Modelling Exchange Rate Volatility Using Asymmetric GARCH Models (Evidence from Sierra Leone)

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Abstract: *This article examines the accuracy and forecasting performance of volatility models for the Leones/USA dollars exchange rate return, including the ARMA, Generalized Autoregressive Conditional Heteroscedasticity (GARCH), and Asymmetric GARCH models with normal and non-normal (student's t and skewed Student t) distributions. In fitting these models to the monthly exchange rate returns data over the period January 2004 to December 2013, we found that, the Asymmetric (GARCH) and GARCH model better fits under the non-normal distribution than the normal distribution and improve the overall estimation for measuring conditional variance. The GJR-GARCH model using the skewed Student t- distribution is most successful and better forecast the Sierra Leone exchange rate volatility. Finally, the study suggests that the given models are suitable for modeling the exchange rate volatility of Sierra Leone and the Asymmetric GARCH models shows asymmetric in exchange rate returns, resulting to the presence of leverage effect. Given the implication of exchange rate volatility, the study would be of great value to policy makers, investors and researchers at home and abroad in promoting development of the capital market and foreign exchange market stability in emerging economies.*

Keywords: Asymmetric GARCH Model, Exchange rates volatility, Leverage effect, Financial Market, Forecasting

1. Introduction

Modeling and forecasting of exchange rate volatility has become a relevant aspect and task in financial markets and the economy. The is because the volatility of exchange rate return can be seen as a measurement of the risk for investment, asset allocation and provides essential information for investors to make the correct decisions. It has gained considerable attention to market participants, investors, policy makers in order to fully understand the changes and the financial stability of an economy. Extensive research reflects its important in the volatility of investment analysis, security valuation, and risk management, trading and hedging strategy, stock market and monetary policy decision making.

In finance, researchers always put a lot of interests in modeling and forecasting volatility of exchange rate returns, failing which could lead to crisis and possible failure in the financial market. A crucial part of risk management is measuring the potential future losses of a portfolio of assets, and in order to measure these potential losses, estimates must be made for future volatilities and correlations. This research does not undermine the implications on financial markets exchange rate volatility. However, studies on exchange rate volatility are rear in emerging financial markets like Sierra Leone.

Every economy uses both monetary and fiscal policies for stabilization. In Africa economies, banks as well as other financial institutions plays important role as depositories and provides financial instrument for household wealth, maintaining payment system and used as vehicles for implementing monetary and fiscal policies for maintaining confidence in the financial sector and hence economic growth. They usually invest in foreign exchange instruments thus the need for accurate modeling and forecasting of volatility. This is the case for Sierra Leone, the Ministry of

Finance and the Bank of Sierra Leone are the authorities that manage fiscal and monetary policies respectively. The fiscal policy aim at enhancing domestic revenue mobilization, reduces the overall budget deficit as well as reduce domestic debt. Monetary policy focuses on maintaining price stability, consistent with high and sustainable economic growth. The Bank of Sierra Leone strengthens its implementation in the aspect of monetary policy through improving its liquidity forecasting , develop and deepen the interbank market, through proper liquidity management strategy and introduce a reserve requirement on foreign currency deposits to control the growth in the growing size of total commercial banks deposits. The Sierra Leone economy is very sensible to fluctuations in the Leones/USD exchange rate given the fact that the country generally import in US dollars. In general, studies have attempted to examine exchange rate and macroeconomic management (monetary and fiscal policies) in Sierra Leone, but there is no specific study on the modeling and forecasting of exchange rate volatility in Sierra Leone, this therefore motivate the study. This study is the first laboratory test case, which aims at fitting a volatility models to the Sierra Leone Leones and USA dollar exchange rate and to assess the accuracy of the forecasts produced.

Financial time series exhibit certain characteristic features such as volatility clustering, leptokurtic and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of Bollerslev (1986) has gained in popularity of its ability to address these issues but, sometimes fails to capture the fat-tail property of financial data. This has lead to the use of non-normal distributions (Student-t,Generalized Error Distribution and Skewed Student-t), within many non-linear extensions of the GARCH model which have been proposed by Exponential GARCH (EGARCH) of Nelson (1991), the Glosten, Jagannathan, and Runkle, the so-called GJR (1993) and the Asymmetric Power ARCH (APARCH) of Ding, Granger, and Engle (1993), to better model the fat-tailed (the excess kurtosis), skewness and leverage effect characteristics.

These models assume Gaussian normal distribution in modeling and forecasting the returns of financial time series (Le Baron, 1999). Modeling their volatility is important to asses these reserve assets in banks and in currency portfolio management.

