Comparative Predictive Behaviour of Two Numerical Techniques to Simulator Design of a Reservoir with Surfactant Mixture in Enhanced Oil Recovery Process

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Abstract: ***We consider here the application of orthogonal collocation and finite difference approximation to simulator design for a reservoir with surfactant mixture in enhanced oil recovery process to the solution of the applicable equations for the multidimensional, multicomponent and multiphase system. In this work, we report on the effect of significant reservoir parameters and the amount or nature of surfactant mixture on reservoir simulator design. Some of the novel aspects of this study stem from the actual formulation of the development of the simulator, in particular, the choice of dependent variables, and the treatment of boundary conditions. Numerical results obtained using orthogonal collocation and finite difference computations are used to control oscillatory overshoot. In both orthogonal collocation and finite difference method, general multi-dimensional schemes were applied in the flow simulations. Matlab computer programs were used for the numerical solution of the model equations. The results of the orthogonal collocation solution were compared with those of finite difference solutions. The results indicate that the concentration profiles of surfactants for orthogonal collocation showed more features than the predictions of the finite difference, offering more opportunities for further understanding of the physical nature of this complex problem. Also, comparison of the orthogonal collocation solution with computations based on finite difference method offers possible explanation for the observed differences especially between the methods and the two reservoirs. We found that the effect of surfactant in enhanced oil recovery process in surfactant flooding is in fact the dominant factor in reservoir simulator design.***

Keywords: Reservoir Simulator Design; Multidimensional, Multicomponent and Multiphase Systems; Surfactant Mixture; Orthogonal Collocation Technique; Finite Difference Approximation

1. Introduction

The development of a simulator of a reservoir in a surfactant assisted water flood required the understanding of the porous formation of complex reservoir and multiphase and multicomponent flow taking place in the reservoir. The understanding of the multiphase, multicomponent flow taking place in any displacement process is essential for successful design of simulator in a reservoir. The world energy demand continues to increase significantly and crude oil still remains the major source.

It is very important to at least, maintain or indeed, increase the current production levels of crude oil. These objectives can be accomplished by further investing in exploration and production of new fields or optimizing production from existing fields. Bringing new fields online is very expensive, while recovery from existing fields by conventional methods (i.e. primary and secondary recovery) will not fully provide the necessary relief for global oil demand.

On an average, only about a third of the original oil in place can be recovered by primary and secondary recovery processes. The rest of the oil is trapped in reservoir pores due to surface and interfacial forces. This trapped oil can be recovered by reducing the capillary forces that prevent oil from flowing within the pores of reservoir rock and into the well bores.Due to high oil prices and declining production in many regions around the globe, the application of advance technologies called "Enhanced Oil Recovery"(EOR) has become very attractive for exploration and production of the trapped oil.This technology requires the injection of a fluid or fluids or materials into a reservoir to supplement the natural energy present in a reservoir, where the injected fluids interact with the reservoir rock /oil /brine system to create favourable conditions for maximum oil recovery. Surfactants are injected to decrease the interfacial tension between oil and water in order to mobilize the oil trapped after secondary recovery by water flooding.

In a surfactant flood, a multi-component multiphase system is involved. The theory of multi- component, multiphase flow has been presented by several authors[1].The surfactant flooding is a form of chemical flooding and is represented by a system of nonlinear partial differential equations: the continuity equation for the transport of the components and Darcy’s equation for the phase flow. The system of equations is completed by the equations representing physical properties of the fluids and the rock. From a physico-chemical point of view, there are three components - water, petroleum and chemical. They are in fact, pseudo-components, since each one consists of several pure components. Petroleum is a complex mixture of many hydrocarbons. Water is actually brine, and contains dissolved salts. Finally, the chemical contains different kinds of surfactants. These components are distributed between two phases –the oleic phase and the aqueous phase. The chemical has an amphiphilic character. It makes the oleic

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phase at least partially miscible with water or the aqueous phase, partially miscible with petroleum.

Interfacial tension depends on the surfactant partition between the two phases. Residual phase saturation decrease as interfacial tension decreases. Relative permeability parameters depend on residual phase saturations. In addition, phase viscosities are functions of the volume fraction of the components in each fluid phase. Therefore, the success or failure of surfactant flooding processes depends on phase behaviour. Phase behaviour influences all other physical properties, and each of them, in turn influences oil recovery.

The two different mathematical techniques are to be utilized in identifying a particular type of physical behaviour and thus enabling the understanding of the propagation phenomena. More so, the techniques will in particular be utilized to predict what happens in EOR process and show how the complexity of the problem can be reduced. Systems of coupled, first-order, nonlinear hyperbolic partial differential equations (p.d.e.s) govern the transient evolution of a chemical flooding process for enhanced recovery. The method of characteristics (MOC) provides a way in which such systems of hyperbolic p.d.e.s can be solved by converting them to an equivalent system of ordinary differential equations. In some cases, the characteristic solution has been used to track the flood-front in two-dimensional reservoir problems [2]. Besides, another approach combines the characteristic method with a finite element approach [3]. The MOC and an adjustable number of moving particles to track three-dimensional solute fronts has been used in groundwater systems; adjusting the number of particles serves to maintain an accurate material balance and save computational time [4]. This front-tracking approach has been used in the present work to trace the movement of coherent waves, of both the diffuse and shock variety.

At the simple level, the results of simulation using the two techniques are analogous to the Buckley-Leverett theory for water flooding, the latter being evident in the case of polymer flooding [5], Also for dilute surfactant flooding[6], For carbonated water flooding, [7] and For miscible [8] and immiscible surfactant flooding[9]. For isothermal, multiphase, multicomponent fluid Flow in permeable media [10].While Case studies for the feasibility of sweep improvement in surfactant-assisted water flooding.[11]

High oil prices and declining production in many regions around the globe make enhanced oil recovery (EOR) increasingly attractive. As evident in the work for a new class of viscoelastic surfactants for EOR[12], For microbially enhanced oil recovery at simulated reservoir conditions by use of engineered bacteria[13], for co-optimization of enhanced oil recovery and carbon sequestration[14],while for development of improved surfactants and EOR methods for small operators[15] and many others.

The present work describes the design of a simulator for an Enhanced Oil Recovery process using surfactant assisted water flooding by applying two different mathematical methods, orthogonal collocation and finite difference method, to solve the basic model transport equations. The approach is multidimensional and involves at least three independent variables for mapping the composition routes of the system components.

2. Methodology

This work considered the solution of a multidimensional, multicomponent and multiphase flow problem associated with enhanced oil recovery process in petroleum engineering. The process of interest involves the injection of surfactant of different concentrations and pore volume to displace oil from the reservoir.

The methodology used here is illustrated by the steps utilized in executing the solution using the developed mathematical models describing the physics of reservoir depletion and fluid flow in which one of the main aims is the determination of the areal distribution of fluids in the flooded reservoir. The system is for two or three dimensions, two fluid phases (aqueous, oleic) and one adsorbent phase, four components (oil, water, surfactants 1 and 2).

The reservoir may be divided into discrete grid blocks which may each be characterized by having different reservoir properties. The flow of fluids from a block is governed by the principle of mass conservation coupled with Darcy's law. The following are taken into consideration in the modeling effort:

(i) The simultaneous flow of oil, gas, and water in three dimensions
(ii) The effects of natural water influx, fluid compressibility, mass transfer between gas and liquid phases and
(iii) The variation of such parameters as porosity and permeability, as functions of pressure.

