the state space to the transfer function are start with the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5)$$
$$y(t) = Cx(t) + Du(t) \quad (6)$$

Taken the Laplace transform

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

Or

$$(sI - A)X(s) = BU(s) + x(0)$$
$$X(s) = (sI - A)^{-1}BU(s) + (sI - A)^{-1}x(0)$$

And

$$Y(s) = [C(sI - A)^{-1}B + D]U(s) + C(sI - A)^{-1}x(0) \quad (6)$$

So

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \quad (7)$$

So the equation (6) contents of transfer function of system and the response to initial conditions, then the transfer function of the DC motor speed control are by using equation (7) is

$$G(s) = C(sI - A)^{-1}B$$
$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+2 & 2 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s}{s^2+2s+2} & \frac{-2}{s^2+2s+2} \\ \frac{1}{s^2+2s+2} & \frac{s+2}{s^2+2s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$G(s) = \frac{\theta}{V} = \frac{1}{s^2+2s+2} \quad (8)$$

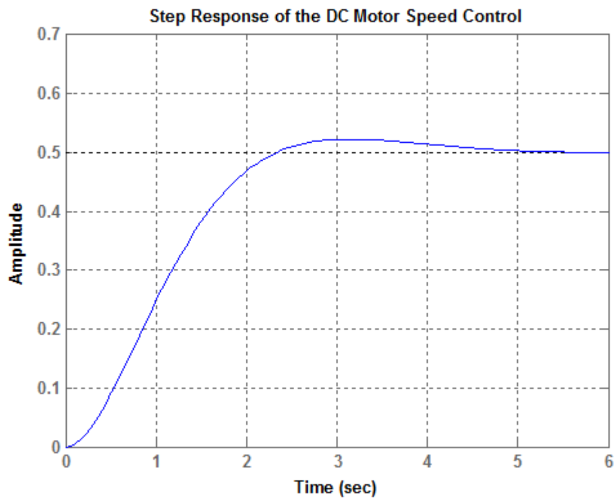

Figure 2: step response the DC motor

The figure 2 its response of the DC motor speed control there are bad response when the DC gain of the transfer function is one, so 0.5 is the final value of the output to an unit step input.
This corresponds to the steady-state error of 0.95 our one, quite large indeed.
Furthermore, the rise t_r time is 1.52second, and the settling time t_s is about 4.22 seconds.

4. InsertionPID Control

The PID Controller Filtering is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

Because most PID controllers are adjusted on-site, many different types of tuning rules have been proposed in the literature. Using these tuning rules, delicate and fine tuning of PID controllers can be made on-site.

The usefulness of PID controls lies in their general applicability to most control systems. In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful. In the field of process control systems, it is well known that the basic and modified PID control schemes have proved their usefulness in providing satisfactory control, although in many given situations they may not provide optimal control.

4.1 Characteristics of PID controllers

A proportional controller Kp will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral control Ki will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control Kd will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

Effects of each of controllersKp,Kd and Ki on a closed-loop system are summarized in the following table

Table 1: Characteristics of Closed Loop Response PID

CL response	Rise time	*overshoot*	Settling time	*SS- error*
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

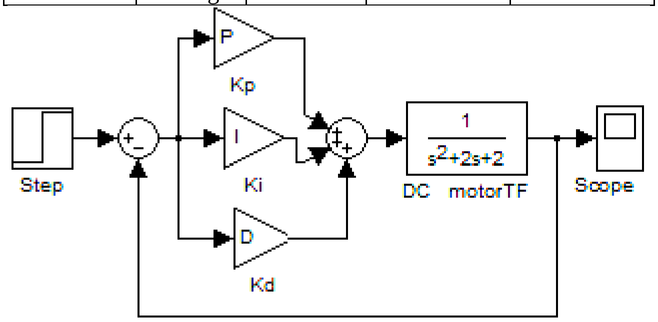

Figure 3: A block diagram of a PID controller

The proportional gain Kp times the magnitude of the error plus the integral gain Ki times the integral of the error plus the derivative gain Kd times the derivative of the error.

$$\mathrm{PID} = \mathrm{Kpe(t)} + \mathrm{Ki}\int \mathrm{e(t)dt} + \mathrm{Kd}\frac{\mathrm{de(t)}}{\mathrm{dt}} \quad (9)$$

This signal u (t) will be sent to the plant (process), and the new output θ (t) will be obtained. This new output θ (t) will be sent back to the sensor again to find the new error signal e (t). This signal u (t) will be sent to the plant, and the new output θ (t) will be obtained. This new output θ (t) will be sent back to the sensor again to find the new error signal e (t). The controller takes this new error signal and computes its derivative and it's integral again, Apply the Laplace transform of equation (9) and rewriting

$$\mathrm{PID} = \frac{\mathrm{Kd}\, s^2 + \mathrm{Kps} + \mathrm{Ki}}{s} \quad (10)$$

Finally the transfer function of the closed loop of DC motor speed control by PID filtering is multiply equation (8), (10) and feedback when the (Kd=100, Kp=250 and Ki=200) then the transfer function of the closed loop