Exporters and importers encountered transaction losses if not managed properly, and thus accurate forecasting models are needed to avoid these losses through hedging and to reduce the cost of foreign exchange transaction. The prominent aspects about time series models is about volatility modeling discussed by (Engle, 1995) and later extensions covered not only volatility (Andersen, 1997) but also excess kurtosis by (Baillie, 1989; Hsieh, 1989) and volatility clustering (Lux, 2000). Another important area is determining the distribution for returns generated by financial time series (Barndorff *et.al* 2001). The returns (shocks) which are created by the changes in prices of stock market or in currency market are to be modeled in order to enhance cost efficient management. Different ideas exist among researchers regarding the shape of the distributions of these returns. The Gaussian normal distribution is the most popular among them is symmetric and thus fail to captures the fat tails (Jensen, 2001), kurtosis and skewness properties (Arifovic and Gencay, 2000) which are widely prevalent in the returns generated by the financial asset price changes. Researchers and practitioners widely used non-normal distributions to draw random effect while analyzing the future exchange rates. The implication is that these various methods in modeling and forecasting volatility are rather in conclusive in nature. Therefore, in this paper, we use normal distribution, student t-distribution and skewed Student t- distribution for modeling Sierra Leone exchange rate volatility.

In view of the literature survey and relevant discussions it clearly reveals that no detailed studies were conducted on modeling of the exchange rate volatility in Sierra Leone. The aim of this article is to contribute to the existing literature on the grounds of modeling exchange rates volatility and find the model that best estimates the volatility of exchange rate, and to evaluate the forecasting performance of the Asymmetric GARCH models with the normal, student's –t and skewed-t distributions for the Leones/United States dollars exchange rates of Sierra Leone. To this end, we ask crucial and important empirical questions (i) how best the model fits exchange rate volatility in Sierra Leone? (ii) why do we need to forecast exchange rate volatility? (iii) how best the study will inform policy marker and international development organizations who are assisting in the development and promotion of monetary and fiscal policies? These therefore also motivate the study, as it is expected that the outcomes of the study would be relevant to academics, policy makers, investors, international organizations such as the World Bank, the International Monetary Fund (IMF) and foreign governments that are interested in facilitating trade and enhance development of the capital market and foreign exchange market in emerging economies. Since stable exchange rate volatility is conventionally required to enhance sustainable economic growth and development of emerging markets in an economy.

The rest of the article is structured as follows: In section 2, is Literature review, section 3 set out the methodology and model specifications, section 4 presents the empirical results and discussions. Finally, conclusion is presented in section 5.

2. Literature Review

This section reviews the body of existing knowledge on the connection between exchange rate volatility and forecasting in an economy.

There are various research and opinions on modeling the volatility of exchange rate in a given economy. Conditional heteroscedasticity models for time series have play a vital role in financial forecasting, risk management and its designs to make financial decisions on the basis of the observed price of asset. Volatility is an important parameter in risk assessment and management and it changes as the market prices of financial products change. To capture the importance of negative returns GJR-GARCH and DGE -GARCH models introduces the leverage parameter. This paper incorporates the GARCH, GJR-GARCH and DGE -GARCH models and integrates with ARMA to compute the return and forecasting exchange rates. Though these models have been thoroughly researched in the last two decades still a large gap is uncovered in the practical application as they all model volatility individually and they come out with their findings. The volatility and ARMA models ultimately ends up in forecasting the financial time series like share prices and exchange rates (Guillaume et. al, 1997) which are actively pursued not only for buying and selling decisions but also for protecting the asset portfolios that is carried out for satisfying the investors, regulators, governments and other development partners who invest in these financial instruments.

Ryan & Worthington (2004) used the GARCH in Mean to assess the impact of market, interest rate and foreign exchange rate risks on the sensitivity of Australian bank Stock Returns. Olowe (2009) examined the volatility of Nigerian Naira / Dollar Exchange rate by fitting six univariate GARCH models with student's t innovations using monthly data and found that the best performing models are the Asymmetric Power ARCH and TS - GARCH. According to Hung-Chung et al. (2009) shown that the GARCH model with an underlying leptokurtic asymmetric distribution outperforms one with an underlying normal distribution for modeling volatility of the Chinese Stock Market and similar studies undertaken by Wilhelmsson (2006) uses nine possible error distributions to model the volatility of the Standard & Poor's 500 stock index with the leptokurtic distributions working out best and also established that the use of fat tailed error distributions within a GARCH (1,1) framework leads to improved volatility forecasts. In Ghana, Adjasi (2008) examined the impact of exchange rate volatility on the local Stock Exchange using an Exponential GARCH model for their purpose and observe that there is a negative relationship between the exchange rate volatility and stock market returns. Balaban (2004) compared the forecasting performance of symmetric and asymmetric GARCH models with the US Dollar/Deutsche Mark returns series was filtered using an AR (1) process and the GARCH (1, 1), GJR-GARCH(1,1) and EGARCH(1,1) volatility equations are used. The author found that the EGARCH model performs better in producing out of sample forecasts with the GARCH (1, 1) closely following whereas the GJR-GARCH fares worst. Also Aggarwal (1981) explore the

relationship between changes in the dollar exchange rates and change in indices of stock prices using monthly data from 1974 to 1978 on stock prices and effective exchange rates for the USA and found that there is a positive correlation and stronger relationship exist in the short run than in the long run among the variables examined. Giovannini and Jorion (1987) also arrived with similar results of Aggarwal (1981) in case of the USA. Rahman and Uddin (2009) examined the relationship between exchange rates and stock prices for three South Asian countries (Bangladesh, India and Pakistan) and found negative relationship among the variables.