The model is developed from the basic law of conservation of mass with assumptions[16] .

The developed partial differential equation is converted to ordinary differential equation using finite difference and orthogonal collocation methods.

The finite difference method is a technique that converts partial differential equations into a system of linear equations. There are essentially three finite difference techniques. The explicit, finite difference method converts the partial differential equations into an algebraic equation which can be solved by stepping forward (forward difference), backward (backward difference) or centrally (central difference).

The orthogonal collocation method converts partial differential equations into a system of ordinary differential equations using the Lagrangian polynomial method. This set of ordinary differential equations generated is then solved with appropriate numerical technique such as the Runge Kutta.

The rock and fluid properties such as density, porosity, viscosity, oil and water etc, and other parameters are listed

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in Tables 1, 2, 3 and 4. Table 1 is the reservoir characteristics from previous work [16]. Table 2 is the reservoir characteristics used for the simulation work [17]. Parameter values used in Trogus adsorption model and for verification runs are shown in Table 3[17], while Table 4 contains additional reservoir parameters presented for the work [16].

In considering the more general form of the multiphase, multicomponent problem, the explicit Runge-Kutta method is chosen for the solution of the problem. The motivation for this explicit method is its simplicity and computational efficiency with regard to the reduction of truncation errors more effectively than other methods. The MATLAB computer program was used to obtain the solutions.

The model encompasses two fluid phases (aqueous and oleic), one adsorbent phase (rock), and four components (oil, water, surfactants 1 and 2). The oil is displaced by water flooding. In-situ interaction of surfactant slugs may occur, with consequent phase separation and local permeability reduction. The model accommodates two (or three) physical dimensions and an arbitrary, nonisotropic description of absolute permeability variation and porosity.

For most of the simulated cases in the work, the reservoir consisted of a rectangular composite of horizontal oil bearing strata, sandwiched above and below by two impervious rocks [16]. Oil is produced from the reservoir by means of water injection at one end and a production well at the other. Data for the hypothetical reservoir simulated are given in Table 1and the model developed [16] is

$$\phi S_w \frac{\partial C_{i,w}}{\partial t} + \rho(1-\phi)\frac{\partial \bar{C}_i}{\partial t} + \phi v_x f_w \frac{\partial C_{i,w}}{\partial x} + \phi v_y f_w \frac{\partial C_{i,w}}{\partial y} = -r_i \left(i = 1,2\right) \quad (1)$$

The term r_i represents the rate of loss of surfactant due to precipitation: for a one-to-one reaction stoichiometry, $r_1 = r_2$. Since reaction occurs instantaneously at a sharp interface, this term may be ignored away from the singular region of the interface.

2.5 Adsorption Model

It is possible to approximate the adsorption isotherm of a pure surfactant on a mineral oxide by use of a simple model. At low concentration the adsorption obeys Henry's law, while above the critical micelle concentration (CMC), the total adsorption remains constant. The Trogus adsorption model [18], [19] is used in this work. The following assumptions are made:

3. Application of Finite Difference to Solution of Model Equations

First-order, finite-difference expressions for the spatial derivatives were substituted into the hyperbolic chromatographic transport equations (Eq. 1), yielding 2 x m coupled ordinary differential equations which may then be integrated simultaneously (also known as the 'numerical method of lines').

$$s_w \frac{\partial C_{i,w}}{\partial \tau} + \sum_{j=1}^{2} m_{ij} \frac{\partial C_{i,w}}{\partial \tau} + f_w\left(\tau, \varepsilon_h\right) \times \left\{ \frac{C_{i,w}\left(\tau, \varepsilon_h\right) - C_{i,w}\left(\tau, \varepsilon_{h-1}\right)}{\Delta \varepsilon} \right\} = 0 \quad (2)$$

where i = 1,2 and h = 1,2,. .. m.

Eqn.2 is the finite-difference form of Eqn.1written for one spatial dimension ε, where m_{ij} are the adsorption coefficients, τ is dimensionless time (injected volume/ pore volume), and ε is dimensionless distance (pore volumes travelled). In two dimensions, the finite-difference terms are multiplied by dimensionless velocities. The distortion of the solution in the τ direction may be neglected by using a 4th order Runge-Kutta method and a sufficiently small time step.

The above equation is now transformed to the original form of Eqns. 1 using the following defined variables:

$$C'_{i,w} = \phi C_{i,w} \quad (3)$$

$$\bar{C}'_i = \rho(1-\phi)\bar{C}'_i \quad (4)$$

$$m_{i,j} = \frac{\partial \bar{C}'_i}{\partial C'_{j,w}} \quad (5)$$

Again, recall that differentiation of a function of another function (chain rule) is of the form

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial x} \quad (6)$$

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Applying the chain rule above, Eqn.2 becomes

$$S_w \frac{\partial C'_{i,w}}{\partial \tau} + \left(\frac{\partial \bar{C}'_i}{\partial C'_{1,w}} \cdot \frac{\partial C'_{1,w}}{\partial \tau} \right) + \left(\frac{\partial \bar{C}'_i}{\partial C'_{2,w}} \cdot \frac{\partial C'_{2,w}}{\partial \tau} \right) + f_w(\tau, \varepsilon_h) \times \left\{ \frac{C'_{i,w}(\tau,\varepsilon_h) - C'_{i,w}(\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (7)$$

Eliminating the primes (') and bars (-) and introducing $m_{i,j}$ terms yield

$$(S_w + m_{11})\frac{\partial C_{1,w}}{\partial \tau} + m_{12}\frac{\partial C_{2,w}}{\partial \tau} + f_w \frac{\partial C_{1,w}}{\partial \varepsilon} = 0 \quad (8)$$

$$(S_w + m_{22})\frac{\partial C_{2,w}}{\partial \tau} + m_{21}\frac{\partial C_{1,w}}{\partial \tau} + f_w \frac{\partial C_{2,w}}{\partial \varepsilon} = 0 \quad (9)$$

Applying the method of lines, a partial transformation to a difference equation, to the equations above yield:

$$(S_w + m_{11})\frac{\partial C_{1,w}}{\partial \tau} + m_{12}\frac{\partial C_{2,w}}{\partial \tau} + f_w \frac{C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}}{\Delta\varepsilon} = 0 \quad (10)$$

$$(S_w + m_{22})\frac{\partial C_{2,w}}{\partial \tau} + m_{21}\frac{\partial C_{1,w}}{\partial \tau} + f_w \frac{C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}}{\Delta\varepsilon} = 0 \quad (11)$$

This can also be written as follows

$$(S_w + m_{11})\frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + m_{12}\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (12)$$

$$(S_w + m_{22})\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + m_{21}\frac{\partial C_{1,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (13)$$

Since we have a set of simultaneous ODE's, we will attempt to solve the equations

$$(S_w + m_{11})\frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + m_{12}\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (14)$$

$$(S_w + m_{22})\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + m_{21}\frac{\partial C_{1,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (15)$$

where

$$m_{11} = \frac{\partial \bar{C}_1}{\partial C_{1,w}}$$
$$m_{12} = \frac{\partial \bar{C}_1}{\partial C_{2,w}}$$
$$m_{21} = \frac{\partial \bar{C}_2}{\partial C_{1,w}}$$
$$m_{22} = \frac{\partial \bar{C}_2}{\partial C_{2,w}}$$

