

# Use of $\ell_{2/3}$ -norm Sparse Representation for Facial Expression Recognition

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**Abstract:** *In this particular paper, we recommend a brand new sparse rendering primarily based group via  $\ell_{2/3}$ -norm minimization with regard to facial Expression Recognition. All of us work with  $\ell_{2/3}$ -norm minimization instead of  $\ell_{2/3}$ -norm minimization with regard to sparser, a lot more identification fee and many correct options, lowest computing moment and optimization dilemma connected with  $\ell_{2/3}$ -norm minimization. This  $\ell_{2/3}$ -norm regularize is actually open to get a wide range of ensuring components such as neutral, eye-sight and sparsity components. All of us work with  $\ell_{2/3}$ -norm minimization ( $\ell_{2/3}$ -SRC) as an alternative to  $\ell_{2/3}$ -norm minimization ( $\ell_{2/3}$ -SRC). Additionally, the active-set primarily based iterative reweighted criteria are actually recommended to fix your  $\ell_{2/3}$ -Norm minimization dilemma. This trial and error effects with JAFFE Database/sources state your productivity connected with  $\ell_{2/3}$ -SRC.*

**Keywords:** Sparse Representation,  $\ell_{1/2}$ -norm minimization,  $\ell_{2/3}$ -norm minimization, Facial Expression Recognition

## 1. Introduction

Facial expression represents emotion, intention and personality of a person. FER is also Non-verbal, natural, human communications. There are three types of classification: face detection, feature extraction, and face recognition.

Facial expression recognition is diagnosed by many classifier i.e. neural network (NN) [3], Support vector machine (SVM), Bayesians Networks [5], the rule-based classifier [6] and the sparse representation based classification (SRC) [7]. Wright et al is proposed a new technique the SRC Algorithm for facial expression recognition [8]. In SRC, the test image is represented as a sparse linear combination of the training samples, and then the classification is conducted by evaluating which class of training samples could result in the minimal reconstruction error to the test image with the corresponding sparse representation coefficients. Intuitively, the sparsity of the representation coefficient vector can be measured by its  $\ell_0$ -norm, which counts the number of nonzero entries in the vector. And the optimization problem of  $\ell_0$ -norm minimization is NP-hard and difficult even to approximate [9]. The problem of  $\ell_1$ -norm minimization and  $\ell_0$ -norm minimization are equivalent if the solution is sparse enough and  $\ell_1$ -norm minimization can be solved in polynomial time. The  $\ell_{1/2}$ -norm regularizer is shown to have many promising properties such as unbiasedness, sparsity, and oracle properties [11].  $\ell_{1/2}$ -norm minimization in SRC that is formulated as

$$x = \arg \min_x \|y - Ax\|^{1/2} + \lambda \|x\|_{1/2} \quad (1)$$

Where  $y$  is a test sample,  $A$  is an over complete dictionary whose basis vector are the training samples,  $x$  is the sparse representation coefficient vector of  $y$  over  $A$ .

Recently, the  $\ell_{2/3}$ -norm proposed as expected option to distinguish the sparsity of the signal. The  $\ell_{2/3}$ -norm regularizer is shown to have many promising properties such as impartial, sparsity and prophesy properties. Based on the

latest evolution of  $\ell_{2/3}$ -norm regularizer, we advise a latest SRC via  $\ell_{2/3}$ -norm minimization in this paper. We show our technique by  $\ell_{2/3}$ -SRC to discriminate it from the conventional SRC which is denoted by  $\ell_{1/2}$ -SRC. In  $\ell_{2/3}$ -SRC, we apply  $\ell_{2/3}$ -norm minimization as an option to  $\ell_{1/2}$ -norm minimization in its place of using  $\ell_0$ -norm minimization in  $\ell_{1/2}$ -SRC. By take on  $\ell_{2/3}$ -norm minimization, we can locate minimum computation time, sparser and more precise solution than  $\ell_{2/3}$ -SRC do and the optimization problem of  $\ell_{2/3}$ -norm minimization is much easier to be solved than that of  $\ell_{1/2}$ -norm minimization.

