International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Profit Maximization of Unbalanced Fuzzy Transportation Problem

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Abstract: ***The aim of this paper is to find out the maximum profit cost of some commodities through a capacited network, when the supply and demand of nodes and the capacity and cost of nodes are represented as triangular fuzzy numbers. Using Yager's Ranking Method fuzzy quantities are transformed in to crisp quantities. Finally a numerical illustration is given to check the validity of the proposal.***

Keywords: Fuzzy Transportation problem, Triangular fuzzy number, Yager's Ranking method, Profit maximization , Vogel's Approximation Method, MODI method for Optimality Test.

1. Introduction

Transportation problem is a particular class of linear programming, which is associated with routine activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements. In other words, transportation problems deal with the transportation of a single product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the demand at destinations from the supply constraints at the minimum possible transportation cost. To reach this objective, we must know the quantity of available supplies and demand. In addition, we must know the location, to find the cost of transporting single homogeneous commodity from the place of origin to the destination. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centres. The objective of the transportation model is to determine the amount to be shipped from each source to each destination to maintain the supply and demand requirements at the lowest transportation cost.

This paper studies fuzzy transportation problem, and introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal or any LR fuzzy number. Thus, some fuzzy numbers are not directly comparable. First, here transform the fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by Yager's ranking method[6] which satisfies the properties of compensation, linearity and additivity, and then by using the classical algorithms , obtain the solution of the problem. This method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether ***maximize or minimize*** objective function.

2. Preliminaries

Zadeh [7] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

3. Fuzzy Set

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1].(ie) $A = \{(x, \mu_A(x); x \in X\}$,here $\mu_A : X \to [0,1]$ is a mapping called the degree of ***membership function*** of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈X in the fuzzy set A.These membership grades are often represented by real numbers ranging from [0,1].

4. Triangular Fuzzy Number

For a triangular fuzzy number A(x),it can be represented by A(a,b,c,;1) with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x \le b \\ 1, x = b \\ \dfrac{c-x}{c-b}, b \le x \le c \\ 0, \text{otherwise.} \end{cases}$$

$\alpha - \text{cut}$

The $\alpha - \text{cut}$ of a fuzzy number A(x) is defined as A(x) = $\{x / \mu(x) \ge \alpha, \alpha \in [0,1]\}$

Defuzzification

Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here use Yager's ranking to defuzzify the TFNs because of its simplicity and accuracy.

Yager's Ranking Technique

Yager's ranking technique [6] which satisfy compensation,linarity, additivity properties and provides results which consists of human intuition. If $\bar{a}$ is a fuzzy number then the Yager's ranking is defined by

$$R(\bar{a}) = \int_0^1 (0.5)(a_\alpha{}^L, a_\alpha{}^U)d\alpha$$

$$\text{where } (a_\alpha{}^L, a_\alpha{}^U) = \{(b-a)\alpha + a, c-(c-b)\alpha\}.$$

5. Numerical Example

Consider the fuzzy transportation problem for maximizing the profit. Due to difference in raw material cost and transportation cost, the profit for unit in rupees differs which is given in the table.

Table 1

(30,40,50)	(20,25,30)	(12,22,32)	(23,33,43)
(33,43,53)	(30,35,40)	(20,30,40)	(20,30,40)
(28,38,48)	(28,38,48)	(18,28,38)	(20,30,40)

Fuzzy availability of the product at sources are (75,100,125),(20,30,40),(60,70,80) and the fuzzy demand of the product at destinations are (30,40,50),(10,20,30),(55,60,65),(20,30,40) respectively. The fuzzy transportation problem is given by

Table 2

	D_1	D_2	D_3	D_4	Fuzzy supply
O_1	(30,40,50)	(20,25,30)	(12,22,32)	(23,33,43)	(75,100,125)
O_2	(33,43,53)	(30,35,40)	(20,30,40)	(20,30,40)	(20,30,40)
O_3	(28,38,48)	(28,38,48)	(18,28,38)	(20,30,40)	(60,70,80)
Fuzzy demand	(30,40,50)	(10,20,30)	(55,60,65)	(20,30,40)	

Since the problem is an unbalanced one, a dummy column with zero cost has to be added to make it balanced.
The balanced table becomes