Substitution of these terms in Eqs.14 and 15 yield

$$\left(S_w + \frac{\partial \bar{C}_1}{\partial C_{1,w}}\right)\frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + \frac{\partial \bar{C}_1}{\partial C_{2,w}}\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (16)$$

and

$$\left(S_w + \frac{\partial \bar{C}_2}{\partial C_{2,w}}\right)\frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{\partial \bar{C}_2}{\partial C_{1,w}}\frac{\partial C_{1,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (17)$$

These on simplification yield

$$S_w \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + \frac{\partial \bar{C}_1}{\partial C_{1,w}} \cdot \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + \frac{\partial \bar{C}_1}{\partial C_{2,w}} \cdot \frac{\partial C_{2,w_{(\tau,\varepsilon_h)}}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

$$S_w \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + \frac{\partial \bar{C}_1}{\partial \tau} + \frac{\partial \bar{C}_1}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (18)$$

$$S_w \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + 2\frac{\partial \bar{C}_1}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

similarly

$$S_w \frac{\partial C_{2,w(\tau,\varepsilon_h)}}{\partial \tau} + 2\frac{\partial \bar{C}_2}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (19)$$

From the Trogus model,

$$\bar{C}_1 = k_1 C_{1,w}$$
$$\bar{C}_2 = k_2 C_{2,w}$$

A final substitution results in the equation below:

$$S_w \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + 2\frac{\partial (k_1 C_{1,w})}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

$$S_w \frac{\partial C_{1,w(\tau,\varepsilon_h)}}{\partial \tau} + 2k_1\frac{\partial C_{1,w}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

$$(S_w + 2k_1)\frac{\partial C_{1,w}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{1,w_{(\tau,\varepsilon_h)}} - C_{1,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

and

$$S_w \frac{\partial C_{2,w(\tau,\varepsilon_h)}}{\partial \tau} + 2\frac{\partial (k_2 C_{2,w})}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0$$

$$(S_w + 2k_2)\frac{\partial C_{2,w}}{\partial \tau} + \frac{f_w}{\Delta\varepsilon}\left[C_{2,w_{(\tau,\varepsilon_h)}} - C_{2,w_{(\tau,\varepsilon_h-1)}}\right] = 0 \quad (21)$$

3.2 Application of Orthogonal Collocation to Solution of Model Equations

Equation7can be written as:

$$S_w \frac{\partial C'_{i,w}}{\partial \tau} + 2\frac{\partial \bar{C}'_i}{\partial \tau} + f_w(\tau,\varepsilon_h) \times \left\{ \frac{C'_{i,w}(\tau,\varepsilon_h) - C'_{i,w}(\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (22)$$

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$$S_w \frac{\partial[\phi C_{i,w}]}{\partial \tau} + 2\frac{\partial[\rho(1-\phi)\bar{C}_i]}{\partial \tau} + f_w(\tau,\varepsilon_h) \times \left\{ \frac{[\phi C_{i,w}](\tau,\varepsilon_h) - [\phi C_{i,w}](\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (23)$$

$$\phi S_w \frac{\partial C_{i,w}}{\partial \tau} + 2\rho(1-\phi)\frac{\partial \bar{C}_i}{\partial \tau} + \phi f_w(\tau,\varepsilon_h) \times \left\{ \frac{C_{i,w}(\tau,\varepsilon_h) - C_{i,w}(\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (24)$$

$$\bar{C}_i = \kappa_i C_{i,w} \quad (25)$$

Now, from the Trogus model,

$$\phi S_w \frac{\partial C_{i,w}}{\partial \tau} + 2\rho(1-\phi)\frac{\partial(\kappa_i C_{i,w})}{\partial \tau} + \phi f_w(\tau,\varepsilon_h) \times \left\{ \frac{C_{i,w}(\tau,\varepsilon_h) - C_{i,w}(\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (26)$$

$$\phi S_w \frac{\partial C_{i,w}}{\partial \tau} + 2\kappa_i\rho(1-\phi)\frac{\partial C_{i,w}}{\partial \tau} + \phi f_w(\tau,\varepsilon_h) \times \left\{ \frac{C_{i,w}(\tau,\varepsilon_h) - C_{i,w}(\tau,\varepsilon_{h-1})}{\Delta\varepsilon} \right\} = 0 \quad (27)$$

$$\phi S_w \frac{\partial C_{i,w}}{\partial \tau} + 2\kappa_i\rho(1-\phi)\frac{\partial C_{i,w}}{\partial \tau} + \phi f_w(\tau,\varepsilon_h)\frac{\partial C_{i,w}}{\partial \varepsilon} = 0 \quad (28)$$

$$[\phi S_w + 2\kappa_i\rho(1-\phi)]\frac{\partial C_{i,w}}{\partial \tau} + \phi f_w(\tau,\varepsilon_h)\frac{\partial C_{i,w}}{\partial \varepsilon} = 0 \quad (29)$$

Let

$$R = [\phi S_w + 2\kappa_i\rho(1-\phi)]$$
$$B = \phi f_w$$

The above equations now become:

$$R\frac{\partial C}{\partial \tau} + B\frac{\partial C}{\partial \varepsilon} = 0 \quad (30)$$

where C is a function of both ε (dimensionless distance) and τ (dimensionless time).

Using the method of orthogonal collocation, let C be approximated by the expression

$$C(\tau,\varepsilon) = \sum_{I=1}^{N+1} C_I(\tau) X_J(\varepsilon_I) \quad (31)$$

Equation 31 can now be expressed as follows:

$$R\frac{\partial C}{\partial \tau} + B\frac{\partial}{\partial \varepsilon}\sum_{I=1}^{N+1} C_I(\tau) X_J(\varepsilon_I) = 0 \quad (32)$$

$$R\frac{\partial C}{\partial \tau} + B\sum_{I=1}^{N+1}\frac{\partial}{\partial \varepsilon}[C_I(\tau) X_J(\varepsilon_I)] = 0 \quad (33)$$

$$R\frac{\partial C}{\partial \tau} + B\sum_{I=1}^{N+1}\frac{\partial}{\partial \varepsilon}[X_J(\varepsilon_I)].C_I(\tau) = 0 \quad (34)$$

$$a_{JI} = \frac{\partial}{\partial \varepsilon} X_J(\varepsilon_I) \quad (35)$$

$$R\frac{\partial C_J}{\partial \tau} + B\sum_{I=1}^{N+1} a_{JI} C_I = 0 \quad (36)$$

$$\frac{\partial C_J}{\partial \tau} + \frac{B}{R}\sum_{I=1}^{N+1} a_{JI} C_I = 0 \quad (37)$$

$$\frac{\partial C_J}{\partial \tau} = -\frac{B}{R}\sum_{I=1}^{N+1} a_{JI} C_I \quad (38)$$

For I = 1, 2, 3, 4… N+1

Therefore,

$$\frac{\partial C_J}{\partial \tau} = -\frac{B}{R}[a_{J1}C_1 + a_{J2}C_2 + a_{J3}C_3 + a_{J4}C_4 + \ldots + a_{JN+1}C_{N+1}] \quad (39)$$