In accordance to (Ken Johnston, 2000) established that exchange rates is more important, but less studied variable when compared to shares, bonds and equities. Financial time series tend to be non-stationary (Hamilton, 1994) meaning that additional data will not only change the mean but also the variance, which is an impediment in forecasting. The argument of non stationary nature is taken care of by natural logarithm differencing. Few studies prove that the return distributions are not-perfectly normal and they are either skewed or with leptokurtic property with fat tails (Lux, 1998) and show t- distribution pattern. Financial time series risk management is concerned about the negative returns at the left tail of a distribution (Beltratti, 1999) and they are to be quantified for effective hedging decisions. Our paper is application oriented and it provides estimation and compares the forecasting accuracy of the three models that forecast the exchange rate volatility (Leones/USA dollar) of the Sierra Leone economy.

A vast number of empirical studies which examine the exchange rate volatility in both developed and developing countries this includes , Marten (2001), McKenzie (1997), McKenzie and Mitchell (2002), Sanchez-Fung (2003), Tse (1998), Andersen and Bollerslev (1998), Vilasuso (2002), Baillie and Bollerslev (1989), Bollerslev, Engle, and Nelson (1994), (Taylor, 1986; Andersen, 1994), and Beine, Laurent and Lecourt (2000). The economic theory suggests that interest rates, inflation, money supply, price level and other macro elements are important variables in understanding the operations of the economy and also for predicting the volatility and trends in exchange rates. According to (Kutty, 2010) suggested that a lower stock price may lead to currency depreciation. There has been considerable attention on whether the exchange rates and stock prices have any empirical relationship. Particularly, this issue has become more attractive among researchers, investors and policy makers after the Asian Financial Crises (1997) and Global Financial Crises (2009). Empirically, it is argued that if the exchange rates and stock prices are inter-related then it is possible to prevent such crises by looking at the direction of causality, given that the causality runs from stock prices to exchange rates then the policy makers should keep an eye on stabilizing the stock markets by enforcing the desirable economic policies.

Based on the overall literature survey indicates that there are studies which have been attempted to modeled exchange rates volatility in both developed and developing economies as well as emerging financial markets. The empirical results of these studies are mostly inconclusive in nature. In view of the above literature survey, the purpose of present study is to contribute to the existing literature on the grounds of modeling exchange rates volatility using asymmetric GARCH models. To the best of our knowledge, in the Sierra Leone context there are some studies have attempted to study the determinants of the real exchange rate in general, and there is no specific study on modeling the volatility of exchange rates using asymmetric GARCH models, in particular. Hence, this motivates us to provide a modeling analysis of exchange rates volatility in the Sierra Leone perspective. The main limitation is the availability of data coverage in terms of in-depth analysis with respect to time. Despite this limitation the data obtain for this study from (January 2004 to December 2013) is appropriate to carry out an empirical study in modeling exchange rate volatility in the context of Sierra Leone.

3. Methodology

In this section, we briefly present the models specification, conditional distributions and forecasting criterias as well as data set we use to model the Leones/US Dollars exchange rate returns volatility in the Sierra Leone economy. This article analyses the volatility of the Sierra Leone exchange rate using various volatility models such as Autoregressive Moving Average (ARMA), GARCH, the Glosten,Jagannathan and Runkle(GJR) GARCH, Asymmetric Power Autoregressive conditional Heteroskedasticity APARCH model of Ding et.al (1993) as well as the conditional distributions such as normal, Student-t and skewed student's-t distributions. In this study three different criteria's, Mean Squared Error (MSE), Mean Absolute Error (MAE) and Adjusted Mean Absolute Percentage Error (AMAPE) are used to evaluate the forecasting performance for the conditional heteroscedasticity models.

3.1 ARMA Model

The ARMA model was established by Box and Jenkins (1970) in order to fit data so as to remove the linear dependence in the series and to obtain the residuals which are uncorrelated. The series $\{y_t\}$ satisfy ARMA (p, q), then $\{y_t\}$ can be described as

$$y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \vartheta_j \varepsilon_{t-j} + \varepsilon_t,$$

where ϕ_i and ϑ_j ,for $i = 1, \cdots, p$ and $j = 1, \cdots, q$ are parameters. The $E(\varepsilon_t) = 0$ and $\mathrm{var}(\varepsilon_t) = \sigma^2$.

3.2 GARCH Model

Empirical evidence has shown that a high ARCH order has to be obtained to catch the dynamics of the conditional variance. The Generalized ARCH (GARCH) model introduced by Bollerslev (1986) tends to address this issue. It is based on an infinite Autoregressive Conditional Heteroscedasticity (ARCH) specification and it allows reducing the number of estimated parameters by imposing non-linear restrictions on them. The standard GARCH (p,q) model specification can be expressed as follows:

$$y_t = x_t \theta + \varepsilon_t, \quad t = 1, 2 \cdots T, \ \varepsilon_t \ \square \ N\left(0, \sigma_t^2\right) \quad (3.1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (3.2)$$

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$\omega > 0, \alpha \geq 0,\ \beta \geq 0,\ \varepsilon_t$ is wide stationary if and only if $\alpha + \beta > 1$.

