Design and Simulation of an Electrochemical ANN-Based Observer for a Lithium-Ion Battery for Estimation of State of Charge

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Abstract: ***Finite Element Methods (FEM) for estimation of State Of Charge (SOC) of the Lithium-ion battery are inaccurate. A more accurate method using Artificial Neural Networks (ANNs) is investigated in this research. The SOC of the Lithium-ion battery was determined from an electrochemical mathematical model of the Lithium-ion battery drawn from current literature. The model turns out to be complex as it is described in terms of non-linear Partial Differential Equations (PDE). Since Luenberger observer design is not applicable to systems described by linear or non-linear PDEs, an observer was designed based on back-stepping approach to estimate SOC of the Lithium-ion battery. The ANNs are implemented and then trained using MATLAB Neural Network Toolbox. The result for undertaking this research is a more accurate estimation of SOC of a Lithium ion battery for real-time applications e.g. in a dual source Pure Electric Vehicle (PEV). The proposed ANN-based design and simulation of the PDE observer has been validated by comparing the error of the observer based on ANNs and the same observer based on FEM. Simulations of the ANN-based observer and FEM-based observer were performed in MATLAB.***

Keywords: Artificial neural networks, finite element method, lithium ion battery, partial differential equations

1. Introduction

In [1], Shi Qingsheng, Zhang Chenghui, Cui Naxin and Zhang Xiaoping says that the continuing rise in international oil prices and people's increasing awareness of environmental protection promote the electric vehicle technology research more and more. The SOC of battery is an important index during the Electric Vehicle (EV) driving process. The battery SOC cannot be directly measured - only the battery output voltage and input current are available for measurement. In control theory, if the state variables of a system are not readily available for measurement, then according to David Luenberger [2] the immeasurable states can be estimated using a state observer. Therefore, how to obtain an accurate estimation of the battery SOC gradually becomes an urgent problem. In [3], S. J. Moura, N. A. Chaturvedi, and M. Krsti´c claim that monitoring battery SOC, which depends on the lithium concentration within each electrode, is particularly challenging for several technical reasons. First, directly measuring Li concentration is impractical outside specialized laboratory environments. Second, the concentration dynamics are governed by partial differential algebraic equations derived from electrochemical principles [3], [4]. As attested to by [3], these equations are solved by Finite Element Methods (FEM), which give slow estimation of SOC as the FEM algorithm is executed sequentially. The only measurable quantities (voltage and current) are related to the states of the battery through nonlinear functions [3]. In this paper, I directly addressed the first two technical challenges. Namely, I designed a state observer using a reduced-order PDE model based upon electrochemical principles. This electrochemical observer is simulated using both FEM and Artificial Neural Networks (ANN) and then compared.

2. Literature Survey

According to [1] and [3], many researchers throughout the world have done a lot of research on the aspect of battery SOC estimation. In the year 2001, Piller Sabine and his colleagues [5] gave a review about methods for battery SOC estimation. These methods mainly were discharge test, ampere-hour counting, open circuit voltage, linear model, impedance spectroscopy, Kalman filtering, back stepping [3] approach and intelligent method. The discharge test could not meet the requirement of online estimation. Linear model and open-circuit voltage method were difficult to guarantee a high accuracy. Impedance spectrum method is costly, which is unwise under the fact that the cars' price was declining. Kalman filtering method needed to establish accurate battery model and its calculation was complicated.

The back stepping approach is used to design an observer which is implemented in [3] using FEM which gives slow estimation of SOC as the FEM algorithm is executed sequentially. Intelligent methods such as neural networks with good nonlinear approximation ability have become more and more promising approach. As [3] observes, the electrochemical observer is described by non-linear Partial Differential Equations (PDE) which can be well approximated using ANNs since ANNs are universal approximators of any given mathematical function whether linear or non linear. This is due to the fact that the ANNs can be used to solve linear and non - ordinary and partial differential equations as Lucie P. Aarts and Peter Van Der Veer report in [7], and as reported by Hornik, K., Stinchcombe, M and White, H. report in [8] and as reported by I.E. Lagaris, A. Likas and D.I. Fotiadis in [9]. The authors of [3] advocates for the use of the electrochemical mathematical model of the Lithium battery since the model is more accurate than the equivalent circuit model used currently. The authors of [1] report that a type of ANN called

Elman neural network was used to estimate SOC of a Li-ion battery. The major drawback of this Elman neural network is that it does not utilize the exact electrochemical model of the Li-ion battery.

