possibility (value of membership function) of being contained in the fuzzy set "young". For example, if we follow the definition of "young" as in the figure, ten year-old boy may well be young. So the possibility for the "age ten" to join the fuzzy set of "young" is 1, also that of "age twenty seven" is 0.9. But we might not say young to a person who is over sixty and the possibility of this case is 0.

Now we can manipulate our last sentence to "Jenny is very young". In order to be included in the set of "very young", the age should be lowered and let us think the line is moved leftward as in the figure. If we define fuzzy set as such, only the person who is under forty years old can be included in the set of "very young". Now the possibility of twenty- seven year old man to be included in this set is 0.5.

That is, if we denote A = "young" and B = "very young", $\mu_A(27)=0.9$, $\mu_B(27)=0.5$.

Definition 2.2: Let X,Y⊆R be universal set, then R={((x,y), μ_R(x,y)): x,y∈X×Y} is said to be a fuzzy relation from X to Y.

Definition 2.3: Let μ be a fuzzy relation on σ. Then μ is said to be reflexive if, μ(x,x) = σ(x), ∀ x∈S. If μ is reflexive, then it follows that $\mu(x,y) \leq \mu(x,x)$ and $\mu(y,x) \leq \mu(x,x)$, ∀ x∈S.

Definition 2.4: A fuzzy relation μ is said to be symmetric if $\mu(x,y) \leq \mu(y,x)$, ∀ x,y∈S.

Definition 2.5: A fuzzy graph(f-graph) is a pair G: (σ,μ) where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. It is assumed that S is finite and nonempty, μ is reflexive and symmetric. Thus if G : (σ,μ) is a fuzzy graph, then σ : S → [0,1] and μ : S×S → [0,1] is such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v ϵ S.
Also, we denote the underlying graph of the fuzzy graph G: (σ,μ) by G*: (σ^*,μ^*) where σ* = {u ∈ S : σ(u) > 0} and μ^* = {(u,v) ϵ S×S : μ(u,v) > 0}. In examples, if σ is not specified, it is chosen suitably. Also G : (σ,μ) is said to be a trivial fuzzy graph if G*: (σ^*,μ^*) is trivial. That is σ^* is a singleton set.

Example2.5:

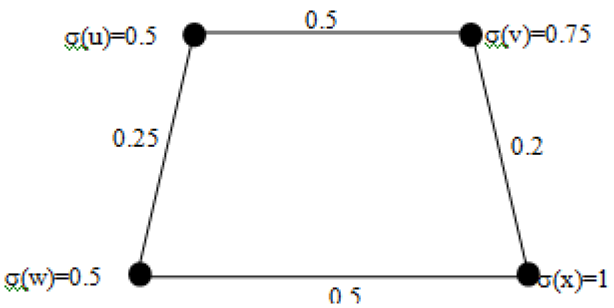

Figure 2: Fuzzy graph

Let G : (σ,μ) be with σ* = {u,v,w,x}. Let $\sigma(u) = 0.5$, $\sigma(v) = 0.75$, σ(w) = 0.5, σ(x) =1and $\mu(u,v) = 0.5$, $\mu(v,x) = 0.2$, μ(w,x) = 0.5, μ(w,u) = 0.25 . Then G is a fuzzy graph. since $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u,v ϵ σ*.

Definition 2.6: In a fuzzy graph G : (σ,μ), a path P of length n is a sequence of distinct nodes $u_0,u_1,\ldots,u_n$ such that $\mu(u_{i-1},u_i) > 0$, i=1,2,…,n and the degree of membership of a weakest arc is defined as its strength.

Example2.6:

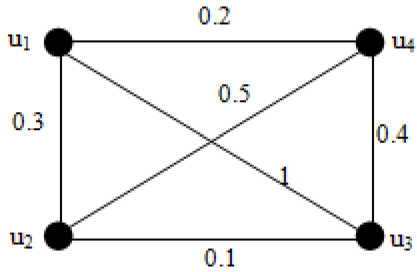

Figure 3: Fuzzy path

Let G: (σ,μ) be a fuzzy graph with $\sigma^* = (u_1,u_2,u_3,u_4)$. Let $\mu(u_1,u_2) = 0.3$, $\mu(u_2,u_3) = 0.1$, $\mu(u_3,u_4) = 0.4$, $\mu(u_4,u_1) = 0.2$. There are three different paths from u_1 to u_3 namely, $P_1 = u_1,u_2,u_3$; P_2 = arc (u_1,u_3); $P_3 = u_1,u_4,u_3$. Now strength of P_1 = min{0.3,0.1} = 0.1, strength of P_2 =1, and strength of P_3 = min{0.2,0.4} = 0.2. So the strongest path joining u_1 and u_3 is the arc (u_1,u_3) with strength 1.