The mean equation (3.1) can be expressed as a function of exogenous variables with error terms. The ε_t is uncorrelated but its conditional variance σ_t^2 is changing over time as the function of the past errors defined in Engle (1982), and then generalized by Bollerslev (1986) who extended the ARCH model to have longer memory and more flexible lag Structure. From (3.2) ω is a constant term, ε_{t-i} is the ARCH term and σ_{t-j}^2 is the GARCH term. This model is widely used in forecasting and modeling the conditional heteroscedasticity in financial time series analysis.

3.3 The APARCH Model

The Asymmetric Power Autoregressive Conditional Heteroskedasticity (APARCH) model was introduced by Ding et al.,(1993). It changes the second order of the error term into a more flexible varying exponent with an asymmetric coefficient takes the leverage effect into account. A time series $\{ y_t, x_t : t = 1, 2, \cdots \}$ satisfies a linear model with APARCH errors, given by

$$y_t = x_t \xi + \varepsilon_t, \quad t = 1, 2 \cdots T \quad (3.3)$$

The mean equation of (3.3) can be written as $y_t = E\left(y_t \mid F_{t-1} \right) + \varepsilon_t$, where $E\left(y_t \mid F_{t-1} \right) = x_t \xi$ such that $x_t \in F_{t-1}$ and $F_t : \sigma \{ y_s, x_{s+1}, \varepsilon_s \ s \leq t \}$ where F_t implies the information set at time t and $E\left(y_t \mid F_{t-1} \right)$ is the conditional mean of y_t given F_{t-1} the information set till time $t-1$. The ε_t term in (1) represent the residual returns which are the innovation of the time series process. .

The conditional variance equation of APARCH (p, q) can be written as

$$\sigma_t^2 = \left\{ \omega + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \right\}^{\frac{2}{\delta}}, (3.4)$$

Where $\omega > 0, \delta > 0, \alpha_i \geq 0, -1 < \gamma_i < 1, i = 1, \cdots, p,\ \beta_j \geq 0,\ j = 1, \cdots, q$. Here

p and q denotes the number of lagged ε^2 terms and σ^2 terms. The parameters $\omega, \alpha_i, \gamma_i, \beta_j$ and δ are to be estimated where γ_i , reflects the leverage effect or the asymmetric response parameter. Positive and negative information with regards γ_i will result in different levels of effect on the price volatility of the exchange rate. As a result, when γ_i is positive it implies negative information that indicates stronger impact on the price volatility of the financial asset than positive information. The conditional variance σ_t^2 is specified by a constant term ω, the parameters $\alpha_i \geq 0$ and $\beta_i \geq 0$ are required to ensure that it is strictly positive. α_i and β_j are weights assigned to the lagged squared returns and lagged variances respectively, which have an impact on the conditional variance. They provide information about volatility of prices of stocks from previous period's in order to have explanatory powers on current volatility of market prices in the financial market. The parameter δ measures the volatility spillover effect based on

the information in financial market. The volatility is directly linked with the rate of information flow between the financial markets which will cause changes of prices in the market.

The APARCH (p, q) process is stationary and entails a general class of models which includes special cases as ARCH by Engle (1982), GARCH by Bollerslev (1986), TS-GARCH by Taylor and Schwert (1986 cited in Ding et al., 1993), GJR-GARCH by Glosten et al. (1993 cited in Ding et al., 1993), and TARCH by Zakoian (1994 cited in Ding et al., 1993).

The GJR-GARCH and DGE-GARCH models are discussed.

a) GJR-GARCH
When $\delta = 2$ we have

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right)^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

For
$$\omega > 0, \alpha_i \geq 0, -1 < \gamma_i < 1, (i = 1, 2, \cdots, p), \beta_j \geq 0, (j = 1, 2, \cdots, q)$$

b) DGE-GARCH

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta ,$$

where
$$\omega > 0, \delta > 0, \alpha_i \geq 0, -1 < \gamma_i < 1, (i = 1, 2, \cdots, p), \beta_j \geq 0, (j = 1, 2, \cdots, q)$$

3.4 Distribution Assumptions

Empirical evidence shows that financial time-series often exhibits non-normality patterns such as excess kurtosis and skewness. It may be expected that excess kurtosis and skewness displayed by the residuals of conditional heteroscedasticity models will be reduced when a more appropriate distribution is used. In this paper: the normal distribution, the Student-t distribution and the skewed Student-t distribution are considered in order to take into account the skewness, excess kurtosis and heavy-tails of return distributions. It is clear that Student-t and skewed Student-t distributions exhibit heavy-tails.

