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Characteristics of Fuzzy Petersen Graph with Fuzzy Rule

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Abstract: ***This work, based on if-then rule in fuzzy Petersen graph. Results related to find distance between two vertices, by applying if-then rules, through shortest paths are presented.***

Keywords: Degree of vertex, Incident graph, Fuzzy Petersen graph, Fuzzy IF-THEN rule.

1. Introduction

The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971). A more elaborate definition is due to Azriel Rosenfeld who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graph in 1975.

During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph. Till now fuzzy graphs has been witnessing a tremendous growth and finds applications in many branches of engineering and technology.

Fuzzy systems based on fuzzy if-then rules have been successfully applied to various theorems in the field of fuzzy control. Fuzzy rule based system has high comprehensibility because human users can easily understand the meaning of each fuzzy if-then rule through its linguistic interpretation.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model.'

2. Basic Concepts

Definition 2.1: Let V be a non-empty set. A fuzzy graph is a pair of functions $G:(\sigma,\mu)$, where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. i.e., $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all u,v in V. Where, uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ & $\sigma(v)$.

Example 2.1:

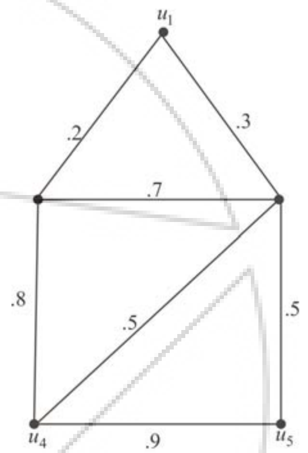

Figure 1: Fuzzy Graph

The Petersen graph G is the simple graph with 10-vertices and 15-edges. The Petersen graph is most commonly drawn as a pentagon inside with five spokes.

Example:

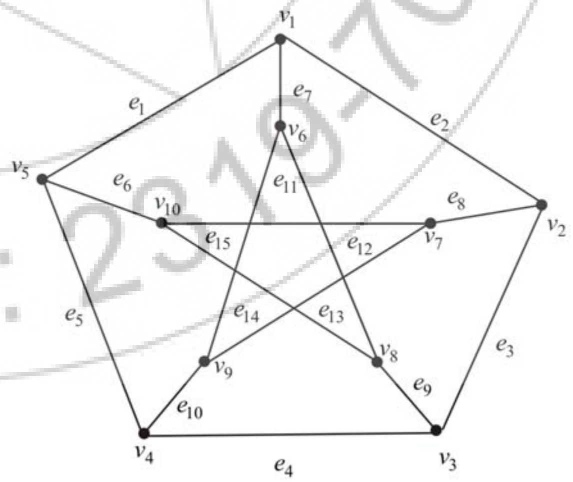

Figure 2: Petersen graph

It is set of vertices
$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$.
It is set of edges

$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$

Definition 2.2: When a vertex $\sigma(u_i)$ is an end vertex of some edges $\mu(u_i, v_j)$ of any fuzzy graph $G:(\sigma,\mu)$. Then $\sigma(u_i)$ and $\mu(u_i, v_j)$ are said to be incident to each other.

Example 2.2:

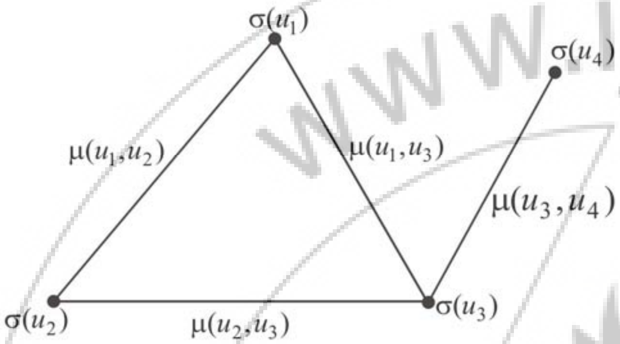

Figure 3: Incident graph

In this diagram $\mu(u_1,u_3), \mu(u_2,u_3)$ and $\mu(u_3,u_4)$ are incident on $\sigma(u_3)$.

Definition 2.3: The degree of any vertex $\sigma(u_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on vertex $\sigma(u_i)$ and is denoted by $d[\sigma(u_i)]$.

Example 2.3:

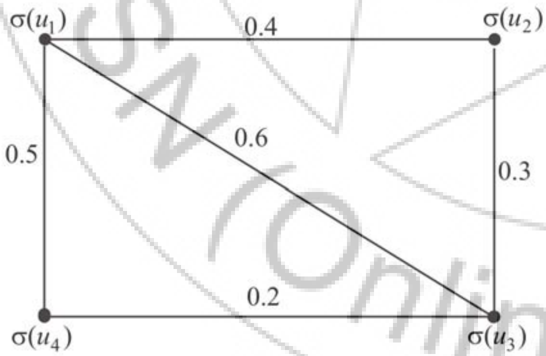

Figure 4: Degree of vertex

Degree of vertex $\sigma(u_1)$ = degree of membership of all those edges which are incident on a vertex $\sigma(u_1)$.
= 0.4 + 0.6 + 0.5
= 1.5
i.e., $d[\sigma(u_1)]$ = 1.5