3. Problem Definition

Finite element methods for estimation of State Of Charge (SOC) of the Lithium-ion battery are inaccurate. This paper proposes the application of ANN to improve accuracy.

4. Methodology / Approach

The authors of [3] proved that the model of the Li-ion battery could also be given by:

$$\frac{\partial c}{\partial t}(r,t) = k\frac{\partial^2 c}{\partial r}(r,t) \quad (1)$$

With the following Dirichlet and Robin boundary conditions:

$$c(0,t) = 0 \text{ and } c(r,t) = 0 \quad (2)$$

$$\frac{\partial c}{\partial t}(1,t) - c(r,t) = \frac{R_s}{D_s F a^- AL^-} \text{ I (t) (12)}$$

According to equation (11), the model equations of the observer are obtained from the system itself i.e. an observer consists of a copy of the plant/system plus another term called boundary state error injection [3], [10].

The authors of [3] give the PDE observer model equations as follows:

$$\frac{\partial \hat{c}}{\partial t}(r,t) = \frac{\partial^2 \hat{c}}{\partial r^2}(r,t) + p_1(r)\,\tilde{c}(1,t) \quad (3)$$

$$\hat{c}(0,t) = 0$$

$$\frac{\partial \hat{c}}{\partial r}(1,t) - \hat{c}(1,t) = -\frac{R_s}{D_s F a^- AL^-} I(t) + p_{10}\,\tilde{c}(1,t) \quad (4)$$

Where the boundary state error is:

$$\tilde{c}(1,t) = \varphi\,(V(t), I(t)) - \hat{c}(1,t) \quad (5)$$

$$p_1(r) = -\frac{\lambda r}{2x}\left[I_1(x) - \frac{2\lambda}{x}I_2(x)\right] \quad (6)$$

$$x = \sqrt{\lambda(r^2 - 1)} \quad (7)$$

$$p_{10} = \frac{3-\lambda}{2} \quad (8)$$

$I_1(x)$ is called the modified Bessel function of first order of first kind. $I_2(x)$ is called the modified Bessel function of second order of first kind. λ is a tunable design parameter [3] used to vary the rate of convergence of the designed PDE observer but for stability reasons λ is given by the range: $\lambda < \frac{1}{4}$.

2) SOC Estimation from the Simplified Battery Model

The authors of [3] also proved that the model of the Li-ion battery can also be given by:

$$\frac{\partial c}{\partial t}(r,t) = k\frac{\partial^2 c}{\partial^2 r}(r,t) \quad (9)$$

With the following Dirichlet and Robin boundary conditions:

$$c(0,t) = 0 \text{ and } c(r,t) = 0 \quad (10)$$

$$\frac{\partial c}{\partial t}(1,t) - c(r,t) = \frac{R_s}{D_s F a^- AL^-} \text{ I (t)} \quad (11)$$

let I (t) = 0

Hence, the exact model that gives the state of charge (SOC) of the battery is given by:

$$\frac{\partial c}{\partial t}(r,t) = k\frac{\partial^2 c}{\partial^2 r}(r,t) \quad (12)$$

With the conditions: $c(0,t) = 0$, $\frac{\partial c}{\partial t}(1,t) = 0$ and

$$c(r,0) = 0.8 \quad (13)$$

Equations (12) and (13) are solved using the Method of Separation of Variables. The solution is given by

$$c_n(r,t) = 0.8 * e\left[-\left(\frac{n\pi}{R}\right)^2 kt\right] \sin\left(\frac{n\pi}{R}r\right) \quad (14)$$

In this paper initial SOC value = 0.8, R =1, *k*=1, n=2 and λ= -5.0

$c_2(r,t) = 0.8 * e[-(2\pi)^2 * t] \sin(2\pi * r)$

Fig.1 below shows surface of the SOC as a function of the radial distance r and time t.