3.4.1 Normal Distribution
The normal distribution is widely used in estimating and forecasting GARCH models. If the error term follows a Gaussian, the log-likelihood function of the standard normal distribution is given by

$$L_T = \ln \prod_t \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{\varepsilon_t^2}{2\sigma_t^2}} = -\frac{1}{2} \sum_{t=1}^{T} \left[\ln(2\pi) + \ln\left(\sigma_t^2\right) + z_t^2 \right] ,$$

Where $z_t = \dfrac{\varepsilon_t}{\sigma_t}$ is independently and identically distributed

(i.i.d) and the $E[z_t] = 0,\ \text{var}[z_t] = 1$ and T is the number of observation with regards the return series.

3.4.2 Student's t-Distribution
The student's t-distribution is used for fitting GARCH model as suggested by Bollerslev (1987) for the standardized error to better capture the observed fat tails in the return series. It is symmetric around mean zero. The log-likelihood function is of the form

$$L_T = T \left\{ \ln \left[\Gamma \left(\frac{v+1}{2} \right) \right] - \ln \left[\Gamma \left(\frac{v}{2} \right) \right] - \frac{1}{2} \ln \left[\pi(v-2) \right] - \frac{1}{2} \sum_{t=1}^{T} \left[\ln \left(\sigma_t^2 \right) + (1+v) \ln \left(1 + \frac{z_t^2}{v-2} \right) \right] \right\}$$

Where $v > 2$ is the shape parameter, $2 < v \le \infty$ and $\Gamma(.)$ is the gamma function. The lower v is, the fatter the tails.

3.4.3 Skewed Student's t-distribution

Fernandez and Steel (1998) as cited in Aberg et al. (2008) provides analysis of the student-t distribution by adding a skew parameter ξ which was applied by Lambert and Laurent (2000,2001) to the GARCH model. The log-likelihood function for the skew t-distribution is given by

$$L_T = T \left(\ln \Gamma \left(\frac{v+1}{2} \right) - \ln \left(\frac{v}{2} \right) - \frac{1}{2} \ln \left(\pi(v-2) \right) \right) + \ln \left(\frac{2}{\xi + \frac{1}{\xi}} \right) + \ln(s) -$$

$$\frac{1}{2} \sum_{t=1}^{T} \left(\ln \left(\sigma_t^2 \right) + (1+v) \ln \left(1 + \frac{(sz_t + m)^2}{v-2} \xi^{-2I_t} \right) \right),$$

here ξ is the skew parameter or asymmetry parameter, v is the degree of freedom of the distribution and

$$I_t = \begin{cases} 1 & \text{if } z_t \ge -\frac{m}{s} \\ -1 & \text{if } z_t < -\frac{m}{s} \end{cases}, \quad m = \frac{\Gamma\left(\frac{v+1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \quad \text{and} \quad s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1 \right) - m^2}.$$

3.5 Data

This study used the monthly data on exchange rates of the Leones against the USA dollars (Le/USD) of the Sierra Leone economy obtained from Central Bank of Sierra Leone from January 2004 to December 2013 and then transformed into logarithmic return series. The corresponding transform price series into monthly logarithmic return are calculated by using the formula: $r_t = \log(E_t) - \log(E_{t-1})$. Where E_t is the exchange rate and r_t denotes the returns.

4. Empirical Result and Discussions

In this section, we present the empirical results as well as discussions of estimation results we obtained to account for the Leones/US Dollars exchange rate returns volatility in Sierra Leone. The analysis was done using the $R-$ package 3.0.3 to provide empirical results of the Sierra Leone monthly exchange rate prices data against United States dollars. The parameter estimation method that we choose is the Maximum Likelihood Estimation (MLE), and estimates the models with the given distributional assumption to determine the best performance of forecasting model of exchange rate volatility of Sierra Leone examined.

Some summary statistics for the monthly exchange rate returns (Le/USD) are displayed in Table 1

Table 1: Summary Statistics of Sierra Leone monthly Exchange rate Returns (rt) (Leones/ USA ($))

Sample size	Mean	Median	Var.	S.Dev.	Min.	Max.	Skew.	Kurt.	Jarque Bera Test	P-value
119	0.0043	0.0013	0.0010	0.0101	-0.0140	0.0603	2.2696	8.7555	502.449	2.20e -16

The summary statistics of this study is presented in table 1. This indicates that the returns series have a daily positive mean of (0.0043) while the daily volatility is (0.0101), without loss of generality the mean grows at a linear rate while the volatility grows approximately at a square root rate. The lowest monthly returns correspond to (-0.0140) and the best monthly exchange rate returns is (0.0603). The returns series of the exchange rate shows positive skewness. This implies that the series is flatter to the right. The kurtosis value is higher than the normal value of perfectly normal distribution in which value for skewness is 'zero' and kurtosis is 'three' and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic. The results of this study reveal that, the series is not normally distributed. Our empirical result is consistent with the Jarque-Bera (JB) tests obtain above which is used to assess whether the given series is normally distributed or not. Here, the null hypothesis is that the series is normally distributed. Results of JB test find that the null hypothesis is rejected for the return series and suggest that the observed series are not normally distributed.