Again J = 1, 2, 3, 4… N+1

Therefore the following system of ODE's can be generated

$$\frac{\partial C_1}{\partial \tau} = -\frac{B}{R}[a_{11}C_1 + a_{12}C_2 + a_{13}C_3 + a_{14}C_4 + \ldots + a_{1N+1}C_{N+1}]$$

$$\frac{\partial C_2}{\partial \tau} = -\frac{B}{R}[a_{21}C_1 + a_{22}C_2 + a_{23}C_3 + a_{24}C_4 + \ldots + a_{2N+1}C_{N+1}]$$

$$\frac{\partial C_3}{\partial \tau} = -\frac{B}{R}[a_{31}C_1 + a_{32}C_2 + a_{33}C_3 + a_{34}C_4 + \ldots + a_{3N+1}C_{N+1}]$$

$$\frac{\partial C_4}{\partial \tau} = -\frac{B}{R}[a_{41}C_1 + a_{42}C_2 + a_{43}C_3 + a_{44}C_4 + \ldots + a_{4N+1}C_{N+1}]$$

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$$\frac{\partial C_{N+1}}{\partial \tau} = -\frac{B}{R}\left[a_{N+11}C_1 + a_{N+12}C_2 + a_{N+13}C_3 + a_{N+14}C_4 + \dots + a_{N+1N+1}C_{N+1}\right] \quad (40)$$

In matrix form, we have the following expression:

$$\begin{bmatrix} \frac{\partial C_1}{\partial \tau} \\ \frac{\partial C_2}{\partial \tau} \\ \frac{\partial C_3}{\partial \tau} \\ \frac{\partial C_4}{\partial \tau} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \frac{\partial C_{N+1}}{\partial \tau} \end{bmatrix} = -\frac{B}{R}\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1N+1} \\ a_{21} & a_{22} & a_{23} & a_{24} & & a_{2N+1} \\ a_{31} & \vdots & & & & a_{3N+1} \\ a_{41} & \vdots & & & & a_{4N+1} \\ \vdots & \vdots & & & & \\ \vdots & \vdots & & & & \\ \vdots & \vdots & & & & \\ \vdots & \vdots & & & & \\ \vdots & \vdots & & & & \\ a_{N+11} & a_{N+12} & \cdots & \cdots & \cdots & a_{N+1N+1} \end{bmatrix}\begin{bmatrix} C_1(\tau) \\ C_2(\tau) \\ C_3(\tau) \\ C_4(\tau) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ C_{N+1}(\tau) \end{bmatrix} \quad (41)$$

Similarly, the following expression defines a_{JI}[20], [21]

$$a_{JI} = \begin{cases} \frac{1}{2}\frac{P^{(2)}_{N+1}(\varepsilon_I)}{P^{(1)}_{N+1}(\varepsilon_I)} & \text{For J = I} \\ \frac{1}{\varepsilon_I - \varepsilon_J}\frac{P^{(1)}_{N+1}(\varepsilon_I)}{P^{(1)}_{N+1}(\varepsilon_J)} & \text{For I} \neq \text{J} \end{cases} \quad (42)$$

where

$$\begin{aligned} &P_J(\varepsilon) = (\varepsilon - \varepsilon_J)P_{J-1}(\varepsilon); J = 1,2,3,\dots,N+1 \\ &P^{(1)}_J(\varepsilon) = (\varepsilon - \varepsilon_J)P^{(1)}_{J-1}(\varepsilon) + P_{J-1}(\varepsilon) \\ &P^{(2)}_J(\varepsilon) = (\varepsilon - \varepsilon_J)P^{(2)}_{J-1}(\varepsilon) + 2P^{(1)}_{J-1}(\varepsilon) \\ &P^{(1)}_0(\varepsilon) = P^{(2)}_0(\varepsilon) = 0 \\ &P_0(\varepsilon) = 1 \end{aligned} \quad (43)$$

Recall that the elements of the matrix can be generated from the following Lagrange polynomial

$$a_{ij} = \frac{dl_j(x_i)}{dx} = \begin{cases} \frac{1}{2}\frac{P^{(2)}_{N+1}(x_i)}{P^{(1)}_{N+1}(x_i)} & j=i \\ \frac{1}{x_i - x_j}\frac{P^{(1)}_{N+1}(x_i)}{P^{(1)}_{N+1}(x_j)} & i \neq j \end{cases} \quad (44)$$

For i = j, the elements here refer to the leading diagonal of the matrix to be generated

For i ≠ j, the elements here refer to all other elements of the matrix

Also, the following recurrence relations are defined below.

$$\begin{aligned} &p_o(x) = 1 \\ &P_j(x) = (x - x_j)P_{j-1}(x) \\ &P^{(1)}_j(x) = (x - x_j)P^{(1)}_{j-1}(x) + P_{j-1}(x) \\ &P^{(2)}_j(x) = (x - x_j)P^{(2)}_{j-1}(x) + 2P^{(1)}_{j-1}(x) \end{aligned} \quad (45)$$

For j = 2, 3, 4, ..., N+1

The following substitutions and manipulations will now be made to redefine Eqn.44. Substituting the recurrence relations into Eqn.44 yields:

$$a_{ij} = \begin{cases} \frac{1}{2}\left[\frac{(x_i - x_j)P^{(2)}_{j-1}(x_i) + 2P^{(1)}_{j-1}(x_i)}{(x_i - x_j)P^{(1)}_{j-1}(x_i) + P_{j-1}(x_i)}\right] & j=i \\ \frac{1}{x_i - x_j}\left[\frac{(x_i - x_j)P^{(1)}_{j-1}(x_i) + P_{j-1}(x_i)}{(x_j - x_j)P^{(1)}_{j-1}(x_j) + P_{j-1}(x_j)}\right] & i \neq j \end{cases} \quad (46)$$

Now, some terms will be cancelled out.
Since j = i,
$(x_i - x_j) = 0$
and
$(x_j - x_j) = 0$

$$a_{ij} = \begin{cases} \frac{1}{2}\left[\frac{2P^{(1)}_{j-1}(x_i)}{P_{j-1}(x_i)}\right] & j=i \\ \frac{1}{x_i - x_j}\left[\frac{(x_i - x_j)P^{(1)}_{j-1}(x_i) + P_{j-1}(x_i)}{P_{j-1}(x_j)}\right] & i \neq j \end{cases} \quad (47)$$

The above becomes:

$$a_{ij} = \begin{cases} \left[\frac{P^{(1)}_{j-1}(x_i)}{P_{j-1}(x_i)}\right] & j=i \\ \frac{(x_i - x_j)P^{(1)}_{j-1}(x_i)}{(x_i - x_j)P_{j-1}(x_j)} + \frac{1}{x_i - x_j}\left[\frac{P_{j-1}(x_i)}{P_{j-1}(x_j)}\right] & i \neq j \end{cases} \quad (48)$$

This becomes:

$$a_{ij} = \begin{cases} \left[\frac{P^{(1)}_{j-1}(x_i)}{P_{j-1}(x_i)}\right] & j=i \\ \frac{P^{(1)}_{j-1}(x_i)}{P_{j-1}(x_j)} + \frac{1}{x_i - x_j}\left[\frac{P_{j-1}(x_i)}{P_{j-1}(x_j)}\right] & i \neq j \end{cases} \quad (49)$$