Table 3

	D_1	D_2	D_3	D_4	D_5	Fuzzy supply
O_1	(30,40,50)	(20,25,30)	(12,22,32)	(23,33,43)	(0,0,0)	(75,100,125)
O_2	(34,44,54)	(30,35,40)	(20,30,40)	(20,30,40)	(0,0,0)	(20,30,40)
O_3	(28,38,48)	(28,38,48)	(18,28,38)	(20,30,40)	(0,0,0)	(60,70,80)
Fuzzy demand	(30,40,50)	(10,20,30)	(55,60,65)	(20,30,40)	(40,50,60)	

In conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form for

Max Z = R(30,40,50)x_{11}+ R(20,25,30)x_{12}+ R(12,22,32)x_{13}+ R(23,33,43)x_{14}+ R(0,0,0)x_{15}+ R(34,44,54)x_{21}+ R(30,35,40)x_{22}+ R(20,30,40)x_{23}+ R(20,30,40)x_{24}+ R(0,0,0)x_{25}+R(28,38,48)x_{31}+ R(28,38,48)x_{32}+ R(18,28,38)x_{33}+R(20,30,40)x_{34}+ R(0,0,0)x_{35}.
Applying Yager's ranking indices for the fuzzy costs

$$R(\bar{a}) = \int_0^1 (0.5)(a_\alpha{}^L, a_\alpha{}^U)d\alpha$$

$$\text{where } (a_\alpha{}^L, a_\alpha{}^U) = \{(b-a)\alpha + a, c-(c-b)\alpha\}$$

$$R(30,40,50) = \int_0^1 (0.5)(10\alpha + 30, 50 - 10\alpha)d\alpha$$

$$R(30,40,50) = \int_0^1 (0.5)(80)d\alpha = 40$$

Similarly
R(20,25,30)=25, R(12,22,32)=22 , R(23,33,43)=33, R(0,0,0)=0, R(34,44,54)= 44, R(30,35,40) = 35, R(20,30,40) = 30, R(20,30,40) = 30, R(0,0,0)=0, R(28,38,48)=38, R(28,38,48)=38,R(18,28,38)=28,R(20,30,40)= 30, R(0,0,0)=0..

Rank of all supply = R(75,100,125)=100, R(20,30,40) = 30,R(60,70,80)=70.

Rank of all demands =R(30,40,50) = 40,R(10,20,30) = 20, R(55,60,65) =60, R(20,30,40)=30,R(40,50,60) = 50 .
Substitute the values in fuzzy transportation problem we get the crisp transportation problem(profit matrix), which is the following table.

Table 4

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	40	25	22	33	0	100
O_2	44	35	30	30	0	30
O_3	38	38	28	30	0	70
Demand	40	20	60	30	50	

Initial Basic Feasible solution: Since the problem is of maximization one (profit matrix), subtract each element from the maximum element [44], and hence the minimum transportation table (loss matrix) can be obtained as

Table 5

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	4	19	22	11	44	100
O_2	0	9	14	14	44	30
O_3	6	6	16	14	44	70
Demand	40	20	60	30	50	

The initial basic feasible solution using Vogel's Approximation Method (VAM) is
x_{13} = 20, x_{14} = 30, x_{15} = 50, x_{21}= 30, x_{31} = 10, x_{32} = 20, x_{33}=40

Table 6

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	40	25	22**(20)**	33**(30)**	0**(50)**	100
O_2	44**(30)**	35	30	30	0	30
O_3	38**(10)**	38**(20)**	28**(40)**	30	0	70
Demand	40	20	60	30	50	

Initial solution is 22 x 20 + 33 x 30 + 0 x 50 + 44x 30+38x10 +38x20+ 28x40 = Rs.5010.

By Modified Distribution Method (MODI) ,the optimal solution is given by

Table 7

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	40 **(20)**	25	22	33**(30)**	0**(50)**	100
O_2	44**(20)**	35	30**(10)**	30	0	30
O_3	38	38**(20)**	28**(50)**	30	0	70
Demand	40	20	60	30	50	

Maximum profit cost = 40x20+33x30+0x50+44x20+30x10+38x20+28x50=Rs. 5130

6. Conclusion

In this paper, the transportation costs are considered as imprecise numbers by fuzzy numbers which are more realistic and general in nature. More over fuzzy transportation problem of triangular numbers has been transformed in to crisp transportation problem using Yager's ranking indices. Numerical examples show that by this method we can have the fuzzy optimal solution (maximum profit).This technique can also be used for solving profit maximization of balanced/unbalanced Fuzzy Assignment problems.

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