Table 2 presents the parameter estimation results of ARMA (1, 1) - GARCH (1, 1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1) models with the normal, student's-t and skewed Student-t distributions and their corresponding p-values. The results show that the parameters estimated in these three models are all significant under the given conditional distributions except for the coefficients of Mu under the three conditional distributions for the GARCH and GJR-GARCH model which are not significant. Under the normal distribution, the sum of the GARCH parameter estimates $(\alpha_i + \beta_j)$ is greater than 1, implying that the volatility rate model is strictly stationary while that of DGE-GARCH model is less than 1, which indicates that the model is well fitted. For the student's- t distribution, the sum of the GARCH parameter for GJR-GARCH model is less than 1, which indicate that the volatility is limited and the data is stationary and the model is well fitted, while in the case of the GARCH and DGE-GARCH models the sum of the GARCH parameters is greater than 1. Similarly the sum of the GARCH parameter is less than 1 for the GJR-GARCH and DGE-GARCH models with the skewed Student t-distribution which also show that the shocks in volatility is limited and stationary and the model is well fitted, the sum is greater than 1 in case of the GARCH model. The leverage effect term (gamma) in both the GJR model and the DGE is statistically significant but it is negative, implying that negative shocks results to a higher next period conditional variance than positive shocks of the same sign, it indicates that the bad news (negative shocks) effect the volatility more than the good news. The table shows that the estimated δ of the DGE-GARCH model under the normal distribution is 0.743 which is significantly different from 2(GJR-GARCH) and decreases from 0.6087 to 0.5869 as the conditional distribution changes to student's- t distribution and skewed Student-t distribution respectively.

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Table 2: Parameter Estimation of the ARMA (1, 1)-GARCH (1, 1), GJR (1, 1) and DGE (1, 1) Models with the Conditional Distributions

	GARCH			GJR-GARCH			DGE-GARCH		
Conditional Distribution	Normal	Student t	Skewed Student t	Normal	Student t	Skewed Student t	Normal	Student t	Skewed Student t
Mu(μ)	6.39e -05	2.28e -05	3.55e -06	8.13e -05	5.31e -05	7.06e -05	1.85e -04	6.31e -05	1.59e -04
	(0.4066)	(0.7471)	(0.9646)	(0.3490)	(0.4772)	(0.4754)	(0.00028)	(0.01047)	(0.00506)
ar1 (ϕ_1)	0.8126	0.8314	0.8339	0.8587	0.8717	0.8737	0.8259	0.8832	0.8657
	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)	(< 2e-16)
Ma1 (ϑ_1)	-0.702	-0.6524	-0.6544	-0.735	-0.6941	-0.6955	-0.6239	-0.6733	-0.6661
	(7.40e-08)	(7.00e-08)	(2.51e-08)	(1.69e-11)	(5.39e-13)	(5.35e-13)	(< 2e-16)	(< 2e-16)	(< 2e-16)
Omega (ω)	9.36e-07	4.63e-07	4.73e-07	8.79e-07	3.73e-07	3.70e-07	1.73e-03	1.93e-03	2.90e-03
	(0.0288)	(0.0221)	(0.0024)	(0.0049)	(0.0018)	(0.0158)	(0.0111)	(0.0161)	(0.0015)
Alpha ($\alpha 1$)	0.8543	0.7119	0.7941	0.4424	0.3156	0.2666	0.3443	0.2089	0.1849
	(0.00260)	(0.03810)	(0.00719)	(0.02803)	(0.01463)	(0.02678)	(0.00608)	(0.00011)	(0.00012)
Beta($\beta 1$)	0.3891	0.5325	0.5294	0.5596	0.6839	0.7012	0.6515	0.8182	0.8178
	(0.000217)	(7.92e -05)	(5.81e -05)	(0.0121)	(4.97e-14)	(9.08e -13)	(3.23e -06)	(< 2e-16)	(< 2e-16)
Gamma($\gamma 1$)				-0.3166	-0.4532	-0.5137	-0.5878	-0.143	-0.146
				(0.00308)	(0.01608)	(0.00237)	(0.01174)	(< 2e-16)	(< 2e-16)
Delta (δ)				2	2	2	0.7439	0.6087	0.5869
							(0.00677)	(0.00660)	(0.00574)
Shape (v)		4.181	3.728		4.11	4.401		4.263	6.065
		(0.0205)	(0.0276)		(0.00159)	(0.0444)		(0.0134)	(0.0101)
Skew (ξ)			0.9374			1.049			1.144
			(1.24e-14)			(5.12e-09)			(2.62e-13)

Note: The coefficients reported as shown in the table are the maximum likelihood estimates of the parameters and the p-values are in parentheses for the ARMA (1, 1) - GARCH (1, 1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1), models. The estimation results of the models with the conditional distributions, including log-likelihood value, the Box-Pierce statistics of lags 10, 15 and 20 of the standardized and squared standardized residuals, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the ARCH test and their respective p-values are listed in Table 3. Comparing the log-likelihood, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values among these models GARCH and GJR-GARCH models better estimate the exchange rate return series than the DGE-GARCH model with the skewed Student t-distribution assumption gives better results. The results also show that, the skewed student t-distribution and the student's t-distribution outperforms the normal distribution, with the skewed Student t-distribution outperforms the other two conditional distributional assumptions discussed in this paper. Among these models, GJR-GARCH with skewed Student t-distribution gives the highest log-likelihood value of 459.820. The AIC and BIC values of the GARCH and GJR-GARCH models under the three conditional distribution gives the lowest values when compared to the DGE-GARCH model and that the GJR-GARCH model with the student's t-distribution provides the smallest values of AIC (-7.5929) and BIC (-7.4061) respectively, this implies that GJR-GARCH model under the student's t-distribution provides a better fit for the monthly exchange rate returns according to this criterions.