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Rewriting the above in terms of epsilon, (ε):

$$a_{ij} = \begin{cases} \left[\dfrac{P_{j-1}^{(1)}(\varepsilon_i)}{P_{j-1}(\varepsilon_i)}\right] & j=i \\ \dfrac{P_{j-1}^{(1)}(\varepsilon_i)}{P_{j-1}(\varepsilon_j)} + \dfrac{1}{\varepsilon_i - \varepsilon_j}\left[\dfrac{P_{j-1}(\varepsilon_i)}{P_{j-1}(\varepsilon_j)}\right] & i \neq j \end{cases} \quad (50)$$

The matrix now looks like this:

$$a_{11} = \frac{P_0^{(1)}(\varepsilon_1)}{P_0(\varepsilon_1)}$$

$$a_{12} = \frac{P_1^{(1)}(\varepsilon_1)}{P_1(\varepsilon_2)} + \frac{1}{\varepsilon_1 - \varepsilon_2}\frac{P_1(\varepsilon_1)}{P_1(\varepsilon_2)}$$

$$a_{13} = \frac{P_2^{(1)}(\varepsilon_1)}{P_2(\varepsilon_3)} + \frac{1}{\varepsilon_1 - \varepsilon_2}\frac{P_2(\varepsilon_1)}{P_2(\varepsilon_3)}$$

$$a_{21} = \frac{P_0^{(1)}(\varepsilon_2)}{P_0(\varepsilon_1)} + \frac{1}{\varepsilon_2 - \varepsilon_1}\frac{P_0(\varepsilon_2)}{P_0(\varepsilon_1)}$$

$$a_{22} = \frac{P_1^{(1)}(\varepsilon_2)}{P_1(\varepsilon_1)}$$

$$a_{23} = \frac{P_2^{(1)}(\varepsilon_2)}{P_2(\varepsilon_3)} + \frac{1}{\varepsilon_2 - \varepsilon_3}\frac{P_2(\varepsilon_2)}{P_2(\varepsilon_3)}$$

$$a_{31} = \frac{P_0^{(1)}(\varepsilon_3)}{P_0(\varepsilon_1)} + \frac{1}{\varepsilon_3 - \varepsilon_1}\frac{P_0(\varepsilon_3)}{P_0(\varepsilon_1)}$$

$$a_{32} = \frac{P_1^{(1)}(\varepsilon_3)}{P_1(\varepsilon_2)} + \frac{1}{\varepsilon_3 - \varepsilon_2}\frac{P_1(\varepsilon_3)}{P_1(\varepsilon_2)}$$

$$a_{32} = \frac{P_2^{(1)}(\varepsilon_3)}{P_2(\varepsilon_3)} \quad (51)$$

The recurrence relations below will again be used to evaluate the terms of the matrix.

$$\begin{aligned} p_o(\varepsilon) &= 1 \\ P_j(\varepsilon) &= (\varepsilon - \varepsilon_j)P_{j-1}(\varepsilon) \\ P_j^{(1)}(\varepsilon) &= (\varepsilon - \varepsilon_j)P_{j-1}^{(1)}(\varepsilon) + P_{j-1}(\varepsilon) \\ P_0^{(1)}(\varepsilon) &= 0 \end{aligned} \quad (52)$$

Let ε assume the range:
ε = [0:0.01:0.09]
where

$\varepsilon_1 = 0$ (53)
$\varepsilon_2 = 0.01$ (54)
$\varepsilon_3 = 0.02$ (55)

4. Results

The reservoir response, as predicted by the simulation on the basis of orthogonal collocation is compared with the numerical predictions obtained using traditional finite difference method. The case studies are chosen to be both hypothetical and using of existing Nigerian well data with simple representative of the important elements of the simulator. The main objective of these case studies has been to demonstrate that the mathematical techniques of orthogonal collocation and finite difference in the context of application of the simulator can be used to obtain wave behaviour in a reservoir. A gradually increasing level of complexity is introduced, representing a range of systems from aqueous phase flow, to surfactant chromatography in two phase flow, to surfactant chromatography in two dimensional porous medium. The initial and injected surfactant compositions corresponding to cases 1,2 and3 are shown in Table 5. The rock and fluid properties are listed in Table 1, 2, 3 and4. These were taken as uniform for convenience.

The two fluid phases consisted of a water phase and an oil phase, which, for convenience are considered incompressible. The density of oil, the viscosity of oil, the salinity of water, and the formation volume factor of oil and water are listed in Table 2. All cases mentioned above were run by using anionic sodium dodecyl sulfate (SDS) and cationic dodecyl pyridinium chloride (DPC) as surfactants.

The system of equations is complete with the equations representing physical properties of the fluids and the rock. Physical properties described here are: (i) phase behaviour (ii) interfacial tension between fluid phases, (iii) residual phase saturations, (iv) relative permeabilities, (v) rock wettabiliy, (vi) phase viscosities, (vii) capillary pressure, (viii) adsorption and (ix) dispersion. From a physico-chemical point of view, there are three components: water, petroleum and chemical. As stated earlier on, these are all pseudo-components, since each one consists of several pure components. Petroleum is a complex mixture of many hydrocarbons. Water is actually brine, and contains dissolved salts. Finally, the chemical contains different kinds of surfactants.

These three pseudo-components are distributed between two phases –the oleic phase and the aqueous phase. The chemical has an amphiphilic character. It makes the oleic phase at least partially miscible with water or the aqueous phase at partially miscible with petroleum.

Interfacial tension depends on the surfactant partition between the two phases. Residual phase saturation decreases as interfacial tension decreases. Relative permeability parameters depend on residual phase saturations. Phase viscosities are functions of the volume fraction of the components in each fluid phase. Therefore, the success or failure of surfactant flooding processes depends on phase behaviour. Phase behaviour influences all other physical properties, and each of them, in turn influences oil recovery.

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4.1Results of Reservoir Prediction in an Aqueous Phase Chromatographic Flow in One Dimension

Figure 1a is the result obtained for solving Equation 2 using the numerical technique for both orthogonal collocation and finite difference. The graph is for the bed composition profile for one dimensional aqueous-phase chromatography (case 1) at one half pore volume injected.

If a one-dimensional, adsorbing porous medium is initially equilibrated with an aqueous composition C_1 = 0.21, C_2 = 0.181 (concentrations normalized as moles in solution per m^3 off bed) and is then injected with a composition C_1 = 0.17, C_2 = 0.013 (Riemann-type problem: case 1, refer to Table 5), the composition upstream of this injected fluid and composition downstream of the initial or previously injected fluid follows the slow "path" from the injected composition to the junction with the "fast path" from the final composition, where it switches to this "fast" path. In Figure 1a, the profile C_1 of finite difference (FD)shows a steady rise from C_1 = 0.17 to C_1 = 0.21 and then attainecd a constant state. Also the profile C_1 of the orthogonal collocation (OC) increased steadily from C_1 = 0.17 to C_1 = 0.21 after which it started depressing from C_1= 0.2 at distance 0.3 epsilon to C_1 = 0.07 at distance 0.5 epsilon before rising back to attain a constant state with the finite difference method. Similarly, the C_2 of finite difference (FD) increased steadily from C_2 =0.017 to a constant state as for C_1.The constant state is at C_2 = 0.18. The orthogonal collocation (OC) for C_2first moves at constant state before rising steadily to C_2 = 0.18 and then declined from C_2 = 0.18 to a minimum of C_2 = 0.08 before rising to a constant state. The profiles for finite difference (FD) and that of orhogonal collocation (OC) agree except for the depressions of the orthogonal collocation profiles.