Degree of vertex $\sigma(u_2)$ = 0.4 + 0.3
= 0.7
i.e., $d[\sigma(u_2)]$ = 0.7

Degree of vertex $\sigma(u_3)$ = 0.6 + 0.3 + 0.2
= 1.1
i.e., $d[\sigma(u_3)]$ = 1.1

Degree of vertex $\sigma(u_4)$ = 0.5 + 0.2
= 0.7
i.e., $d[\sigma(u_4)]$ = 0.7

Definition 2.4: A fuzzy rule is defined as a conditional statement in the form:

IF x is *A*, THEN y is B; where x and y are linguistic variables; A and B are linguistic values determined by fuzzy sets on the universe of discourse *X* and *Y*, respectively.

- A rule is also called a fuzzy implication.
- "*x* is A" is called the antecedent or premise.
- "*y* is B" is called the consequence or conclusion.

Example 2.4:

(i) IF pressure is high, THEN volume is small.
(ii) IF the speed is high, THEN apply the brake a little.

3. Fuzzy Petersen graph with fuzzy rule

Result 3.1:
Let $G:(\sigma,\mu)$ is a fuzzy Petersen graph, then the distance $d[\sigma(v_i),\sigma(v_j)]$ between two of its vertices $\sigma(v_i)$ and $\sigma(v_j)$ is the length of shortest path between them.

i.e., $d[\sigma(v_i),\sigma(v_j)] = \min\left[\sum_{i,j\in\wedge} \mu(u_i, v_j)\right]$

Example 3.1:

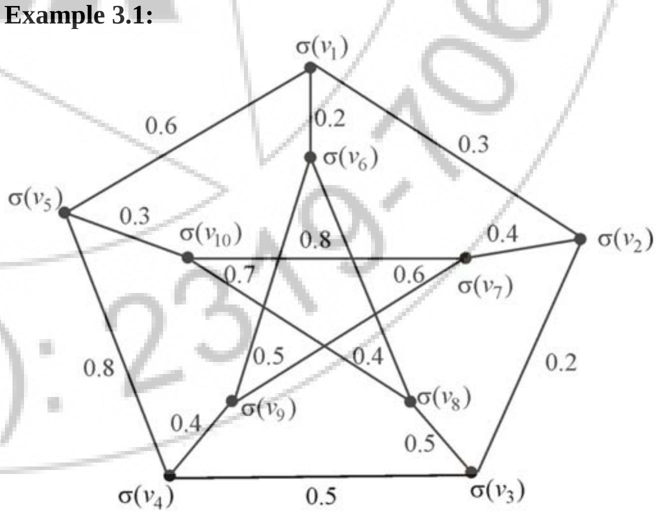

Figure 5: Fuzzy Petersen graph

In this fuzzy Petersen graph, there is four path between $\sigma(v_1)$ and $\sigma(v_4)$.

(i) $\sigma(v_1)-\sigma(v_6)-\sigma(v_9)-\sigma(v_4)$ i.e., 3 path
(ii) $\sigma(v_1)-\sigma(v_5)-\sigma(v_4)$ i.e., 2 path

(iii) $\sigma(v_1)-\sigma(v_2)-\sigma(v_3)-\sigma(v_4)$ i.e., 3 path

(iv) $\sigma(v_1)-\sigma(v_6)-\sigma(v_8)-\sigma(v_3)-\sigma(v_4)$ i.e., 4 path

IF we find the shortest path from $\sigma(v_1)$ *to* $\sigma(v_4)$, THEN the shortest path will be,

$\sigma(v_1)-\sigma(v_5)-\sigma(v_4)$.

IF membership grades are assigned to edges, THEN the length of the shortest path is,

$$d[(\sigma(v_1),\sigma(v_4)]=\sigma(v_1)-\sigma(v_5)-\sigma(v_4)$$
$$= 0.6 + 0.8$$
$$= 1.4$$

i.e., $d[\sigma(v_1),\sigma(v_4)]=1.4$

Theorem 3.2:

In fuzzy Petersen graph *G*, the sum of degrees of vertices of even degree is equal to twice the degree of membership of all the edges and the difference of the sum of degrees of vertices of odd degree.

Proof

Let $G:(\sigma,\mu)$ is a fuzzy Petersen graph. Consider 10-vertices $\{\sigma(v_1),\sigma(v_2),\sigma(v_3),...,\sigma(v_{10})\}$ of fuzzy Petersen graph $G:(\sigma,\mu)$.