Definition 2.7: A complete fuzzy graph (CFG) is a fuzzy graph G : (σ,μ) such that $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all x and y. Note that if G is a Complete fuzzy graph, then G* is also a complete graph.

Example2.7:

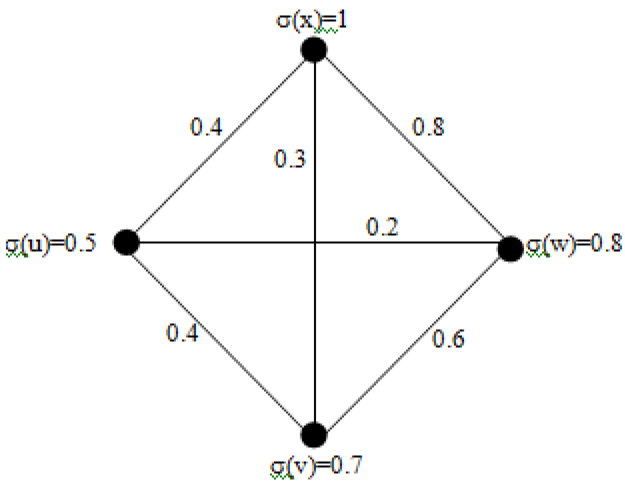

Figure 4: Complete Fuzzy graph

Let G: (σ,μ) be a fuzzy graph with σ^* = (u,v,w,x). σ(u)=0.5, σ(v)=0.7, σ(w)=0.8, σ(x)=1. μ(u,v) = 0.5, μ(v,w) = 0.7, μ(w,x) = 0.8, μ(x,u) = 0.5, μ(u,w) = 0.5, μ(v,x) = 0.7. Then G is a Complete fuzzy graph.

Definition 2.8: The degree of any node $\sigma(v_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on a node $\sigma(v_i)$. And is denoted by $d(\sigma(v_i))$. A node of odd degree is an odd node and a node of even degree is an even node.

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Example 2.8:

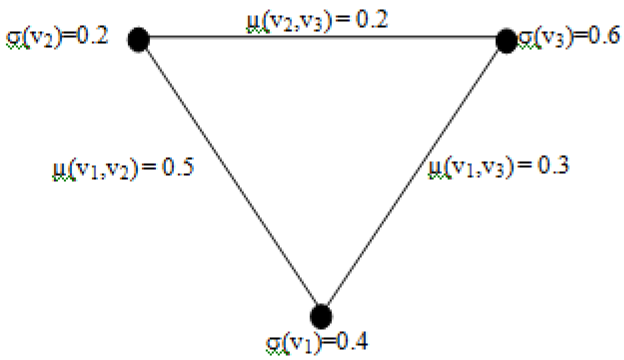


Figure 5: G

Degree of vertex $\sigma(v_2)$ = degree of membership of all those edges which are incident on a vertex $\sigma(v_2)$

$= \mu(v_1,v_2)+ \mu(v_2,v_3)$

$= 0.5+0.2=0.7$

i.e. $d[\sigma(v_2)] = 0.7$.

Definition 2.9: Let G : (σ,μ) be a fuzzy graph. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and is denoted by $CONN_G(x,y)$. An x-y path P is called a strongest x-y path if its strength equals $CONN_G(x,y)$. An fuzzy graph $G:(\sigma,\mu)$ is connected if for every x,y in σ^*, $CONN_G(x,y) > 0$. A maximal connected fuzzy subgraphs are called components. Clearly if G is connected, then any two nodes are joined by a path.