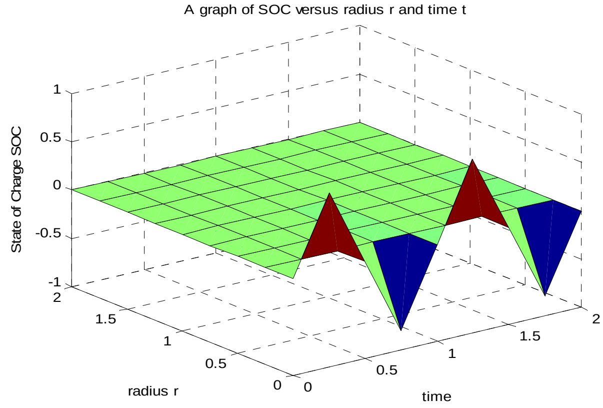

Figure 1: Surface of the SOC as a function of the radial distance r and time t.

3) Finite Element Method

The partial differential equations above that describe the dynamics of the electrochemical observer are in this section solved used the Partial Differential Equation Toolbox for MATLAB. This toolbox utilizes the Finite Element Methods (FEM) to solve a given partial differential equation. The observer is described by two PDEs hence the procedure used in solving the equations are as follows: (i) treat the current input to the battery as equal to zero. (ii) Create a MATLAB m-file that solves the remaining equation the first one. Call this *eqn1.m*. (ii) Run the m-file called *pde.m* in the MATLAB workspace. Because of step (i), the state variable error becomes

$$\tilde{c}(1,t) = -\hat{c}(1,t) \quad (15)$$

Again for convenience we let λ= -5; hence p10 = 4. Hence, this reduces the second equation to:

$$\frac{\partial \hat{c}}{\partial r}(1,t) - \hat{c}(1,t) = -p_{10}\hat{c}(1,t) \quad (16)$$

$$\frac{\partial \hat{c}}{\partial r}(1,t) - \hat{c}(1,t) = -4\hat{c}(1,t) \quad (17)$$

Then $\frac{\partial \hat{c}}{\partial r}(1,t) = -3\hat{c}(1,t)$

$c_{ss}^- = \hat{c}(1,t)$ is the Li concentration at the particle surface = constant= 1mol / m^3(say)

Hence, $\frac{\partial \hat{c}}{\partial r}(1,t) = -3$

The value of

$$p_1(r) = 5\frac{r}{2x}\left[I_1(x) + \frac{10}{x}I_2(x)\right] \quad (18)$$

Let the value of x = 0.5 meters, hence $p_1(r)$ = 4.4795r

Hence, the new PDE that we need to solve is given by (30) and (31) below:

$$\frac{\partial \hat{c}}{\partial t}(r,t) = \frac{\partial^2 \hat{c}}{\partial r^2}(r,t) - 4.5r \quad (19)$$

Subject to the conditions,

$$\frac{\partial \hat{c}}{\partial r}(1,t) = -3\;\; \hat{c}(r,0) = 0 \text{ and } \hat{c}(0,t) = 0 \quad (20)$$

We obtained the graphs below.

The Fig. 2 below shows a surface of the SOC as a function of the radial distance r and time t.

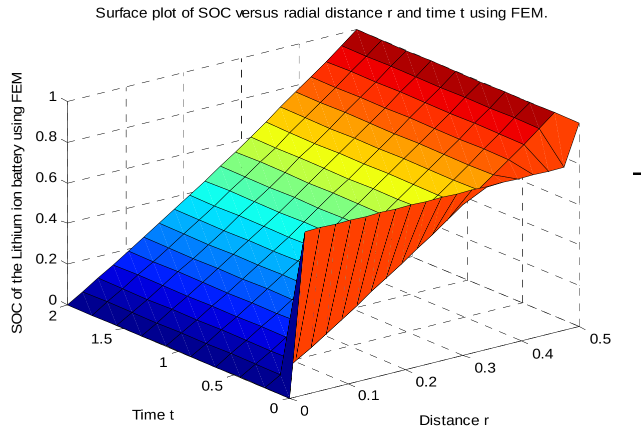

Figure 2: A surface of the SOC as a function of the radial distance r and time t using FEM.

4) Artificial Neural Networks

i) My contributions

The PDE that we are required to solve using ANN is given below:

$$\frac{\partial c}{\partial t}(r,t) = \frac{\partial^2 c}{\partial r^2}(r,t) - 4.5r \quad (21)$$

With the boundary conditions:

$$\frac{\partial c}{\partial r}(1,t) = -3 \quad c(r,0) = 0 \text{ and } c(0,t) = 0 \quad (22)$$

According to [7], the solution to the equations (21) and (22) that govern the observer can be assumed to be approximated by a neural network represented by (23).

$$\phi(r,t) = \sum_{i=1}^{m}[\alpha_i f(w_{i1} r + w_{i2} t + b_i)] \quad (23)$$

Where; m is the number of neurons in the hidden layer, α_i is a constant, w_{i1} and w_{i2} are the weights for the *i-th* neuron, b_i is the bias for the i-th layer, r is the radial distance of the active particle of the lithium ion battery measured from the centre of the Li-ion battery, $t =$ time in seconds, and $\phi(r,t)$= a function that approximates the c (r, t), the state of charge of the Li-ion battery.