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{100s^2 + 250s + 200}{s^3 + 102s^2 + 252s + 200} \quad (11)$$

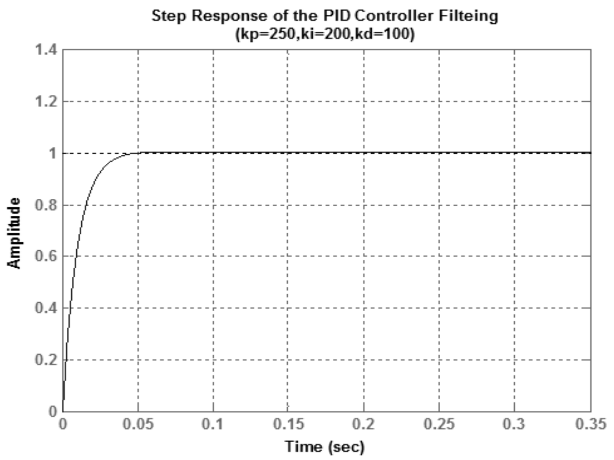

Figure 4: Step Response of DC motor By PID

The new characteristics of DC motor If the (Kd=100, Kp=250 and Ki=200) The characteristics of system is no over shoot, fast the rise time, decrease the settling time and the last value of steady state error is one shown the figure 4.

Table 2: Response of PID Controller

	Rise time	*overshoot*	Settling time	*SS- error*
PID	0.0217	0.407	0.0372	0

5. Transition Matrix

We now look at another technique for solving the state equations. Rather than using the Laplace transform, we solve the equations directly in the time domain using a method closely allied to the classical solution of differential equations.

We will find that the final solution consists of two parts that are different from the forced and natural responses.

The solution in the time domain is given directly by

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \quad (12)$$

$$x(t) = \varphi(t)x(0) + \int_0^t \varphi(t-\tau)Bu(\tau)d\tau \quad (13)$$

Where φ (t) = e^{At} by definition, and which is called the state-transition matrix. Notice that the first term on the right-hand side of the equation (12) is the response due to the initial state vector, x (0). Notice also that it is the only term dependent on the initial state vector and not the input. We call this part of the response the zero input response, since it is the total response if the input is zero.

The second term, called the convolution integral, is dependent only on the input, u (t), and the input matrix, B, not the initial state vector. We call this part of the response the zero-state response, since it is the total response if the initial state vector is zero. Thus, there is a partitioning of the response different from the forced natural response we have seen when solving differential equations. In differential equations, the arbitrary constants of the natural response are evaluated based on the initial conditions and the initial values of the forced response and its derivatives. Thus, the natural response's amplitudes are a function of the initial conditions of the output and the input, the zero-input response is not dependent on the initial values of the input and its derivatives. It is dependent only on the initial conditions of the state vector.

Then the state transition matrix

$$\varphi(t) = e^{At} = L^{-1}[(sI - A)^{-1}] \quad (14)$$

The initial condition are $x_1(0) = x_2(0) = x_3(0) = 1$ the time response of the system is given by solution of the differential equation in equation (12) and equation (13).

The matrixes of the equation (11)

$$A = \begin{bmatrix} -102 & -31.5 & -12.5 \\ 8 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [6.25 \quad 1.953 \quad 0.7813]$$

$$D = [0]$$

$$\varphi(t) = \begin{bmatrix} \frac{-1}{139608}\text{sum}(4\alpha^2+1879\alpha+4150)e^{\alpha t} & \frac{-1}{1116864}\text{sum}(1879\alpha^2+194800\alpha+422500)e^{\alpha t} & \frac{-25}{1116864}\text{sum}(83\alpha^2+8450\alpha+13400)e^{\alpha t} \\ \frac{1}{69804}\text{sum}(83\alpha^2+8450\alpha+13400)e^{\alpha t} & \frac{1}{279216}\text{sum}(4225\alpha^2+427192\alpha+675100)e^{\alpha t} & \frac{25}{139608}\text{sum}(67\alpha^2+6751\alpha+8434)e^{\alpha t} \\ \frac{-1}{34902}\text{sum}(67\alpha^2+6751\alpha+8434)e^{\alpha t} & \frac{-1}{279216}\text{sum}(6751\alpha^2+680152\alpha+846868)e^{\alpha t} & \frac{1}{279216}\text{sum}(4217\alpha^2+413434\alpha+387584)e^{\alpha t} \end{bmatrix}$$

$$a_{11} = \frac{-1}{139608}\text{sum}(4\alpha^2+1879\alpha+4150)e^{\alpha t}$$
$$a_{12} = \frac{-1}{1116864}\text{sum}(1879\alpha^2+194800\alpha+422500)e^{\alpha t}$$
$$a_{13} = \frac{-25}{1116864}\text{sum}(83\alpha^2+8450\alpha+13400)e^{\alpha t}$$
$$a_{21} = \frac{1}{69804}\text{sum}(83\alpha^2+8450\alpha+13400)e^{\alpha t}$$
$$a_{22} = \frac{1}{279216}\text{sum}(4225\alpha^2+427192\alpha+675100)e^{\alpha t}$$
$$a_{23} = \frac{25}{139608}\text{sum}(67\alpha^2+6751\alpha+8434)e^{\alpha t}$$
$$a_{31} = \frac{-1}{34902}\text{sum}(67\alpha^2+6751\alpha+8434)e^{\alpha t}$$
$$a_{32} = \frac{-1}{279216}\text{sum}(6751\alpha^2+680152\alpha+846868)e^{\alpha t}$$
$$a_{33} = \frac{1}{279216}\text{sum}(4217\alpha^2+413434\alpha+387584)e^{\alpha t}$$