Table 3: Analysis of standardized residuals and fitted parameters

	GARCH			GJR-GARCH			DGE-GARCH		
Conditional Distribution	Normal	t-distribution	Skewed t-distribution	Normal	t-distribution	Skewed t-distribution	Normal	t-distribution	Skewed t-distribution
Log likelihood	454.414	457.862	457.975	456.237	459.779	459.820	302.537	437.113	437.790
Jarque-Bera Test	6.301	38.953	46.364	5.340	32.229	27.613	15883.960	52492.640	41332.130
	(0.0043)	(0.0000)	(0.0000)	(0.0069)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Ljung-Box Test R (Q10)	14.388	12.348	11.784	12.871	9.881	10.198	18.758	18.016	15.894
	(0.1560)	(0.2625)	(0.2998)	(0.2310)	(0.4510)	(0.4233)	(0.5227)	(0.0647)	(0.1027)
Ljung-Box Test R (Q15)	25.435	24.204	23.980	25.041	22.720	22.879	14.599	24.782	23.255
	(0.0644)	(0.0617)	(0.0654)	(0.4939)	(0.0903)	(0.0867)	(0.4807)	(0.0630)	(0.0789)
Ljung-Box Test R (Q20)	28.536	25.962	25.694	28.583	25.400	25.709	16.113	27.582	25.968
	(0.0973)	(0.1671)	(0.1762)	(0.0963)	(0.1865)	(0.1756)	(0.7096)	(0.1197)	(0.1669)
Ljung-Box TestR^2(Q10)	7.670	7.432	6.834	7.077	5.692	5.935	15.249	14.809	11.075
	(0.6610)	(0.6842)	(0.7411)	(0.7181)	(0.8405)	(0.8207)	(0.1232)	(0.1392)	(0.3517)
Ljung-Box TestR^2(Q15)	14.436	19.101	19.079	13.758	16.138	16.343	28.621	18.968	14.865
	(0.4928)	(0.2092)	(0.2102)	(0.5439)	(0.3730)	(0.3596)	(0.0553)	(0.2152)	(0.4612)
Ljung-Box TestR^2(Q20)	18.211	22.239	22.086	16.783	19.278	19.649	30.397	21.422	17.356
	(0.5735)	(0.3277)	(0.3359)	(0.6670)	(0.5038)	(0.4801)	(0.0637)	(0.3727)	(0.6297)
LM Arch Test	10.486	13.978	14.093	8.194	11.988	11.933	19.308	16.457	11.842
	(0.5734)	(0.3021)	(0.2948)	(0.7698)	(0.4467)	(0.4511)	(0.0814)	(0.1712)	(0.4585)
AIC	-7.5364	-7.5775	-7.5626	-7.5502	**-7.5929**	-7.5768	-4.9502	-7.1952	-7.0721
BIC	-7.3962	-7.4040	-7.3758	-7.3867	**-7.4061**	-7.3666	-4.7634	-6.9850	-6.8386

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Note: The table shown the t-statistics and p-values are in parentheses for ARMA (1, 1)- GARCH(1,1),GJR(1,1) and DGE(1, 1) models.(AIC) represent Akaike Information Criterion, (BIC) is Bayesian Information Criterion (BIC), Ljung-Box Test R (Standardized Residuals and Ljung-Box TestR^2 (Square Standardized Residual)

The Jarque-Bera statistic to test the null hypothesis of whether the standardized residuals are normally distributed. The results presented in table 3 show that the standardized residuals are leptokurtic and the Jarque-Bera statistic strongly rejects the hypothesis of normal distribution which means that the fat-tailed asymmetric conditional distributions outperform the normal for modeling and forecasting the Sierra Leone exchange rates volatility returns. The Ljung Box tests for the residuals have p-values that are statistically not significant indicating that no serial correlation exists. The Ljung-Box statistics for up to twentieth-order serial correlation of squared residuals are not significant suggesting that no significance correlation exist. As for the LM-ARCH test the results reveals that the conditional heteroskedasticity that existed in the exchange rate returns time series have successfully removed, indicating that no significant appearance of the ARCH effect.

4.1 Forecasting

The forecasting ability of the GARCH models has been discussed precisely by Poon and Granger (2003). We use the R- package 3.0.3 with the "ets" function to evaluate a ten step ahead forecast using 119 observations for the monthly exchange rate returns. The forecasts are evaluated using three different measures which provide robustness in choosing the optimal predicts models for the return series. We consider the following measures.

