Performance Evaluation in State Estimation by Discrete Full Order UI Observer and Discrete Kalman Filter for L-1011 Aircraft

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Abstract: Unknown input full order observer using projection operator method is used to estimate five states of the fifth order lateral axis model of L-1011 system in discrete domain in noiseless and noisy environment in this paper. After that Kalman filter has been designed to estimate the state of L-1011 system when noise is considered. Finally error between the actual and estimated states of the system by UI observer and Kalman filter is compared. It has been shown that Kalman filter is good estimator than UI full order observer in noisy environment. The simulations have been done using MATLAB/Simulink Toolbox & the results of the simulations have been displayed in this paper.

Keywords: Unknown input observer (UIO), projection operator method, estimated state, Full order UI observer, Kalman filter.

1. Introduction

Observer and Kalman filter is widely used in estimation of different linear and non linear systems. Many authors have designed unknown input observer in different approach and Kalman filter for estimation of linear and non linear systems. Many significant works have been carried out over the past three decades on the construction of observers for linear and nonlinear systems in the control system domain. The observer was first proposed and developed by Luenberger in the early sixties of the last century [2],[3],[6]. Full order and reduced order observer was introduced to estimate the states of the linear and non linear systems[1],[21]. Sliding mode observers were proposed for uncertain systems[10],[15],[17]. Later, Sliding mode observer was introduced for robust model-based fault detection and isolation [17],[20]. Alexander Stotsky and Ilya Kolmanovsky designed an observer to estimate the unknown input from available state measurements in automotive control application [19]. Talel Bessaoudi, Karim Khemiri, Faycal Ben Hmida and Moncef Gossa [27] estimate the states of a linear discrete time systems using recursive least-square approach. Kalyana C. Veluvolu and Soh Yeng Chai designed a high gain observer with multiple sliding mode for state and unknown input estimations[26]. Simultaneous estimation [14] of states and unknown input in a class of nonlinear systems has been proposed by Q. P. Ha and H. Trinh. Thierry Floquet and Jean-Pierre Barbot designed a state and unknown input delayed estimator for discrete-time linear systems [25]. Abhijit Banerjee and Prof. G. Das [29] used the reduced order observer [28] to estimate the unknown input of a linear time invariant system. Ashis De, Abhijit Banerjee and Prof. G. Das [30] estimate the unknown input of an LTI system using full order observer constructed by the method of generalized matrix inverse [31].

R.E. Kalman published his famous paper [1] describing a recursive solution to the discrete-data linear filtering problem in the year 1960. In practice, linear system with white Gaussian noises is commonly taken as the standard model of a Kalman filter. The filter is said to be very powerful as it can estimate the state of a process even when the precise model of the system is unknown. Many researchers are using different Kalman filters like discrete Kalman filter , Extended Kalman filter[32], Unscented Kalman filter for estimation of tracking[18],[32], military system, flight control system, image and video tracking etc in a real time system.

Stefen Hui and Stanislaw H. Zak[21] designed both unknown input full order and reduced order observer using projection operator method in continuous domain. They did not estimate the states of the L-1011 system in noisy environment. M.S Das and S.K Singh estimates the state of L-1011 system in discrete domain in noisy environment using UI reduced order observer and Kalman filter[33]. In this paper a discrete full order UI observer using projection operator method and Kalman filter has been designed to estimate the states of L-1011 system in noisy environment. Finally accuracy in estimation by discrete Kalman filter and Full order UI observer is compared.

Here full order UI observer using projection operator has been designed in discrete domain. Projection operator method for full order observer is described in [22]. Here u1 is the known element and u2 is the unknown element of the input matrix u=[u1 u2] and also we divided the input matrix B corresponding to the known and unknown input as described above. From the division we get B1 and B2, where we consider B2 has full column rank. As the system output y is known it would seem reasonable to disintegrate the state x as follows,

\[ x= (I-MC)x + MCx 
= (I-MC)x + My \]

Where, M is a real matrix. So in the above equation only (I-MC)x is unknown part. We assume q=(I-MC)x and then we can write,

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\[ \dot{q} = (I - MC) \dot{x} \]

\[ = (I - MC)(Ax + B1u1 + B2u2) \]

\[ = (I - MC)(Ax + B1u1) + (I - MC)B2u2 \]

\[ = (I - MC)(Aq + AMy + B1u1) \]

If matrix M is chosen so that \((I - MC)B2 = 0\) then we can write the above equation as,

\[ \dot{q} = (I - MC)(Aq + AMy + B1u1) \]

To improve the convergence rate of the above equation we add an extra term to the right hand side of the equation and we obtain,

\[ \dot{q} = (I - MC)(Aq + AMy + B1u1 + L(y - Cq - CMy)) \]

Where, L is the gain matrix which depends upon desired pole location. Let, \(P = (I - MC)\) then if \(P\) is a projection, then it also said that \(P^2 = P\).