When

$$\alpha = \text{Root Of}\,(s^3+102s^2+252s+200) \qquad (15)$$

For the given system with input u (t) =0 and $x_1(0)=x_2(0)=x_3(0)=1$ at any time t, the state transition matrix is computed as (0≤t≥2), and using lsim method command we found the (X_1, X_2, X_3)

```
>> A= [-102 -31.5 -12.5; 8 0 0; 0 2 0];
>> B= [16; 0; 0];
>> C= [6.25 1.953 0.7813];
>> D= [0];
>> X0= [1 1 1];
>> t= [0:0.2:2];
>> u=0*t;
>> [y, x] = lsim(A,B,C,D,u,t,x0)
```

Table 3: State Response at time

dt	X_1	X_2	X_3
0.0	1.0000	1.0000	1.0000
0.2	-0.3183	0.4891	1.3107
0.4	-0.2025	0.0775	1.4179
0.6	-0.1197	-0.1764	1.3937
0.8	-0.0621	-0.3190	1.2915
1.0	-0.0233	-0.3852	1.1487
1.2	0.0016	-0.4010	0.9901
1.4	0.0165	-0.3854	0.8320
1.6	0.0245	-0.3518	0.6841
1.8	0.0279	-0.3094	0.5517
2.0	0.0281	-0.2643	0.4370

```
>> subplot(3,1,1),plot(t,x(:,1));xlabel('time sec');ylabel('x1');grid;subplot(3,1,2),plot(t,x(:,2));xlabel('time sec');ylabel('x2');grid;subplot(3,1,3),plot(t,x(:,3));xlabel('time sec');ylabel('x3');grid
```

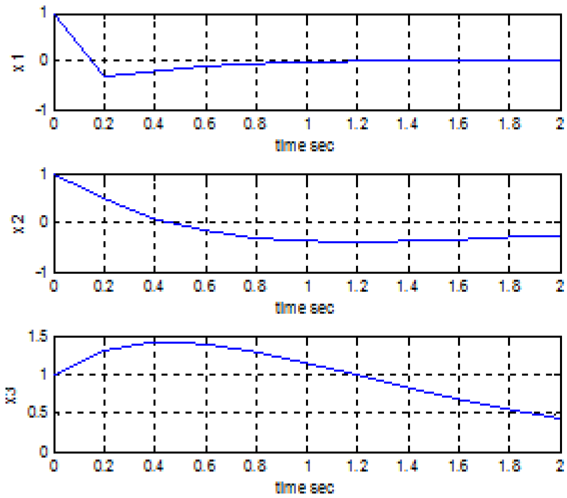

Figure 5: Lsim Method for (X_1, X_2, X_3)

6. Conclusion

The paper aimed to explain the improved of dealing with control systems in there phase variable form. The phase variable representation of the DC motor was derived form it is original transfer function. The state variable of the phase variable form ware computed at different is leased at time to check the behavior of the system.The graph of this behavior was clock also for the sans state variables. The results of plotting the state variable behavior match those obtained by direct computation. The pen feet it these is to show and asses the control design the specific of the behavior planes state variable.

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Author Profile

Musa Adam received the B.Sc in Electronics engineering from the EL-Masherq College for Science and Technology in 2008, And now, M.Sc. degrees in Control System Engineering from Al-Neelain University during 2013-2015. He was born in Sudan, he has researched and has interest in Control Systems, for one years he worked on National Electricity Corporation (Combined Cycle Power Plant Control and Instrumentation Duration: From 2009-2010). He now teaching assistant, Faculty of Engineering, EL-Mashreq College for Science & Technology – Electronics Engineering Program and Communication Engineering Program, December 2009 – Present. (Digital Electronics Circuits - Analogue Electronics Circuits – Electrical Circuits - Digital Communication – Analogue Communication – Instrument and measurement - Electronics Circuits – Electrical Circuits - Power Electronics Circuits).

Associate Professr **Muawia Mohamed Ahmed Mahmud** awarded the B.Sc degree in1987, M.Sc in 2000, and Ph D in 2006. He has taken many managerial responsibilities including Dean of Engineering Faculties, Head of Departments, and programs coordinator. He works now as Head of Control Engineering at Engineering Faculty in AL-Neelain University. He has a remarkable contribution in the research area in Sudan.