Example2.9:

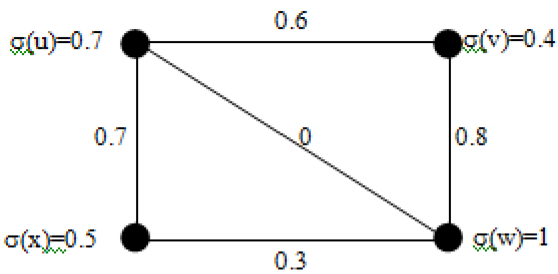


Figure 6: Connected Fuzzy Graph

The path between u and v nodes are {uv, uxwv, uwv}

$CONN_G(u,v)$ = max { uv, uxwv, uwv }

$CONN_G(u,v)$ = max [0.6, 0.3, 0}

$CONN_G(u,v) = 0.6$. Similarly, $CONN_G(u,w)=0.6$, $CONN_G(u,x)= 0.7$, $CONN_G(v,w) = 0.8$, $CONN_G(v,x) = 0.6$, $CONN_G(w,x) = 0.6$

Definition 2.10: The modus ponens (MP)

Fact : x is A

Rule : IF x is A THEN y is B

Result : y is B

is a well- known deduction rule in (Boolean) logic. From the fact "x is A" and the IF- THEN rule "IF x is A THEN y is B", we can derive a new fact, namely "y is B,". However, if we do not exactly known that "x is A", we cannot make any deduction concerning y, even if we would have tons of other information on x. This implies that if we want to develop a useful derivation system (e.g.. for a computer- controlled car) based on this MP, we have provide an IF-THEN rule for each possible A. Needless to say this would be highly inefficient, if not impossible.

3. Fuzzy If-Then Rule with Modus Ponens Method

Theorem3.1: If a fuzzy graph G : (σ,μ) has exactly two nodes of odd degree, then they must be connected by a path.

Proof:

We can prove this theorem by using Modus ponens Method:

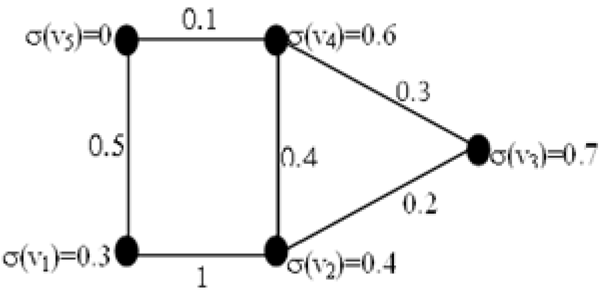
Rule 1:

Fact : x is a fuzzy graph G : (σ,μ) with all its nodes of even degree, except for $\sigma(v_1)$ and $\sigma(v_2)$ which are of odd degree and also G_1 is the component with $\sigma(v_1)$.

Rule :**IF** x is a fuzzy graph G : (σ,μ) with all its nodes of even degree, except for $\sigma(v_1)$ and $\sigma(v_2)$ which are of odd degree and also G_1 is the component with $\sigma(v_1)$ **THEN** y is a fuzzy graph in which G_1 has an even number of nodes of odd degree.

Result: y is a fuzzy graph in which G_1 has an even number of nodes of odd degree.

Example:



$d[\sigma(v_1)] = 1+0.5 =1.5$

$d[\sigma(v_2)] = 1+0.4+0.2 =1.6$

$d[\sigma(v_3)] = 0.2+0.3 =0.5$

$d[\sigma(v_4)] = 0.4+0.3+0.1 =0.8$

$d[\sigma(v_5)] = 0.5+0 .1 =0.6$

Here the nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree and the remaining nodes $\sigma(v_3),\sigma(v_4),\sigma(v_5)$ are of even degree.

Consider the component G_1

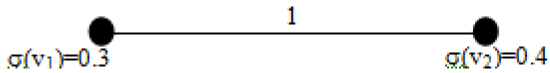


Figure 8:G_1

Here $\sigma(v_1) \in G_1$ and G_1 has an even number of nodes of odd degree.

Rule 2:

Fact: x is a fuzzy graph having component G_1 with node $\sigma(v_1)$.

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Rule: **IF** x is a fuzzy graph having component G_1 with node $\sigma(v_1)$ **THEN** y is also a fuzzy graph having component G_1 must contain the node $\sigma(v_2)$, the only other node of odd degree.

Result: y is also a fuzzy graph having component G_1 must contain the node $\sigma(v_2)$, the only other node of odd degree.
Example:

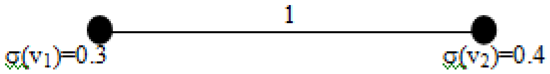

Figure 9: G_1

i.e. $\sigma(v_2) \in G_1$
$d[\sigma(v_1)] = 1$
$d[\sigma(v_2)] = 1$

Rule3:
Fact: x is a fuzzy graph with exactly two nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree.