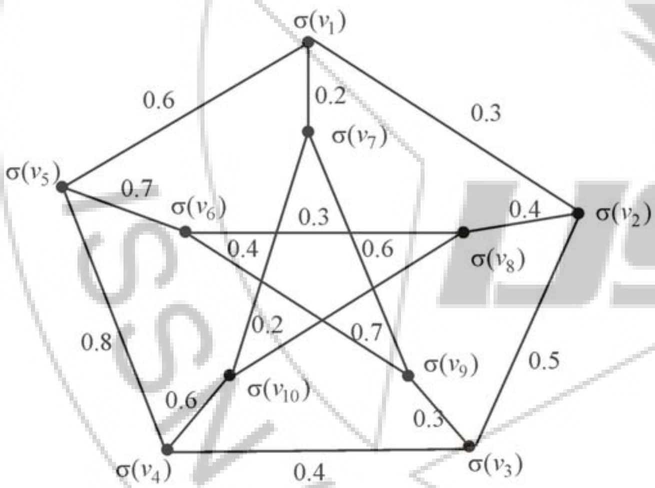

Figure 6: Fuzzy Petersen graph

IF the membership grades of edges which are incident on any degree of vertex $\sigma(v_i)$ are added, THEN the sum of corresponding membership values of vertices vary.

$d[\sigma(v_1)]$ = 0.6 + 0.3 + 0.2 = 1.1

$d[\sigma(v_2)]$ = 0.3 + 0.5 + 0.4 = 1.2

$d[\sigma(v_3)]$ = 0.5 + 0.4 + 0.3 = 1.2

$d[\sigma(v_4)]$ = 0.6 + 0.8 + 0.4 = 1.8

$d[\sigma(v_5)]$ = 0.6 + 0.8 + 0.7 = 2.1

$d[\sigma(v_6)]$ = 0.7 + 0.3 + 0.2 = 1.2

$d[\sigma(v_7)]$ = 0.2 + 0.4 + 0.6 = 1.2

$d[\sigma(v_8)]$ = 0.4 + 0.3 + 0.7 = 1.4

$d[\sigma(v_9)]$ = 0.3 + 0.6 + 0.2 = 1.1

$d[\sigma(v_{10})]$ = 0.6 + 0.4 + 0.7 = 1.7

IF the membership grades of edges are added, THEN we find the degree of edges,

$$\sum_{i=1}^{10}\mu(u_i,v_{i+1}) = 0.6 + 0.3 + 0.5 + 0.4 + 0.8 + 0.2 + 0.4 + 0.3 + 0.6 + 0.7 + 0.4 + 0.3 + 0.6 + 0.7 + 0.2$$
$$= 7$$

$$\therefore \sum_{i=1}^{10}\mu(u_i,v_{i+1})=7$$

$\sum_{i=1}^{n}d[\sigma(v_i)]$ = Twice the sum of degree of membership of (u_i,v_{i+1})

$$\therefore \sum_{i=1}^{n}d[\sigma(v_i)]=2\sum_{i=1}^{n}\mu(u_i,v_{i+1})$$

But here,

The deg $[\sigma(v_i)]$ has been splited into two parts.

$$\text{i.e., } \sum_{i=1}^{k}\deg v_i+\sum_{i=1}^{n}\deg w_k=2\sum_{i=1}^{n}\mu(u_i,v_{i+1})$$

Here, $\sum_{i=1}^{k}\deg v_i$ is the sum over even degree vertices, i.e., $\sigma(v_2),\sigma(v_4),\sigma(v_6),\sigma(v_8),\sigma(v_{10})$.

$$\sum_{i=1}^{k}\deg v_i=d[\sigma(v_2)]+d[\sigma(v_4)]+d[\sigma(v_6)]+d[\sigma(v_8)]+d[\sigma(v_{10})]$$
$$= 1.2 + 1.8 + 1.2 + 1.4 + 1.7$$
$$= 7.3$$

$$\therefore \sum_{i=1}^{k}\deg v_i=7.3$$

Now, $\sum_{K=1}^{n}\deg w_k$ is the sum over odd degree vertices, i.e., $\sigma(v_1),\sigma(v_3),\sigma(v_5),\sigma(v_7),\sigma(v_9)$.

$$\sum_{K=1}^{n}\deg w_k=d[\sigma(v_1)]+d[\sigma(v_3)]+d[\sigma(v_5)]+d[\sigma(v_7)]+d[\sigma(v_9)]$$
$$= 1.1 + 1.2 + 2.1 + 1.2 + 1.1$$
$$= 6.7$$

$$\therefore \sum_{i=1}^{k}\deg w_k=6.7$$

IF the sum of degrees of vertices of even degree is 7.3, THEN it is equal to twice the degree of membership of all edges and the difference of the sum of degrees of vertices of odd degree.

$$\text{i.e., } \sum_{i=1}^{k}\deg v_i=2\sum_{i=1}^{n}\mu(u_i,v_{i+1})-\sum_{k=1}^{n}\deg(w_k)$$

$$\sum_{i=1}^{k} \deg(v_i) = 2(\mu_i, v_{i+1}) - \sum_{k=1}^{n} \deg(w_k)$$

$7.3 = 2(7) - 6.7$
$= 14 - 6.7$
$7.3 = 7.3$

Hence the theorem.

4. Conclusion

We have applied fuzzy IF-THEN rule in fuzzy Petersen graph to arrive distance between two vertices. Finally, IF-THEN rules applied through shortest paths are shown with different analysis.

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