

Design and Analysis of Phase Lead Compensator for DC Motor Speed Control Using State Space Forms

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Abstract: comparison to classical cascade control architecture in state space of DC motor speed control, the state feedback control offers advantages in terms of design complexity, hardware realization. This paper presents a methodical approach to state space control of a DC motor speed model and phase lead compensator of this model. The state space model identified from experimental data provides the basis for canonical form and Continuous-time model of state space design. The phase lead compensator is successfully applied and the closed loop behavior is evaluated on the project tested under unit step reference input.

Keywords: phase lead compensator, DC motor, speed control, state space.

1. Introduction

In general, in electric drives systems, the electric motor is coupled with the load through various mechanical transmission systems, which can be characterized by transmission ratio, stiffness, and backlash, Considerable cases are the following assumptions:

A rigid coupling between motor and load, the drive system shown in Figure.1 has a constant combined motor and load inertia of the rotor, (J).

The damping ratio of the mechanical system (b) will be taken into account. All the designed is calculation with no load.

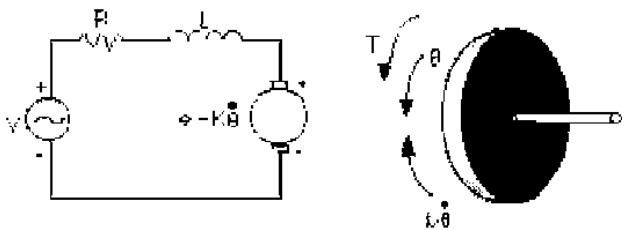


Figure 1: DC Motor Model

2. Methodology

2.1 Procedure

The design will include the following required steps:

2.1.1 Mathematical Model

To develop the mathematical modelling equations of the dc motor drive, and have to draw an explanatory figure.1 (the chosen dc electric motor can be either with permanent magnets or with wound field excitation). Formulate the canonical form of state-space matrix of the dc motor model with the following state variables.

2.1.2 Design Matrix Model

The MATLAB code is expanded with the DC motor state

model's matrices A, B, C, and D, the provide the verification of the phase lead compensator.

2.1.3 Phase Lead Compensator

To design the state feedback assuming that all state are available, consider the open loop and closed loop phase lead compensator system to find the new characteristics equation and response.

2.2 DC Motor and Load Parameters

2.2.1 Required Parameters

We will assume the following values for the physical parameters.

- Dc motor inertia of the rotor ($J=1 \text{ kg m}^2/\text{s}^2$).
- Electromotive force constant ($k=1 \text{ Nm/Amp}$).
- Electric resistance ($r=1 \text{ ohm}$).
- Electric inductance ($l=1 \text{ h}$).
- Input (v): source voltage=unit step.
- Output (θ): speed of shaft.

2.2.2 Desired Reference Input

The unit step reference input for all simulation.

2.3 Mathematical Analysis

From the figure.1 above we can write the following equations based on Newton's law combined with Kirchoff's law:

$$j\ddot{\theta} + b\dot{\theta} = ki \quad (1)$$

$$li + Ri = v - k\dot{\theta} \quad (2)$$

Taken the transfer function is:

$$s(js + R)\theta(s) = kiI(s) \quad (3)$$

$$(ls + R)I(s) = V - ks\theta(s) \quad (4)$$

$$G(s) = \frac{\theta}{V} = \frac{k}{(js + b)(ls + R) + k^2} \quad (5)$$

$$G(s) = \frac{\theta}{V} = \frac{\frac{k}{j_1}}{s^2 + \frac{j_2 + b_1}{j_1}s + \frac{b_2 R + k^2}{j_1}} \quad (6)$$

Taking the unity coefficients, then the transfer function of the DC motor speed is:

$$G(s) = \frac{\theta}{V} = \frac{1}{s^2 + 2s + 2} \quad (7)$$

2.4 State Space Representation

$$A = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 1]$$

$$D = [0]$$

The response by using the MATLAB had shown the figure.2.

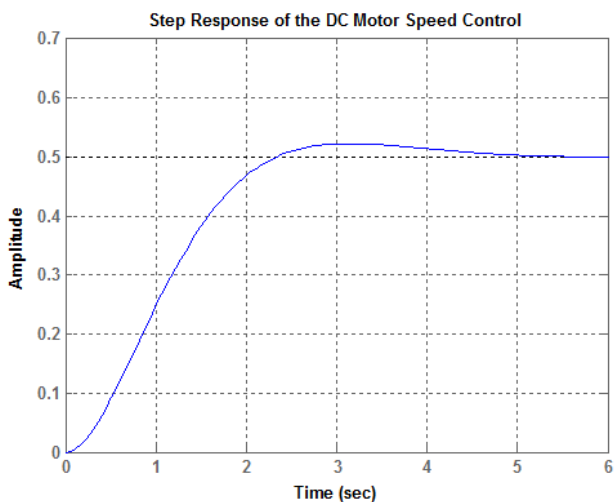


Figure 2: the response of DC motor state space model

2.5 Phase Lead Compensator

Are use to alter the response of a control system in order to accommodate the set design criteria. By introducing additional poles and/or zeros to a system, the response of the system will change significantly. The compensator is an additional device or a component to improve the system performance. (My be beside the original controller). Lead compensation alters the transient response of system. This includes overshoot, rise time (TR), settling time (TS), and peak time (TP). Many steps in design of phase-lead compensator:

Step1.

