Analysis of the DC Motor Speed Control Using State Variable Transition Matrix

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Abstract: In this paper the dynamics in DC motor speed control system ware mathematically model and its phase variable for was obtained. The transition matrixes of state variable form was found and used in investigating the performance of the systems. The state variable values of system ware computed of difference time interference. The goal is to determine the state variables change with time, this was approving by graphically.

Keywords: DC motor, speed control, state variable, transition matrix.

1. Introduction

In general, in electric drives systems, the electric motor is coupled with the load through various mechanical transmission system, which can be characterized by transmission ratio, stiffness, and backlash, considerable case are the following assumptions.

A rigid coupling between the motor and load, the drive system shown in Figure 1, has a constant combined motor and load inertia of the rotor, (J).

The damping ratio of the mechanical system (b) will be to taken into account (is not shown in figure 1).

But all the designed is calculation with no load.



Figure 1: DC motor model and parameter

2. Methodology

To develop the mathematical modelling equations of the dc motor drive, and have to draw an explanatory figure1, (the chosen dc electric motor can be either with permanent magnets or with wound field excitation). Formulate the canonical form of state-space matrix of the dc motor model with the following state variables.

2.1 Design Matrix Model

The MATLAB code is expanded with the DC motor state model's matrices A, B, C, and D the provide the verification of the PID controller filtering.

2.2 PID Controller IMPACT

To design the state feedback assuming that all state are available, consider the open loop and closed loop phase lead compensator system to find the new characteristics equation and response.

2.3 Required Parameters

We will assume the following values for the physical parameters.

- Dc motor inertia of the rotor (j=1 k gm^2/s^2).
- Electromotive force constant (k=1 Nm/Amp).
- Electric resistance(r=1 ohm).
- Electric inductance (l=1 h).
- Input (v): source voltage=unit step.
- Output (θ): speed of shaft.

2.4 Mathematical Analysis

From the figure.1 above we can write the following equations based on Newton's law combined with Kirchhoff's law:

$$j\ddot{\theta} + b\dot{\theta} = ki$$
 (1)

$$li + Ri = v - k\theta$$
 (2)

Then the differential equation of the DC motor speed control

$$\ddot{\theta} + \left(\frac{jR+bl}{jl}\right)\dot{\theta} + \left(\frac{bR+k^2}{jl}\right)\theta = \frac{k}{jl}v$$
 (3)

Taken the unity confection, then the differential equation is:

$$\ddot{\theta} + 2\dot{\theta} + 2\theta = v$$
 (4)

3. State Space Phase Variable

The state of a system may be defined as the set of variables (called the state variable) which at some initial time t_o together with the input variables completely determine the behavior of the system for time($t \ge t_o$).

The state variables are the smallest number of states that are required to describe the dynamic nature of the system, and it is not a necessary constraint that they are measurable. The manner in which the state variable change as a function of time may be thought of as a trajectory in a dimensional space, called the state space two-dimensional state space is sometimes referred to as the phase-plane when one state is derivative of the other.

In these systems the differential equation of equation (4) the state of a system is described by a set of second-order differential equation in terms of the variable (θ) and input variables (v), then the matrixes of this system is.

$$A = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$$

$$R = [0]$$

$$C = [0 \ 1]$$

We converting the state space to the transfer function are start with the state equation

$$\dot{x}(t) = Ax(t) + Bu(t)$$
 (5)
 $y(t) = Cx(t) + Du(t)$ (6)

Taken the Laplace transform

$$sX(s) - x(0) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

Or

$$(sI - A)X(s) = BU(s) + x(0)$$

 $X(s) = (sI - A)^{-1}BU(s) + (sI - A)^{-1}x(0)$

And

$$Y(s) = [C(sI - A)^{-1}B + D]U(s) + C(sI - A)^{-1}x(0)$$
(6)
So

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$
(7)

So the equation (6) contents of transfer function of system and the response to initial conditions, then the transfer function of the DC motor speed control are by using equation (7) is

$$G(s) = C(sI - A)^{-1}B$$

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+2 & 2\\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1\\ 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s}{s^2 + 2s + 2} & \frac{-2}{s^2 + 2s + 2} \\ \frac{1}{s^2 + 2s + 2} & \frac{s+2}{s^2 + 2s + 2} \end{bmatrix} \begin{bmatrix} 1\\ 0 \end{bmatrix}$$

$$G(s) = \frac{\theta}{V} = \frac{1}{s^2 + 2s + 2}$$
(8)



The figure 2 its response of the DC motor speed control there are bad response when the DC gain of the transfer function is one, so 0.5 is the final value of the output to an unit step input.

This corresponds to the steady-state error of 0.95 our one, quite large indeed.

Furthermore, the rise t_r time is 1.52second, and the settling time t_s is about 4.22 seconds.

4. InsertionPID Control

The PID Controller Filtering is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

Because most PID controllers are adjusted on-site, many different types of tuning rules have been proposed in the literature. Using these tuning rules, delicate and fine tuning of PID controllers can be made on-site.

The usefulness of PID controls lies in their general applicability to most control systems. In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful. In the field of process control systems, it is well known that the basic and modified PID control schemes have proved their usefulness in providing satisfactory control, although in many given situations they may not provide optimal control.