Figure 1b shows the results obtained for solving Eqn.2 by the use of orthogonal collocation (OC) and finite difference (FD) methods. The graph is for the bed composition profile for one dimensional aqueous phase chromatography for case 1 at one pore volume injected.In this case also, the adsorbing porous medium is initially equibrated with an aqueous composition concentrations.C_1 = 0.21, C_2 = 0.181(concentrations normalized as moles in solution per m^3 off bed) and is then injected with a composition C_1 = 0.17, C_2 = 0.013 (Riemann-type problem: case 1,(refer to Table 5)). The profile C_1 of finite difference (FD) indicates rise in concentration fromC_1 = 0.17 to 0.21 after which the concentration maintained a constant state. The profile of C1 of the orthogonal collocation (OC) also rise from C_1 = 0.17 to C_1 = 0.21 but falls to 0.03 at distance 0.4 epsilon and then increased steadily to constant state as for C_1 finite difference (FD). The C_2 of finite difference increased steadily from C_2 = 0.02 to attain constant state at 0.18. Also the profile of C_2 of the orthogonal collocation (OC) increase gradually from C_2 = 0.02 to C_1 = 0.18 at distance 0.2 epsilon for short constant state and thendecline to C_2 = 0.02 at distance 0.4 epsilon before rising back to reach constant state with the finite difference.

The bed composition profile for one dimensional aqueous phase chromatography for case 1 at two pore volume injected is shown in Figure 1c. This is the result obtained for solving Eqn.2by using orthogonal collocation (OC) and finite difference (FD) methods. The adsorbing porous medium is initially equibrated with an aqueous composition concentrations. C_1 = 0.21, C_2 = 0.181 (concentrations normalized as moles in solution per m^3 off bed) and is then injected with a composition C_1 = 0.17, C_2 = 0.013 (Riemann-type problem: case 1,(refer to Table 5)). The profile C_1 of finite difference (FD) and the profile C_1 of orthogonal collocation (OC) indicate that there is steady increase from C_1 = 0.17 to C_1 = 0.21 at distance 0.1 epsilon and then attained a constant state for both profiles. Similarly, the profile C_2 of finite difference (FD) shows a steady rise from C_2 = 0.02 to C_2 = 0.18 and then maintained a constant state. Also, the profile C_2 for orthogonal collocation (OC), follows the same pattern, which indicate an increase from C_2 = 0.02 to C_2 = 0.18 and then attained a constant state. The orthogonal collocation (OC) profiles match the finite difference (FD) profiles.

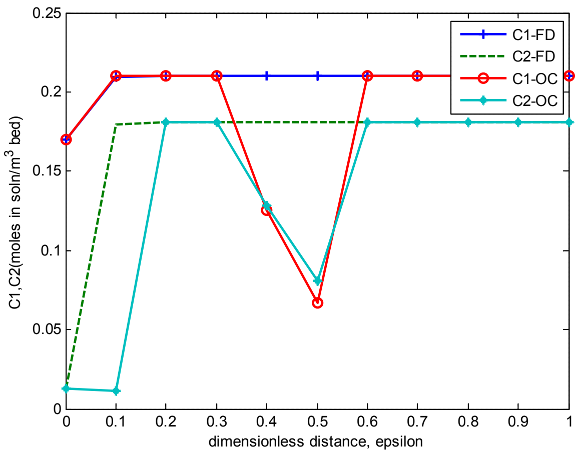

Figure 1a: CASE 1 C_1,C_2 vs epsilon at τ= 0.5. Bed composition profile for one-dimensional aqueous-phase chromatography; case 1, at one-half pore volume injected. The plots are for two methods: Orthogonal collocation (OC), and finite difference (FD).

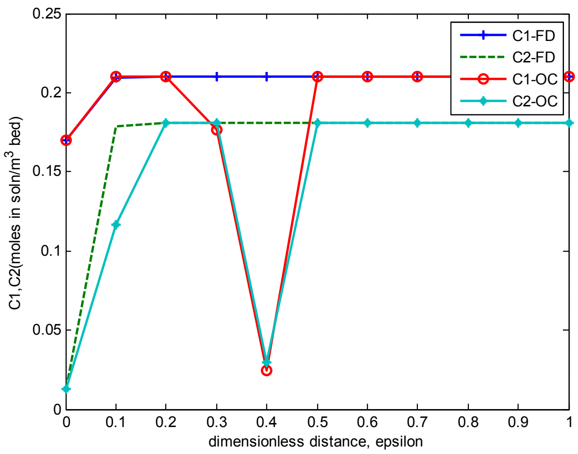

Figure 1b:. CASE 1 C_1,C_2 vs epsilon at τ = 1.0. Bed composition profile for one-dimensional aqueous-phase chromatography; case 1, at one pore volume injected. The plots are for two methods: Orthogonal collocation (OC), and finite difference (FD).

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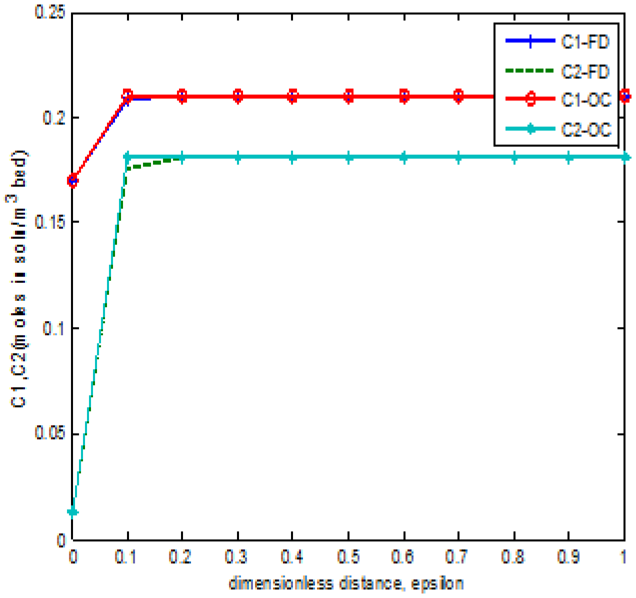

Figure 1c: CASE 1 C_1,C_2 vs epsilon at $\tau = 2.0$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 1, at two pore volumes injected. The plots are for two methods: Orthogonal collocation (OC), and finite difference (FD).

Figure 2a shows the bed concentration profiles for one dimensional aqueous phase chromatography for case 2 at one-half pore volume injected in the adsorbing porous medium initially devoid of surfactant and then injected with a mixture $C_1 = 0.042$, $C_2 = 0.115$ (Riemann-type problem: case 2 (refer to Table 5)),with the numerical result obtained for solving Eqn.2 by using orthogonal collocation (OC) and finite difference (FD) as the numerical technique.The profile C_1 of finite difference (FD) indicates a steady fall from in concentration from $C_1 = 0.04$ to a constant state of zero. The profile of C_1 of the orthogonal collocation (OC) falls steadily from $C_1 = 0.04$ but however oscillatesbetween 0.01 and 0.04 jumping to its injection value before attaining constant state with the finite difference (FD). Similarly the C_2 of finite difference (FD) decreased steadily from $C_2 = 0.119$ to a constant state as for C_1. Also the profile C_2 of orthogonal collocation (OC) decreases steadily from $C_2 =0.119$ but however gives a more pronounced oscillation from $C_2 = 0.02$ and $C_2 = 0.119$ jumping to its injection value before attaining constant state with the finite difference(FD).