The rest of the paper is organized as follows. Section II describes the proposed model in detail and presents an active set based iterative reweighted algorithm to solve the  $\ell_{2/3}$ -norm minimization problem. We conduct the experiments in section III, and conclude this paper in section IV.

## 2. Sparse representation VIA $\ell_{2/3}$ - norm minimization

In this category, we launch our latest model of sparse representation via  $\ell_{2/3}$ -norm minimization. After that we represent an active-set based iterative reweighted algorithm to resolve the  $\ell_{2/3}$ -norm minimization problem, and review the classification procedure of  $\ell_{2/3}$ -SRC.

### A. Sparse Representation via $\ell_{2/3}$ -norm Minimization

The  $\ell_{2/3}$ -norm minimization is usually running in SRC as an alternative to  $\ell_{2/3}$ -norm minimization. However, it is not constantly the case that the optimization problem of  $\ell_{1/2}$ -norm minimization can discover the sparsset solution. Recently,  $\ell_{2/3}$ -norm regularizer, which is shown to comprise many hopeful properties such as equity, sparsity and inadequacy and prophecy properties, has been projected as an option to minimum computation time, sparser and more precise solution  $\ell_{2/3}$ -norm regularizer for variable selection and compressive sensing. By achieving these properties of  $\ell_{2/3}$ -norm regularizer. We recommend a new model of SRC via  $\ell_{2/3}$ -norm minimization, which can be formulated as

$$x = \arg \min_x \|y - Ax\|^{2/3} + \lambda \|x\|_{2/3} \quad (2)$$

Where  $\|x\|_{2/3}$  indicate the  $\ell_{2/3}$ -norm of  $x$ ,  $\lambda$  is the regularization parameter which is used to control the sparsity of the sparse representation coefficient vector  $x$ .

**B. An efficient Iterative Algorithm for  $\ell_{2/3}$ -SRC**

The optimization problem of  $\ell_{2/3}$ -norm minimization in (2) is non-convex and is expressed as awkward in the literature. Luckily, we appreciate that the iterative reweighted method can be applied to solve the  $\ell_{2/3}$ -norm minimization problem. Adopting an iterative algorithm to create the weights tends to permit for successively enhanced assessment of the locations of nonzero coefficients.

Let  $F$  and  $G$  be two subsets of  $\{1, 2, \dots, n\}$  which satisfy that  $F \cup G = \{1, 2, \dots, n\}$  and  $F \cap G = \emptyset$ , where  $F$  and  $G$  express the active set and inactive set, respectively. The coefficients in active set are nonzero, whereas the coefficients in inactive set are zeros. We represent an active-set based iterative reweighted algorithm to comfortably solve the optimization problem of  $\ell_{2/3}$ -norm minimization. The whole recognition method of  $\ell_{2/3}$ -SRC is compiled as follows:

- 1. Initialization:** the iteration number  $x = t, x_i^0 = 0, \theta_i^0 = 0, w_i^0 = 1, i = 1, 2, \dots, n$ ,  
Where  $\theta_i^t = \text{sign}(x_i^t) \in \{-1, 0, 1\}$  express the sign of  $x_i^t, \mu^t = \frac{1}{t}, t > 0$ , the active set  $F = \emptyset$ .
- 2. Update the active set:** From zero coefficients of  $x^t$  select the  $r$ th element which satisfies:

$$r = \arg \max_j \left| \frac{\partial \|y - Ax^t\|^{2/3}}{\partial x_j^t} \right|$$

Add  $r$  to the active set, only if it locally improves the objective, namely:

If  $\lambda$ , then set  $\theta_r^t = \frac{\partial \|y - Ax^t\|}{\partial x_r^t}$  and update the active set  $F = F \cup \{r\}$ .