4.1 Characteristics of PID controllers

A proportional controller Kp will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral control Ki will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control Kd will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

Effects of each of controllers *Kp*, *Kd* and *Ki* on a closed-loop system are summarized in the following table

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CL response	Rise time	overshoot	Settling time	SS- error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small	Decrease	Decrease	Small Change
	Change			
Step Ki DC motorTF Sc				TF Scope

Table 1: Characteristics of Closed Loop Response PID

Figure 3: A block diagram of a PID controller

The proportional gain Kp times the magnitude of the error plus the integral gain Ki times the integral of the error plus the derivative gain Kd times the derivative of the error.

$$PID = Kpe(t) + Ki \int e(t)dt + Kd \frac{de(t)}{dt}$$
(9)

This signal u (t) will be sent to the plant (process), and the new output θ (t) will be obtained. This new output θ (t) will be sent back to the sensor again to find the new error signal e (t). This signal u (t) will be sent to the plant, and the new output θ (t) will be obtained. This new output θ (t) will be sent back to the sensor again to find the new error signal e (t). The controller takes this new error signal and computes its derivative and it's integral again, Apply the Laplace transform of equation (9) and rewriting

$$PID = \frac{Kd s^2 + Kps + Ki}{s}$$
(10)

Finally the transfer function of the closed loop of DC motor speed control by PID filtering is multiply equation (8), (10) and feedback when the (Kd=100, Kp=250 and Kt=200) then the transfer function of the closed loop

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{100s^2 + 250s + 200}{s^2 + 102s^2 + 252s + 200}$$
(11)



Figure 4: Step Response of DC motor By PID

The new characteristics of DC motor If the (Kd=100, Kp=250 and Ki=200) The characteristics of system is no over shoot, fast the rise time, decrease the settling time and the last value of steady state error is one shown the figure 4.

Table 2:	Response of PID Controller	
		-

	Rise time	overshoot	Settling time	SS- error
PID	0.0217	0.407	0.0372	0

5. Transition Matrix

We now look at another technique for solving the state equations. Rather than using the Laplace transform, we solve the equations directly in the time domain using a method closely allied to the classical solution of differential equations.

We will find that the final solution consists of two parts that are different from the forced and natural responses.

The solution in the time domain is given directly by

$$x(t) = e^{\operatorname{At}} x(0) + \int_{0}^{t} e^{\operatorname{A}(t-\tau)} \operatorname{Bu}(\tau) d\tau \qquad (12)$$

$$x(t) = \varphi(t)x(0) + \int_0^t \varphi(t-\tau)Bu(\tau)d\tau \qquad (13)$$

Where $\varphi(t) = e^{At}$ by definition, and which is called the statetransition matrix. Notice that the first term on the right-hand side of the equation (12) is the response due to the initial state vector, x (0). Notice also that it is the only term dependent on the initial state vector and not the input. We call this part of the response the zero input response, since it is the total response if the input is zero.

The second term, called the convolution integral, is dependent only on the input, u (t), and the input matrix, B, not the initial state vector. We call this part of the response the zero-state response, since it is the total response if the initial state vector is zero. Thus, there is a partitioning of the response different from the forced natural response we have seen when solving differential equations. In differential equations, the arbitrary constants of the natural response are evaluated based on the initial conditions and the initial values of the forced response and its derivatives. Thus, the natural response's amplitudes are a function of the initial conditions of the output and the input, the zero-input response is not dependent on the initial values of the input and its derivatives. It is dependent only on the initial conditions of the state vector.

Then the state transition matrix

$$\varphi(t) = e^{At} = L^{-1}[(sI - A)^{-1}]$$
 (14)

The initial condition are $x_1(0) = x_2(0) = x_3(0) = 1$ the time response of the system is given by solution of the differential equation in equation (12) and equation (13). The matrixes of the equation (11)

	-102	-31.5	-12.5]	
A =	8	0	0	
	L O	2	0]	

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$$\mathbf{B} = \begin{bmatrix} \mathbf{16} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

C = [6.25 1.953 0.7813]

D = [0]





When

$$\alpha = \text{Root Of}(s^3 + 102s^2 + 252s + 200)$$
 (15)

For the given system with input u (t) =0 and $x_1(0) = x_2(0) = x_3(0) = 1$ at any time t, the state transition matrix is computed as $(0 \le t \ge 2)$, and using lsim method command we found the (X_1, X_2, X_3)

>> A= [-102 -31.5 -12.5; 8 0 0; 0 2 0]; >> B= [16; 0; 0]; >> C= [6.25 1.953 0.7813]; >> D= [0]; >> X0= [1 1 1]; >> t= [0:0.2:2]; >> u=0*t; >> [y, x] = lsim(A,B,C,D,u,t,x0)

Table 3:	State	Response	at time
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dt	X_{I}	X_2	X_3	
0.0	1.0000	1.0000	1.0000	
0.2	-0.3183	0.4891	1.3107	
0.4	-0.2025	0.0775	1.4179	
0.6	-0.1197	-0.1764	1.3937	
0.8	-0.0621	-0.3190	1.2915	
1.0	-0.0233	-0.3852	1.1487	
1.2	0.0016	-0.4010	0.9901	
1.4	0.0165	-0.3854	0.8320	
1.6	0.0245	-0.3518	0.6841	
1.8	0.0279	-0.3094	0.5517	
2.0	0.0281	-0.2643	0.4370	

>> subplot(3,1,1),plot(t,x(:,1));xlabel('time sec');ylabel('x1');grid;subplot(3,1,2),plot(t,x(:,2));xlabel('tim e

sec');ylabel('x2');grid;subplot(3,1,3),plot(t,x(:,3));xlabel('tim
e sec');ylabel('x3');grid



Figure 5: Lsim Method for (X₁, X₂, X₃)

6. Conclusion

The paper aimed to explain the improved of dealing with control systems in there phase variable form. The phase variable representation of the DC motor was derived form it is original transfer function. The state variable of the phase variable form ware computed at different is leased at time to check the behavior of the system. The graph of this behavior was clock also for the sans state variables. The results of plotting the state variable behavior match those obtained by direct computation. The pen feet it these is to show and asses the control design the specific of the behavior planes state variable.

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