Figures 2b and 2c compare the bed concentration profiles expected at one and two pore volume injected with a mixture $C_1 = 0.042$, $C_2 = 0.115$ in the adsorbing porous medium initially devoid of surfactant (Riemann-type problem: case 2,(refer to Table 5)). The graph shows the results obtained using the numerical technique; finite difference (FD) and orthogonal collocation (OC)

In Figure 2b, the profile C_1 of finite difference (FD) shows steady decline from from $C_1 = 0.04$ to a constant state. Also the C_1 of orthogponal collocation falls steadily from C_1= 0.04 to a constant state as for finite difference (FD). The profile C_2 of finite difference decreased steadily from C_2 = 0.119 to a constant state as for C_1. Similarly, the C_2 of orthogonal collocation (OC) falls steadily from $C_2 = 0.119$ to a constant state.

In Figure 2c, the profiles C_1 of orthogonal collocation (OC) follow the same pattern as that in Figure 2b. Similarly, the profiles C_2 of finite difference (FD) and orthogonal collocation (OC) have the same pattern as in Figure 2b.

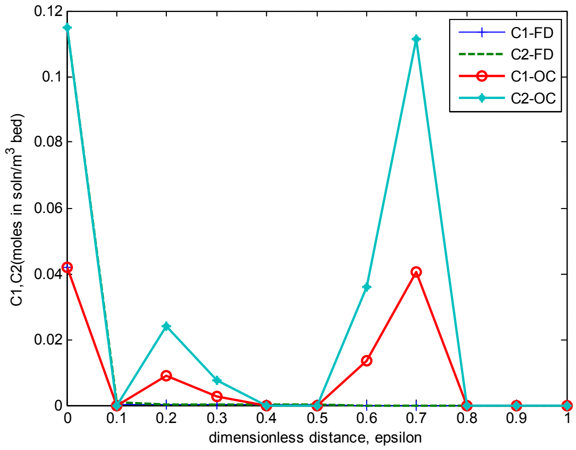

Figure 2a: CASE 2. C_1,C_2 vs epsilon at $\tau = 0.5$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 2, at one-half pore volume injected. The plots are for two methods:Orthogonal collocation (OC), and finite difference (FD).

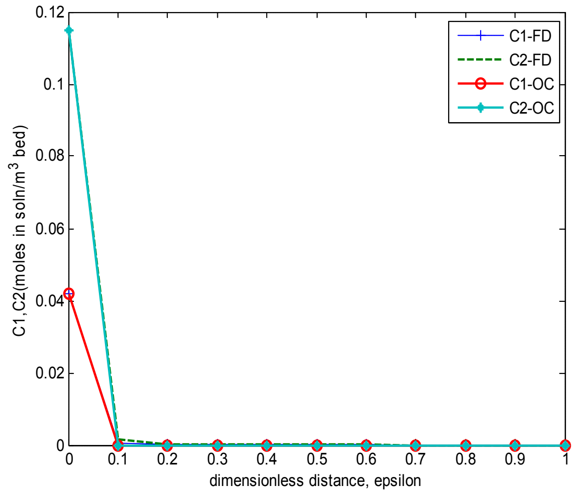

Figure 2b: CASE 2: C_1,C_2 vs epsilon at $\tau = 1.0$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 2, at one pore volume injected. The plots are for two methods:Orthogonal collocation (OC), and finite difference (FD).

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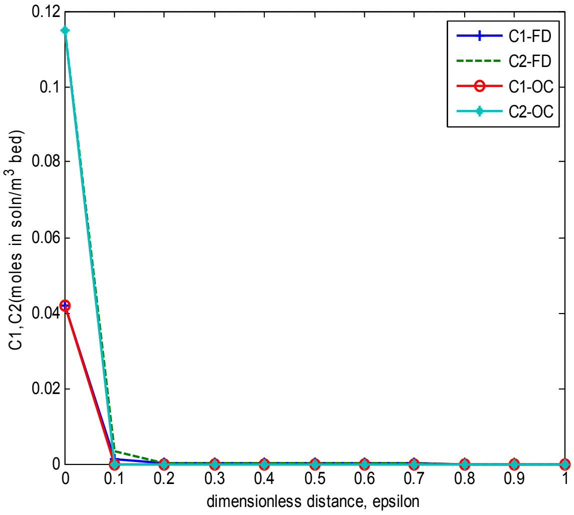

Figure 2c: CASE 2. C_1,C_2 vs epsilon at $\tau = 2.0$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 2, at two pore volumes injected. The plots are for two methods:Orthogonal collocation (OC) and finite difference (FD).

Figure 3a is the result obtained for solving equation 2 using finite difference (FD) and orthogonal collocation numerical technique. The graph shows the bed concentration profiles expected at one-half pore volume injected for a one dimensional aqueous phase chromatography. The response is as a result of injecting a mixture $C_1 = 0.66$, $C_2 =0.875$ into a bed equilibrated with $C_1 =0.35$, $C_2 = 0.15$ (Riemann type problem, case 3(refer to Table 5)).The profile C_1 of finite difference decline steadily from $C_1 = 0.67$ to $C_1 = 0.35$ and maintained a constant state at this concentration, while the profile C_1 of orthogonal collocation decreases steadily from $C_1= 0.67$ to $C_1 = 0.05$ and then declined further with little oscillation before rising to $C_1 = 0.35$ at distance of 0.6 for it to remain at a region of constant state with the finite difference technique, For the initial C_1 concentration in the reservoir. Similarly, the C_2 of finite difference decreases steadily from $C_2 = 0.88$ to $C_2 = 0.15$, it then declines further with oscillation before rising back to $C_2 = 0.15$ to attain constant state of initial C_2 reservoir concentration.

Figure 3b shows the plots for two methods; finite difference(FD) and orthogonal collocation (OC) for one dimensional aqueous phase chromatography for injecting a mixture $C_1 = 0.66$, $C_2 = 0.875$ into a bed equilibrated with $C_1 = 0.35$ $C_2 = 0.15$ (Riemann type problem, case 3(refer to Table 5)) at one pore volume injected. The profile C_1 of finite difference decreases gradually from $C_1 = 0.67$ to $C_1 = 0.35$ and continued with a constant concentration. The profile C_1 of orthogonal collocation declines gradually from $C_1 = 0.67$ to $C_1 = 0.01$. It then increased steadily to $C_1 = 0.35$ with small depression and then later remain constant after attaining $C_1 = 0.35$ again. Also the C_2 of finite difference decreases steadily from $C_2 = 0.88$ to $C_2 = 0.15$ to attain constant state. The C_2 of orthogonal collocation decreases steadily from $C_2 = 0.88$ to $C_2 = 0.15$ then short constant statebut with a small depression before continuing the region of constant state again with the finite difference.