Choose gain K to satisfy steady-state requirements:
 Determine the required value of k for steady state error.
 For unit step $ess = 1 / (1 + kp)$
 $0.01 = 1 / (1 + kp)$ from which $kp \cong 1/0.01 = 100$
 Then $k=100$

Step2.

Draw Bode-diagram of KG(s) and find PM = (current phase margin):

Check the current phase margin of kG(s) (with k=0.01) from which the current, shown the figure.3.

PM = 11.5

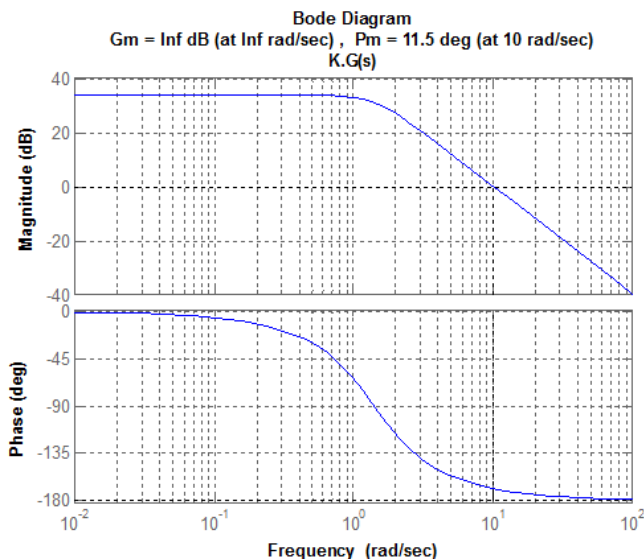


Figure 3: K.G(s) Bode diagram before Compensation

Step3.

Find the difference of phase margin = Required PM – Current PM, (Let the open-loop phase margin of at least 45°):
 So a difference of $45 - 11.5 = 33.5$ should be compensated.

Step4.

Give a margin of 5 degree and apply sin formula to find α :

$$\sin(38.5) = \frac{\alpha - 1}{\alpha + 1} \quad (8)$$

Then $\alpha = 4.26$

So the transfer functions of compensator:

$$G_c(s) = \frac{1 + 4.26 Ts}{1 + Ts} \quad (9)$$

Step5.

Find the new crossover frequency $\omega_c = \omega_{nc}$ at

$$\omega_c = (0.5) 20 \log(\alpha) \quad (10)$$

$$\omega_c = 6.29$$

And apply

$$T = \frac{1}{\omega_c \sqrt{\alpha}} \quad (11)$$

Now the value of T must be found:

We have ω_c of G_c at max phase shift the equation (10)

$$= (0.5) 20 \log \alpha = 20 \log 4.26 = 12.588 \text{ dB}$$

And we know meets the half of the magnitude of the max phase-lead shift = $(0.5) (12.588) = 6.29 \text{ dB (-ve)}$

Now from Bode $\omega_{nc} = 6.29 \text{ rad/sec}$ and hence the equation (11)

$$T = \frac{1}{6.29 \sqrt{4.26}}$$

Then

$$T = 0.077$$

Step6.

Construct $G_c(s)$:

Check the new cascaded $G_c(s).G(s)$ (and closed loop) Bode to ensure the required PM=47.7 so $G_c(s)$ will be is

$$G_c(s) = \frac{1 + 0.328s}{1 + 0.077s} \tag{12}$$

To check the effect of $G_c(s)$ see the Bode & response of the open loop with and without the compensator. There is no effect of phase margin in open loop, shown the figure.4 and figure.5.

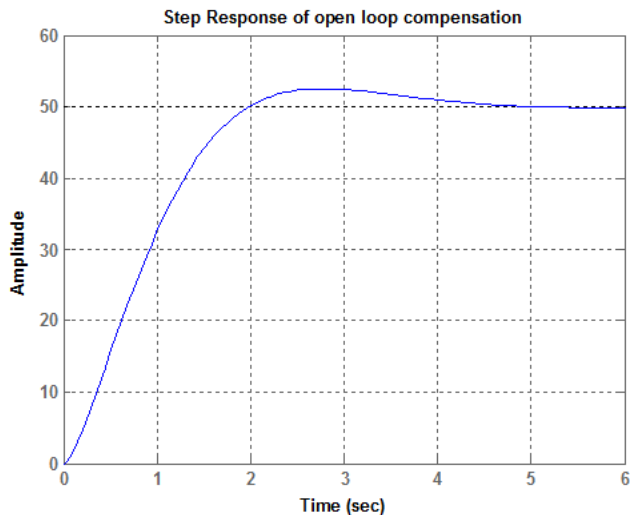


Figure.4: Step response for open loop after compensation of the state space (with $k=100$)