Obtaining the second derivative of the equation (23) that approximates the neural network and substituting in the simplified equation (21) we get an ANN that is described by the following equations (24) and (25)

$$\sum_{i=1}^{m}\alpha_i w_{i1}^2 f(w_{i1} r + w_{i2} t + b_i) - \sum_{i=1}^{m}\alpha_i w_{i2} f(w_{i1} r + w_{i2} t + b_i) + p_1(r)\, c(1,t) = 0 \quad (24)$$

$$\sum_{i=1}^{m}\alpha_i w_{i1} f(w_{i1} r + w_{i2} t + b_i) - \sum_{i=1}^{m}[\alpha_i f(w_{i1} r + w_{i2} t + b_i)] - 4.5r = 0 \quad (25)$$

Let the number of neurons, m, be equal to three, i.e. m =3.

Then equation (24) becomes:

$$\sum_{i=1}^{3}\alpha_i w_{i1}^2 f(w_{i1} r + w_{i2} t + b_i) - \sum_{i=1}^{3}\alpha_i w_{i2} f(w_{i1} r + w_{i2} t + b_i) - 1 = 0 \quad (26)$$

Applying i=1, 2 and 3 to (37) we have:

$$\alpha_1 w_{11}^2 f(w_{11} r + w_{12} t + b_1) + \alpha_2 w_{21}^2 f(w_{21} r + w_{22} t + b_2) + \alpha_3 w_{31}^2 f(w_{31} r + w_{32} t + b_3) - \alpha_1 w_{12} f(w_{11} r + w_{12} t + b_1) - \alpha_2 w_{22} f(w_{21} r + w_{22} t + b_2) - \alpha_3 w_{32} f(w_{31} r + w_{32} t + b_3) - 4.5r = 0 \quad (27)$$

ii) TRAINING OF THE ANN

In this section, the ANN will be trained so that the initial conditions are satisfied. For convenience, the ANN has been reproduced below:

$$\alpha_1 w_{11}^2 f(w_{11} r + w_{12} t + b_1) + \alpha_2 w_{21}^2 f(w_{21} r + w_{22} t + b_2) + \alpha_3 w_{31}^2 f(w_{31} r + w_{32} t + b_3) - \alpha_1 w_{12} f(w_{11} r + w_{12} t + b_1) - \alpha_2 w_{22} f(w_{21} r + w_{22} t + b_2) - \alpha_3 w_{32} f(w_{31} r + w_{32} t + b_3) - 4.5r = 0 \quad (28)$$

These ANNs are trained such that *r*= 0 for all values of time, *t*, the output of the neural network is always equal to zero. Training of the ANN is done using MATLAB's Neural Network Toolbox. The ANN is trained with a training set that contains inputs:(r, t) $\in$ {(0,0),(0,0.1),(0,0.2),(0,0.3),(0,0.4),(0,0.5),(,0.6),(0,0.7),(0,0.8),(0,0.9),(0,1) } . In addition, the corresponding outputs are: Outputs $\subset$ {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

5. Results & Discussion

5.1 Architecture of the Proposed ANN

The proposed neural network has five layers with three neurons in the first and second layers, one neuron in the third and fourth layers. The activation function of the first layer is the first derivative of the log sigmoid transfer function while that of the second layer is the second derivative of the log sigmoid transfer function. The third, fourth and fifth layers have linear activation functions. The proposed ANN has been designed using NEURAL NETWORK Graphical User Interface (GUI) for MATLAB. This GUI is accessed through the MATLAB command *nntool.*

The complete view of the designed ANN is shown in the Fig. 3 below.