Full Order observer and Kalman filter is used to estimate the states of L-1011 aircraft in noisy environment and their performance in estimation is compared in this paper.

2. L-1011 Model in State Space Form

The fifth order linear system representing the lateral axis model of an L-1011 fixed wing aircraft with actuator dynamics removed is given below. Where the state vector is represented by

\[ x = [\begin{bmatrix} \theta \\ r \\ p \\ \beta \\ \gamma \end{bmatrix}, \text{ and the output vector } y = [\begin{bmatrix} r w_0 \\ \beta \end{bmatrix}], \text{ and inputs are } u = [\begin{bmatrix} \delta r \\ \delta \beta \end{bmatrix}] \]

\[ x_1 = \theta \text{ bank angle (rad/sec) }, x_2 = r \text{ yaw rate (rad/sec) }, x_3 = p \text{ roll rate (rad/sec) } , x_4 = \beta \text{ side slip-angle (rad) } , x_5 = \text{ washed out filter state and } \delta \beta = \text{ rudder deflection (rad) and } \delta \alpha = \text{ aileron deflection (rad) } . \]

\[ A = \begin{bmatrix} 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.3540 & -0.0042 & 1.5400 & 0.0000 \\ 0.0000 & 0.2490 & -1.0000 & 1.5400 & 0.0000 \\ 0.0386 & -0.9960 & -0.0003 & -0.1170 & 0.0000 \\ 0.0000 & 0.5000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \]

\[ B_2 = \begin{bmatrix} 0.0000 & 0.0000 \\ -0.7440 & 0.0320 \\ 0.3370 & -1.1200 \\ 0.0200 & 0.0000 \\ 0.0000 & 0.0000 \end{bmatrix} \]

\[ C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \text{ and } u = \begin{bmatrix} \delta r \\ \delta \alpha \end{bmatrix} = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} \]

3. Mathematical Model of full order UI Observer using Projection Operator Method

The dynamics of the full order observer in continuous-time domain as follows,

\[ \dot{q} = (I - MC)(Aq + AMy + B1u1) + L(y - Cq - CMy) \]

\[ = (T - MA)(Ax + B1u1 + (I - MC)B2u2) \]

\[ = (T - MA)(Ax + B1u1) + (T - MA)B2u2 \]

\[ \dot{q} = (T - MA)q + Ky + TB1u1 \]

\[ \dot{x} = q + My \]

Where, \(T = (I - MC), k_1 = L, k_2 = (TA - LC)M, K = k_1 + k_2 \text{. } T = (I - MC) \text{ is called the projector operator} \]

3.1 State Space Model of discrete full Order UI Observer

Previously we describe the dynamics of full order unknown input observer in continuous domain . Full order observer from continuous-domain is discretised assuming sampling time 0.01 sec.

\[ q(k+1) = A_{def} q(k) + B_{def} y(k) \]

\[ \dot{x}(k) = C_{def} q(k) + D_{def} y(k) \]

Where,

\[ A_{def} \]
This discrete UI observer is used to estimate the state of discrete system (fifth order lateral axis model of L-1011 Aircraft). Discrete UI order observer is applied on the L-1011 system to estimate the state in noisy and without noisy condition.

3.2. Estimation by full order UI observer in noisy environment

All systems are affected by unwanted noise that changes the actual output or actual state. Here it is assumed that the noise that is introduced with the system (Fifth order lateral axis model of L-1011) is zero mean white Gaussian in nature. UIO is applied on noisy system to estimate state of the system.

3.2.1 Simulation result

Actual state of the noisy system estimated state by reduced order observer using projection operator method in discrete domain has been shown in below.

4. State estimation by discrete Kalman Filter

The Kalman filter produces estimates of the true values of measurements and their associated calculated values by predicting a value, estimating the uncertainty of the predicted value, and computing a weighted average of the predicted value and the measured value. Weight is chosen such that uncertainty is minimum. Process equation can be written as

\[ x(k+1) = \Phi(k) x(k) + Bu(k) + w_k \]

Where \( \Phi_k \) is the state transition matrix of nxn dimension taking the state \( x_k \) from time \( k \) to time \( k + 1 \). The process noise \( w_k \sim N(0,Q) \). Where Q process noise covariance matrix defined by \( E(w_n^* w_k^{'}) = Q_k \) when \( n-k \) otherwise zero.