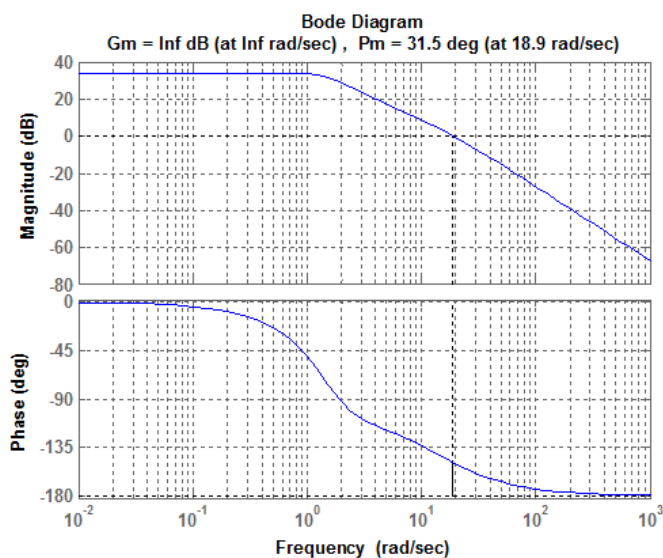


Figure 5: bode diagram of open loop compensation

In figure.4 Step response for open loop after compensation in the state space this includes Minimum overshoot in peak amplitude =52.5, Fast rise time (sec) (T_R) =1.34, decreased the settling time (T_S), and No steady-state error. And in figure.5 the phase margin in open loop compensator is 31.5. And to check the effected of $G_c(s)$ see the Bode diagram and responses of the closed loop with and without the compensator, shown the figure.6, and figure.7.

$$TF = \frac{32.8s + 100}{0.077s^4 + 1.154s^3 + 34.95s^2 + 102s} \tag{13}$$

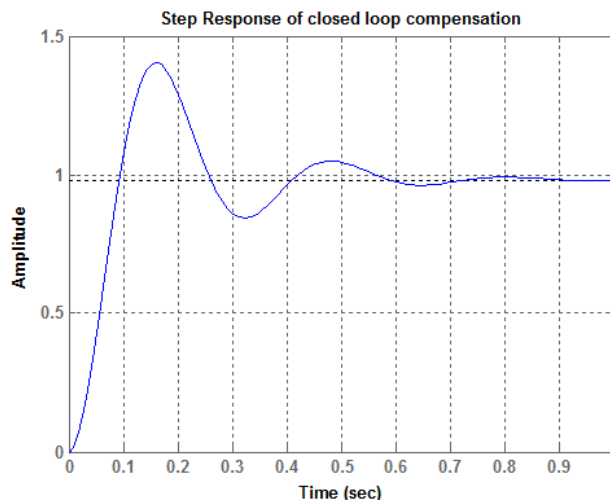


Figure 6: Step response of closed loop after compensation of state space (with $k=100$)

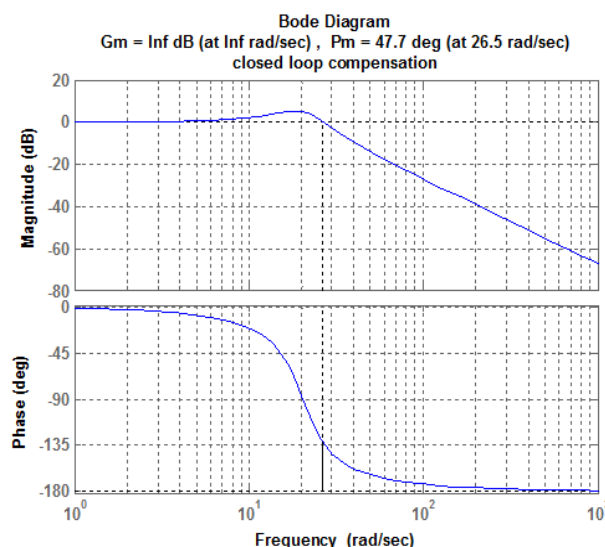


Figure 7: bode diagram of compensated closed loop

Now, we have obtained the system with Minimum overshoot in peak amplitude =1.4, fast rise time, and no steady-state error, shown the figure.6 and figure.7.

3. Conclusion

In this paper the state space of the DC motor system was obtained and the step response has shown bat system characteristics such as the steady state absent. A phase lead compensation was designed to improve its characteristics which was connected the DC motor system. The resulted have shown that adding the phase lead compensator has got a great effect on the DC motor performance in the closed loop phase lead compensator when the system with Minimum overshoot in peak amplitude is 1.4, fast the rise time is 0.06sec, decrease the settling time and the last value of steady state is 0.98.

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Author Profile



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