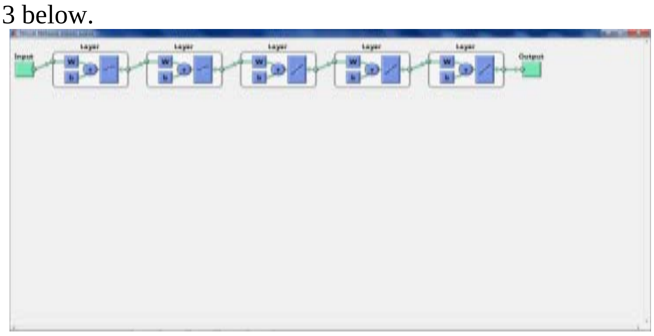

Figure 3: The complete view of the designed ANN.

The proposed ANN was trained using MATLAB Neural network Toolbox. The results for the training exercise are shown in Fig. 4 below.

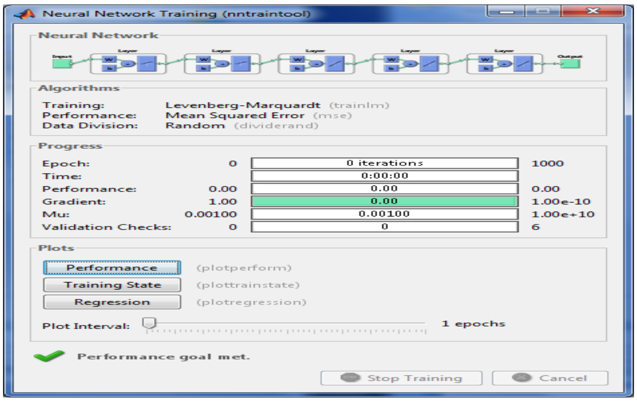

Figure 4: The results for the training exercise

6. Conclusion

To check whether the proposed ANN has been trained without any error, the ANN is simulated in the **Network: pde** dialogue box. The results for simulation are stored in the **Data: pde_errors** dialogue box shown in the Fig. 5 below.

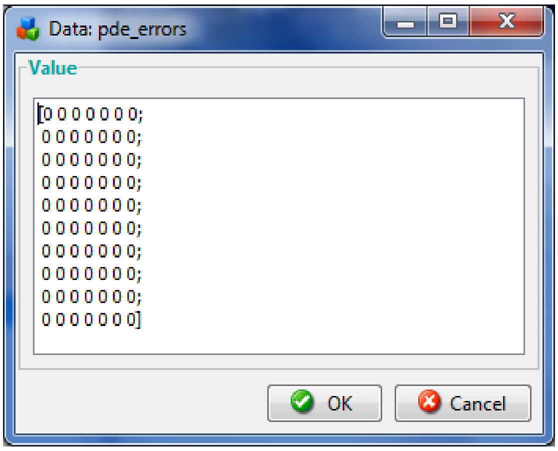

Figure 5: Errors

Fig.6 below shows the graph of error versus r and t.

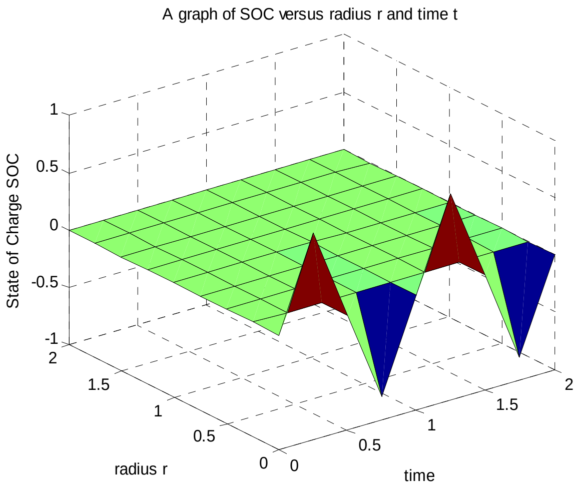

Figure 6: Graph of error versus radius r and time t.

From Fig. 6 above it is evident that the error is almost equal to zero using ANN implying that the ANN has accurately estimated the SOC of the Li-ion battery with a higher degree of accuracy that the Finite Element Method. The paper presented a method to estimate battery SOC of Electric Vehicle using ANN. From the above simulation studies in MATLAB, it is evident that the error for FEM is greater than that of ANNs for the same PDE model. Hence, an ANN provides a more attractive method for estimation of state of charge of a Li-ion battery than the FEM method.

7. Future Scope

SOC can be used as an input to a fuzzy logic controller (FLC) that regulates the distribution of power between a Li-ion battery and a super capacitor for power management in a dual source pure electric vehicle (PEV).

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