Measurement equation can be written as

\[ Z_k = Hx_k + v_k \]
where, $v_k$ is the measurement noise of $m \times 1$ vector which is denoted by $v_k \sim N(0,R)$ where $R$ measurement noise covariance matrix Filter estimate state in two phase one is predict and another is update. Predict Equations and update equations are given below. Predicted state (a priori state) -
\[
\hat{x}_{k|k-1} = A \hat{x}_{k-1|k-1} + Bu_{k-1}
\]
And predicted (a priori state) error covariance
\[
P_{k|k-1} = AP_{k-1|k-1}A^T + Q
\]
The current a priori prediction is combined with current observation information to refine the state estimate. Typically, the two phases alternate, with the prediction advancing the state until the next scheduled observation and the update incorporating the observation. Updates equations are Optimal Kalman gain:
\[
K_k = P_{k|k-1}H(HP_{k|k-1}H^T + R)^{-1}
\]
Updated (a posteriori) state estimate:
\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(Z_k - H\hat{x}_{k|k-1})
\]
and Updated estimate covariance:
\[
P_k = (I - K_kH)P_{k|k-1}
\]
The discrete Kalman filter is applied on noisy fifth order lateral axis model of L-1011 system to estimate the state of the system. Here it is assumed zero mean white Gaussian noise is mixed with system and output equation. It means process noise $W_k$ and measurement noise $V_k$ all are zero mean white Gaussian noise. So process noise $W_k$ and measurement noise $V_k$ can be represented as $W_k \sim N(0,Q_k)$ and $V_k \sim N(0,R_k)$ Where $Q_k$ is process noise covariance matrix and $R_k$ measurement noise covariance matrix. It is assumed that process noise covariance matrix $Q_k$ and measurement noise covariance matrix $R_k$ is identity matrix. Initial estimated states by Kalman filter is $\hat{x}(0) = 0$.

4.1 Simulation Results
Kalman filter is applied to estimate the states of the noisy system. Actual State and estimated states are shown in figure.

5. Comparison in Estimation between Kalman Filter and Full Order UI observer
In this thesis the comparison of Kalman Filter and Unknown Input Observer is shown. These two techniques i.e discrete Kalman Filter and discretised Unknown Input Observer (using projection operator approach) are applied on the same noisy system to estimate the state of the system. These two techniques are applied on the fifth-order lateral axis model of an L-1011 fixed wing aircraft model. Here it is considered that zero mean white Gaussian noise is introduced to the system. First Unknown Input observer and secondly Kalman Filter is applied on the same the noisy system to estimate the state and hence estimated error is calculated for each cases. Estimated state error by UI observer and The Kalman Filter is given below. There are five states in the system i.e five error plots is shown below. Five states are as follows $x_1=\Phi$, bank angle (rad), $x_2=yaw$ rate (rad/sec), $x_3=roll$ rate (rad/sec), $x_4=side$ slip-angle (rad) and $x_5=wash$ ed out filter state.

5.1 Simulation Result
Actual state of a noisy system, estimated state by UI observers using projection operator method and estimated state by Kalman filter in noisy environment are shown in the following figure.
6. Comparison of RMS error of Kalman filter and UI full order observer

Finally we have compared the RMS error of UI observer and RMS error of Kalman Filter and error covariance. Simulated outputs are shown in the following figures.

Figure 11: Comparison – Bank angle estimation (rad/sec)

Figure 12: Comparison- yaw rate estimation (rad/sec)

Figure 13: Comparison- roll rate estimation (rad/sec)

Figure 14: Comparison - side slip angle (rad)

Figure 15: Comparison- washed out filter state

Figure 16: Comparison - RMS of Bank angle error e1 of UI observer, Kalman Filter and Error covariance P1 (rad/sec)

Figure 17: Comparison - RMS of Yaw rate error e2 of UI observer, Kalman Filter and Error covariance P2 (rad/sec)

Figure 18: Comparison - RMS of Roll rate error e3 of UI observer, Kalman Filter and Error covariance P3 (rad/sec)

Figure 19: Comparison - RMS of side slip angle error e4 of UI observer, Kalman Filter and Error covariance P4 (rad)

Figure 20: Comparison - RMS of Washed out filter state error e5 of UI observer, Kalman Filter and Error covariance P5

7. Conclusion

A number of conclusions can be drawn from this paper. Full order unknown input observer is used here to estimate the state of the fifth order lateral axis model of an aircraft model of L-1011 in continuous and discrete domain. Also it is seen that UIO is capable of estimating state of a system in noisy environment in discrete domain. Here initial value of UIO observer q(0) is assumed to be zero. It is seen that estimated error using UIO using projection operator method is low in noise free system. It is a good estimator for noise free system. But it gives poor performance in estimation in noisy environment. Finally discrete Kalman filter is used to estimate the state of the fifth order lateral axis model of an aircraft model of L-1011 system in noisy environment. Kalman filter is good minimum variance estimator and it gives satisfactory result in noisy environment. It is easy to conclude from the fig.11 to fig. 15 and RMSE plot in fig16.
to fig20) that Kalman filter is more efficient estimator than unknown input observer in noisy field. In this paper process noise $W_i$ and measurement noise $v_i$ is assumed to be zero mean white Gaussian noise. The value of $P$, $Q$, $R$ is assumed as identity matrix. We have applied Kalman filter and full order UI observer to estimate state for the linear system only not for non-linear system. In future Extended Kalman Filter, Unscented Kalman Filter, UI observer, Sliding mode observer, particle filter and Disturbance observer will be applied to estimate states of non-linear systems and compared their performance in estimation.

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References


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