- 3. Feature-sign Step:**
  - 1) Let  $A_F$  be a sub matrix of  $A$  that contains only the columns corresponding to the active set  $F$ . Accordingly,  $x_F^t$  and  $\theta_F^t$  are the sub vectors of  $x^t$  and, respectively.
  - 2) Compute the analytical solution to the unconstrained quadratic problem
  - 3) Perform a discrete line search on the close line segment from  $x_F^t$  to  $x_{new}^t$ : check the objective value at  $x_{new}^t$  and all points where any coefficient changes sign; update  $x_F^t$  to the point with the lowest objective value.
  - 4) Remove zero coefficient of  $x_F^t$  from the active set  $F$  and update  $\theta^t = \text{sign}(x^t)$
- 4. Check the optimality state:**
  - (a) Optimality state for nonzero coefficients:

$$\frac{\partial \|y - Ax^t\|^{2/3}}{\partial x_i^t} + \lambda \theta_i^t = 0, \forall x_i^t \neq 0$$

- (b) Optimality condition for zero coefficient:

$$\left| \frac{\partial \|y - Ax^t\|^{2/3}}{\partial x_i^t} \right| + \lambda, \forall x_i^t \neq 0$$

If state (b) is not satisfied go to step 2; else check state (c).

- (c) Optimality state for  $\ell_{2/3}$ -norm minimization

$$t > t_{max} \text{ or } \left| \sum_{i=1}^n (|x_i^t|^{2/3} - |x_i^{t-1}|^{2/3}) \right| \sum_{i=0}^n |x_i^{t-1}|^{2/3} < \eta$$

$t > 1$

If state (c) is not satisfied:

Let  $t = t + 1$ , update the weights:

$$w_i^t = 1 / (|x_i^{t-1}| + \mu^t) \text{ and set } x_i^t = 0, \theta_i^t = 0, i = 1, 2, \dots, n, = \emptyset, \text{ go to step 2;}$$

Otherwise  $x = x^t$  is the optimal solution.

**5. Calculate the Residuals**

$$r_c(y) = \|y - A\delta_c(x)\|_{2/3}, \text{ for } c = 1, 2, \dots, k$$

Where  $\delta_i(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^n$  is the characteristic function which selects the coefficients from the  $c$ th class.

**6. Output:** identity  $(y) = \arg \min_c r_c(y)$

Some of the coding coefficients  $x_i^t, i=1, 2, \dots, n, t > 0$  could be zeros during the iteration, so we introduce the parameter  $\mu^t$  to ensure the stability and feasibility of the algorithm. The parameter sequence of  $\mu^t$  is a non-increasing sequence of positive numbers when  $t > 0$ , which contributes to ensure the convergence of the optimization problem of (2). The active-set based iterative reweighted algorithm described above finds an approximation solution of the  $\ell_{2/3}$ -norm minimization. Using an iterative algorithm to update the weights tends to allow for successively better estimation of the nonzero coefficient, causing the  $\ell_{2/3}$ -norm of the objective function value of (2) decreases with the iteration. Furthermore, the  $\ell_{2/3}$ -norm of a coding vector is larger than its  $\ell_\infty$ -norm (the maximum absolute value of entries in a vector). As a result,  $\ell_{2/3}$ -norm minimization is bounded and monotonically decreasing function, so it will converge during the iterative procedure.

Besides the maximal number of iteration, we consider  $\ell_{2/3}$ -norm of the coding vectors from adjacent iterations as the condition of convergence. The convergence is achieved when the following condition is met

$$\frac{\left| \sum_{i=1}^n (|x_i^t|^{2/3} - |x_i^{t-1}|^{2/3}) \right|}{\sum_{i=0}^n |x_i^{t-1}|^{2/3}} < \eta, t > 1 \quad (3)$$

Where  $\eta$  is a small positive number.

It is currently well recognized that  $\ell_{2/3}$ -norm minimization can get sparser solution than  $\ell_{1/2}$ -norm minimization. The enhancement on sparsity of  $\ell_{2/3}$  sparse representation over  $\ell_{1/2}$  sparse representation can be recognized to the decrease of nonzero coefficients which corresponds to the training samples from classes that are dissimilar to the test sample. Therefore, it will be easier to precisely resolve the identity of the test sample. He following experiments on FER additional demonstrates the efficiency of  $\ell_{2/3}$ -SRC for classification.