Rule: **IF** x is a fuzzy graph with exactly two nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree **THEN** y is also a fuzzy graph in which of $\sigma(v_1)$ and $\sigma(v_2)$ are must be connected by a path.

Result: y is also a fuzzy graph in which of $\sigma(v_1)$ and $\sigma(v_2)$ are must be connected by a path.

Example:

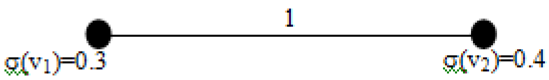

Figure 10: G_1

i.e. $\sigma(v_1), \sigma(v_2) \in G_1$.
Since a component is connected, there is a path between $\sigma(v_1)$ and $\sigma(v_2)$.

Hence a fuzzy graph G : (σ,μ) of the graph G : (V,E) has exactly two nodes of odd degree, they must be connected by a path.

Hence proved the Theorem.

Lemma3.2: For any set of positive integers $n_1,n_2,\ldots,n_k$. If a fuzzy graph G : (σ,μ) of a graph G : (V,E). Then
$\sum_{i=1}^{k}\sigma(n_i^2) \leq (\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)(2\sum_{i=1}^{k}\sigma(n_i) - k)$.

Proof:

We have, $\sum_{i=1}^{k}(\sigma(n_i) - 1) = \sum_{i=1}^{k}\sigma(n_i) - k$

Squaring on both sides, we get

$[\sum_{i=1}^{k}(\sigma(n_i) - 1)]^2 = [\sum_{i=1}^{k}\sigma(n_i) - k]^2$,

or $[(\sigma(n_1)-1) + (\sigma(n_2)-1) + \ldots + (\sigma(n_k)-1)]^2 = [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

or $(\sigma(n_1)-1)^2+(\sigma(n_2)-1)^2+\ldots+(\sigma(n_k)-1)^2+\sum_{i=1}^{k}\sum_{j=1, i\neq j}^{k}\sigma(n_i - 1)$ $\sigma(n_j - 1) = [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i)+k^2$

Therefore,
$\sum_{i=1}^{k}\sigma(n_i^2) - 2\sum_{i=1}^{k}\sigma(n_i) + k + \sum_{i=1}^{k}\sum_{j=1, i\neq j}^{k}\sigma(n_i - 1)\sigma(n_j - 1)$
$= [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

or $\sum_{i=1}^{k}\sigma(n_i^2) \leq 2\sum_{i=1}^{k}\sigma(n_i) - k + [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

or $\sum_{i=1}^{k}\sigma(n_i^2) \leq [\sum_{i=1}^{k}\sigma(n_i)]^2 - (k-1)(2\sum_{i=1}^{k}\sigma(n_i) - k)$.

Apply Modus Ponens Method:

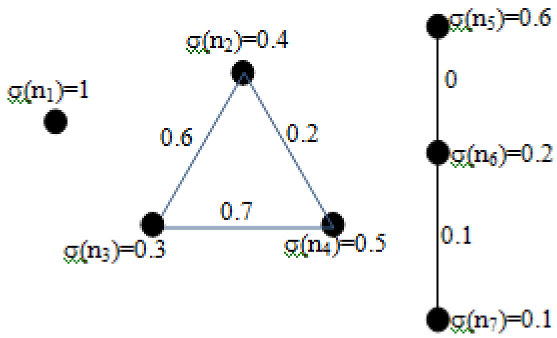

Figure 11: G
Figure 11: Gives a disconnected fuzzy graph with three components.

Rule1:
Fact: x is $\sum_{i=1}^{k}\sigma(n_i^2)$
Rule: **IF** x is $\sum_{i=1}^{k}\sigma(n_i^2)$ **THEN** y is 1.91
Result: y is 1.91
Example:

$\sum_{i=1}^{k}\sigma(n_i^2) = \sigma(n_1)^2 + \sigma(n_2)^2 + \sigma(n_3)^2 + \sigma(n_4)^2 + \sigma(n_5)^2 + \sigma(n_6)^2 + \sigma(n_7)^2$
$= (1)^2 + (0.4)^2 + (0.3)^2 + (0.5)^2 + (0.6)^2 + (0.2)^2 + (0.1)^2$
$= 1 + 0.16 + 0.09 + 0.25 + 0.36 + .04 + 0.01$
$= 1.91$