1). Mean Squared Error (MSE): It quantifies the difference between values with regards the estimator and the true values of the quantity being estimated within a given sample. It is defined as follows

$$MSE = \frac{1}{h+1}\sum_{t=s}^{S+h}\left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2$$

2). Mean absolute error (MAE): It takes into consideration the average of the absolute value of the residuals. It is similar to the MSE but is less sensitive to large errors.

$$MAE = \frac{1}{h+1}\sum_{t=S}^{S+h}\left|\hat{\sigma}_t^2 - \sigma_t^2\right|$$

3). Adjusted mean absolute percentage error: Adjusted Mean Absolute Percentage Error (AMAPE) is a measure based on percentage errors.

$$AMAPE = \frac{1}{h+1}\sum_{t=S}^{S+h}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 - \sigma_t^2}\right|$$

Here h is the number of lead steps, S the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual variance.

Table 4: Forecasting Analysis for the Exchange rate returns with the Conditional distributions

Exchange rate returns (Le /USA($))	GARCH			GJR-GARCH			DGE-GARCH		
	Normal	Student-t	Skew-t	Normal	Student-t	Skew-t	Normal	Student-t	Skew-t
MSE	0.033	0.032	0.030	0.023	0.021	0.0202	0.029	0.0312	0.0335
MAE	0.574	.524	0.500	0.354	0.327	0.322	0.414	0.400	0.493
AMAPE	0.672	0.662	0.652	0.621	0.612	0.604	0.650	0.649	0.624

Note: MSE denotes the Mean Squared Errors, MAE is Mean Absolute Error, and AMAPE is Adjusted Mean Absolute Percentage Error

The results, as shown in the table 4 above, indicate that the forecasting performance of the GJR-GARCH and DGE-GARCH models, especially when fat-tailed asymmetric conditional distributions are taken into account in the conditional volatility, is better than the GARCH model. However, the comparison between the models with normal, student-t and skewed Student-t distributions shows that, according to the different measures used for evaluating the performance of volatility forecasts, the GJR –GARCH model provides the best forecasts and clearly outperforms DGE-GARCH and GARCH models and the DGE-GARCH model provides less satisfactory forecast results while the poorest forecast results was registered for the GARCH model. Moreover, it is found that the skewed Student-t distribution is more appropriate for modeling and forecasting the exchange rate returns volatility.

5. Conclusion

Modeling exchange rate volatility has received considerable attention from academies, market participant, policy makers, investors and practitioners in recent years as it provide a measure of risk in the financial market. It is important to note that, portfolio selection, asset valuations, risk management, option pricing and hedging strategies provides the importance of modeling and forecasting the conditional volatility of exchange rate returns. This article contributes to the existing literature of volatility modeling and forecasting by the following aspects by estimating and evaluate the forecasting performance of volatility models for exchange rate returns including the ARMA, GARCH, GJR-GARCH, and DGE-GARCH with normal, student-t and skewed Student-t distributions for modelling the Sierra Leones/USA dollar exchange rate returns volatility.

The results show that the forecasting performance of asymmetric GARCH Models (GJR and DGE), especially when fat-tailed asymmetric conditional distributions are taken into consideration in the conditional volatility, is better than GARCH model. The estimated parameters of the Models are statistically significant except, the coefficients of Mu for the GARCH and GJR- GARCH models under the three conditional distributions. Also, the coefficients on the standardized residuals and squared residuals of 10, 15 and 20 and the ARCH effect are not statistically significance which implies that no serial correlation exists in the exchange rate return series and that no significant appearance of the ARCH effect in the returns series and the variance equation is correctly specified.

In relation to the results found with regards the exchange rate markets, the leverage effect term (gamma) in both the GJR-GARCH model and the DGE-GARCH model is statistically significant but negative, indicating that the existence of leverage effect is observed in returns of the exchange rate market . However, according to the different measures used for evaluating the performance of volatility forecasts the GJR-GARCH model under the skewed Student t-distribution provides the best forecasts and clearly outperforms the GARCH and DGE-GARCH models. Moreover, it is found that the skewed Student-t distribution is more appropriate for modeling and forecasting the Sierra Leone/USA dollars exchange rate volatility in particular. Among all of these models, the GJR-GARCH with skewed Student t-distribution gives the highest log-likelihood value and the smallest AIC and BIC values under the student's t-distribution. The overall results, clearly suggests that GJR-GARCH model coupled with a skewed Student t-distribution performs very well with the dataset, and will be more suitable and relevant in facilitating risk management strategy for the Sierra Leone/USA dollars exchange rate returns volatility, which will provide useful information and benchmarks' to investors and policy makers for appropriate decision making in understanding investment strategies and enhancing exchange rate stability in the economy.

Finally, future researches directions could be investigated to improve the modeling Sierra Leone/USA dollars exchange rate volatility which could be better estimated by selecting shorter time intervals as well as introducing long run persistence of shocks in the volatility with fractionally integrated models and asymmetric models (FIGARCH, FIAPARCH, TS-GARCH and EGARCH) that allow to better capture the dynamics of the return series

6. Acknowledgements

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