### 3. Experimental Results

In this section, we will calculate the certainty of the  $\ell_{2/3}$ -SRC algorithm for FER on JAFFE databases. We evaluate  $\ell_{2/3}$ -SRC with four associated algorithms: the conventional  $\ell_1$ -norm minimization based SRC ( $\ell_1$ -SRC), the iterative reweighted  $\ell_{1/2}$ -norm minimization algorithm in [11] (IRL<sub>1/2</sub>), the recently developed maximum correntropy based sparse representation (CESR) [14]. We put the parameters for  $\ell_{2/3}$ -SRC as follows: the maximal number of iteration  $t_{max} = 10$ , the nonincreasing sequence of positive numbers  $\mu^t = 1/t$  for  $t > 0$ , the threshold parameter  $\eta = 0.001$ . The positive parameter  $\varepsilon$  is set to 0.1 for IRL<sub>1/2</sub>. The sparsity parameter  $\lambda$  is set to 0.01 for all the challenging methods. Moreover, principal component analysis (PCA)[15] is took to plot the original facial expression images to the feature space with the dimensions of 128 for all the computing methods.

### 4. FER on JAFFE Database

The JAFFE database [16] consists of 213 images from 10 persons of Japanese female, covering seven categories of basic facial expression. All the original images have the same size of 256×256 pixels with 256-level gray scale. The images are cropped automatically to create two eyes align at the similar situation and then resized to 5×5 pixels. Excluding for the neutral expression, we select 4 images for each expression in our testing. We accept the leave-one-out cross-validation approach to compare different algorithms. One image is elected from each expression for testing, while the left 26 images are used for training. This should be repeated 27 times, and the average recognition rate regarded as the final recognition result. Moreover, we initiate “sparsity ratio” to compute the sparsity of a coding vector  $x$ , which can be defined as

$$S(x) = 1 - \frac{\|x\|_0}{L(x)} \quad (4)$$

Where  $\|x\|_0$  is the  $\ell_0$ -norm of vector  $x$ ,  $L(x)$  denotes the length of vector  $x$ , which counts the number of elements in a vector.  $S(x)$  is the sparsity ratio, which denotes the ratio of the number of zeros to the total number of elements in a vector.

Table 1 lists the average recognition rates and sparsity ratios of the five challenging methods. It can be seen that  $\ell_{2/3}$ -SRC outperforms the other four methods from both perspectives of recognition rate and sparsity ratio. The sparsity ratios of  $\ell_{2/3}$ -SRC and IRL<sub>1/2</sub> are comparable to each other, and much higher than those of  $\ell_1$ -SRC and CESR, which confirm that the solution of  $\ell_{2/3}$ -norm minimization is much sparser than

that of  $\ell_{1/2}$ -norm minimization. The results from Table 1 show that  $\ell_{2/3}$ -SRC can establish the characteristics of a test sample more precisely with sparser representation than IRL<sub>1/2</sub>,  $\ell_{1/2}$ -SRC,  $\ell_1$ -SRC and CESR.

**Table 1:** Experimental Results on Jaffe Database

	$\ell_{2/3}$ -SRC	$\ell_{1/2}$ -SRC	IRL <sub>1/2</sub>	$\ell_1$ -SRC	CESR
Recognition rate (%)	98.36	94.28	93.36	92.59	91.36
Sparsity ratios (%)	99.96	99.18	99.06	94.47	94.71

### 5. Conclusion

In this paper, we recommend a new model of sparse representation based classification via  $\ell_{2/3}$ -norm minimization, called  $\ell_{2/3}$ -SRC, for facial expression recognition. Instead of using  $\ell_{1/2}$ -norm minimization to search for the best sparse representation for the test image over the whole training samples. By adopting  $\ell_{2/3}$ -norm minimization, we can get a sparser and more precise solution than the conventional SRC, and the optimization problem of  $\ell_{2/3}$ -norm minimization is much easier to be solved than that of  $\ell_{1/2}$ -norm minimization. Moreover, an active-set based iterative reweighted algorithm is proposed to efficiently resolve the  $\ell_{2/3}$ -norm minimization problem. The experimental outcome on JAFFE exhibit that  $\ell_{2/3}$ -SRC can get better classification performance with sparser representation.

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