Rule2:
Fact: x is $(\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)(2\sum_{i=1}^{k}\sigma(n_i) - k)$
Rule: **IF** x is $(\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)(2\sum_{i=1}^{k}\sigma(n_i) - k)$ **THEN** y is 3.21
Result: y is 3.21.
Example:

$(\sum_{i=1}^{k}\sigma(n_i))^2-(k-1)(2\sum_{i=1}^{k}\sigma(n_i)-k)$
$= (1+0.4+0.3+0.5+0.6+0.2+0.1)^2-(3-1)(2(3.1)-3)$
$= (3.1)^2-(2)(6.2-3)$
$= 9.61-(2)(3.2)$
$= 9.61-6.4$
$=3.21$

Rule3:
Fact: x is $\sum_{i=1}^{k}\sigma(n_i^2)$

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Rule: **IF** x is $\sum_{i=1}^{k}\sigma(n_i^2)$ **THEN** y is lessthan or equal to $\sum_{i=1}^{k}\sigma(n_i))^2 - (k\text{-}1)\ (2\sum_{i=1}^{k}\sigma(n_i) - k))$.
Result: y is lessthan or equal to ($\sum_{i=1}^{k}\sigma(n_i))^2 - (k\text{-}1)\ (2\sum_{i=1}^{k}\sigma(n_i) - k))$.

Example:

$\sum_{i=1}^{k}\sigma(n_i^2) \leq [\ \sum_{i=1}^{k}\sigma(n_i)\ ]^2 - (k\text{-}1)\ (2\sum_{i=1}^{k}\sigma(n_i) - k)$.
$1.91 \leq 3.21$.

Hence prove the Lemma.

4. Conclusion

Finally fuzzy If-Then rule with Modus Ponens method applied on the fuzzy graph results are efficient than crisp results.

References

[1] "An Introduction to Graph Theory" – S. Pirzada, Professor of Mathematics, University of Kashmir.
[2] "Fuzzy If-Then Rules In Computational Intelligence" Theory and applications edited by Da Ruan and Etienne E.Kerre.
[3] "Fuzzy Graphs – Basic, Concepts and Applications" – Sunil Mathew , M.S. Sunitha.
[4] "First Course on Fuzzy Theory and Applications" – Kwang H. Lee.
[5] .Vaishnaw and S.Sharma "Some analoges on fuzzy graph" International Journal of Computational and Mathematical Science 2:4 2008.
[6] "Invitation to Graph Theory" S.Arumugam and S.Ramachandran.
[7] "Fuzzy Set Theory" – By Shiv Raj Singh, Associate Professor, Department of Mathematics, D.N.(P.G) College Meerut, Chaman Singh, Associate Professor, Department of Mathematics, Acharya Narendra Dev College,(University of Delhi)_New Delhi.
[8] .Ramaswamy and Poornima B. "Product fuzzy graphs" IJCSNS International Journal of computer science and network security, vol.9 No.1,2009.
[9] .Dr.G.Nirmala and N.Vanitha "Risk Analysis of River-Type Hydro-Power Plants" Aryabhatta Journal of Mathematics & Informatics Vol.6, No. 1, Jan-July, 2014.
[10] .Dr.G.Nirmala and K.Uma, "Fuzzy Graph-Estimations of strongest weight", The PMU Journal of Humanities and Sciences.
[11] Dr.G.Nirmala and N.Vanitha [2010], "Risk of Construction Project with Fuzzy Characteristics", Naros Publication.
[12] Y. Vaishnaw and A.S. Ranadive "On isomorphism between fuzzy graphs" Chhattisgarh Journal of science and technology, volume 3 & 4 (2007), 41-52.
[13] Nagoor Gani and J. Malarvizhi "Isomorphism on Fuzzy Graphs" International Journal of Computational and Mathematical Sciences 2:4 2008.
[14] Dr.G.Nirmala and G.Suvitha "Implication Relations In Fuzzy Propositions" Aryabhatta Journal of Mathematics & Informatics Vol.6, No. 1, Jan-July, 2014.
[15] Dr.G.Nirmala and G.Suvitha "Comparision of 3 valued Logic using Fuzzy Rule-Fuzzy" International Journal of Scientific and Research Publications, Volume 3, Issue 8, (August 2013) 1 ISSN 2250-3153.

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