Figure 3c indicates the plots for two numerical technique; finite difference and orthogonal collocation for one dimensional aqueous phase chromatography for injecting a mixture $C_1 = 0.66$, $C_2 = 0.875$ into a bed equilibrated with $C_1 = 0.35$, $C_2 = 0.15$ (Riemann type problem, case 3,(refer to Table 5)) at two pore volume injected.The profile C_1 of finite difference decreases steadily from $C_1 = 0.67$ to $C_1 = 0.35$ at distance 0.1epsilon and maintained a constant state at this concentration while the profile C_1 of orthogonal collocation decreases from $C_1 = 0.67$ to $C_1 = 0.35$ at a distance 0.2 epsilon to attain a constant state with the finite difference. Similarly, the C_2 of finite difference decreases steadily from $C_2 = 0.88$ to $C_2 = 0.15$ at distance 0.1epsilon and then continues with the constant concentration. The profile of orthogonal collocation falls from $C_2 = 0.88$ to $C_2 = 0.05$ and then attain a constant state with the finite difference.

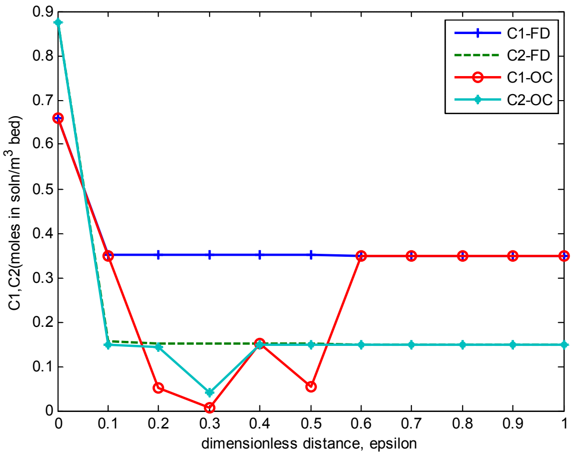

Figure 3a: CASE 3: C_1,C_2 vs epsilon at $\tau = 0.5$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 3, at one-half pore volume injected. The plots are for two methods: Orthogonal collocation(OC), and finite difference(FD).

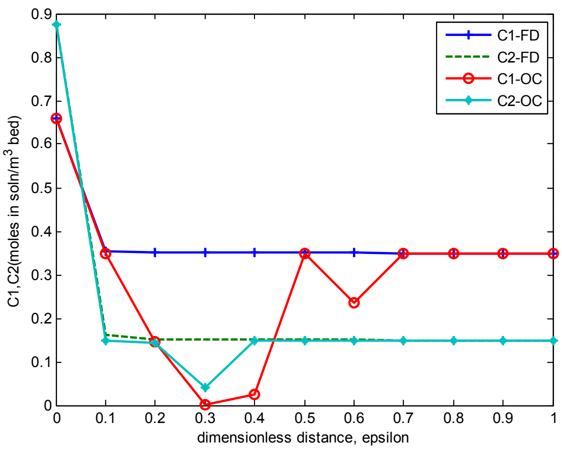

Figure 3b: CASE 3: C_1,C_2 vs epsilon at $\tau = 1.0$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 3, at one pore volume injected. The plots are for two methods: Orthogonal collocation(OC), and finite difference(FD).

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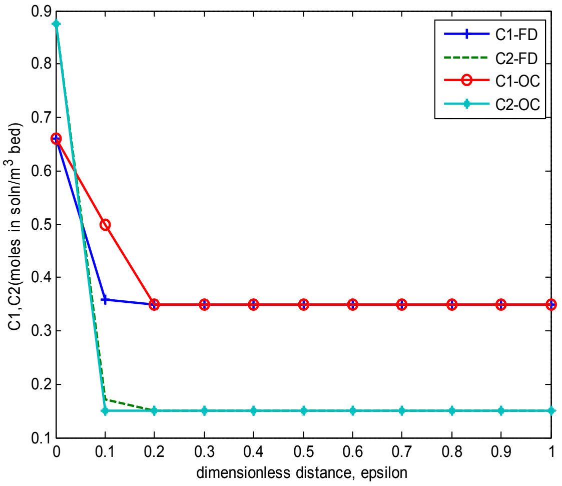

Figure 3c: CASE 3: C_1,C_2 vs epsilon at $\tau = 2.0$. Bed composition profile for one-dimensional aqueous-phase chromatography; case 3 at two pore volumes injected The plots are for two methods:Orthogonal collocation(OC), and finite difference(FD).

5. Discussion of Results

The ultimate objective of the simulator designed here is the prediction of the appropriate surfactant concentration necessary for the required enhanced oil recovery from a chosen reservoir.

The basic physical principle employed by the simulator is that of mass conservation. Usually those quantities are conserved at stock tank conditions and related to reservoir fluid quantities through the pressure dependent parameters. The profiles of two cases, 2 and 3, one dimensional aqueous phase chromatography and two-phase chromatography for one, one-half, and two pore volume injected were developed using simulated solutions to model equations. These equations are solved by finite difference (FD),and orthogonal collocation .The use of these methods permit the determination of the relative efficiency of the methods and how well they predicts the complex characteristics of the enhanced oil recovery process. We will now discuss the significant results of this work.

We did find out that:

(i) For the situation where a mixtureof low concentration aqueous surfactant composition is injected into adsorbing porous medium that is initially injected with high concentration aqueous surfactant composition a variation may exist in the initial profile or be generated by the injection. The initial fluid or previously injected fluid has the composition downstream of the change in amount while the newly injected fluid has the composition upstream of the original variation.The composition route along the bed follows the slow path from the injected composition and then switches to the fast path which leads to the previously injected compòsition. The route passes along paths and follows the paths in the sequence of increasing wave velocities.

(ii) Injecting a mixture of an aqueous composition into a porous medium, initially devoid of surfactant, the expected composition is a self sharpening shock wave. The steepness in all the profiles generated by finite difference (FD), andorthogonal collocation confirms the self sharpening behaviour.It may be noted in all cases of these nature the waves trajectories gradually fall, as a result of a gradual increase in the associated eigenvalues of the waves as salinity increases. The finite difference (FD) and orthogonal collocation (OC) response essentially agrees.The consequence of this steepening is that the flows are sharpening, so that they break through both earlier and over a smaller injected volume. For the dependent variables such as component concentration, common velocity exists at each point in the wave, and the associated composition route remains unchanged and the same during relative shifts of waves associated with other dependent variable waves as shown in all the methods. This is in agreement with was obtained previous author [22].

Injecting a mixture of high concentration of surfactant into adsorbing porous medium that is initially injected with low concentration aqueous surfactant composition yield two types of path. The slow and fast paths. The slow paths eigenvalues are closed to the fast path that has eigenvalues of 1 and the effect of dispersion results in the merging of the two waves. This is due to their spatial position, and loss of intermediate region of constant state. This region later reappear with less dispersion.

6. Conclusions

The applicability of the simulator for the solution of the model equations of multiphase, multicomponent flow and transport in a reservoir has been demonstrated using orthogonal collocation solution. The results of the orthogonal collocation solution were compared with those of finite difference. The results obtained using this methodology revealed certain features unobserved by previous investigators [16]. The results indicate that the concentration of surfactants (C_1, C_2) for orthogonal collocation appear to show more features than the predictions of finite difference. The reason for the difference is the subject of continuing study.

It is obvious that the routes for the compositions of adsorbing surfactants correspond to the simpler case of aqueous phase chromatography, with modified eigenvalues. This observation also holds for “shock” waves. The spatial position of waves and loss of intermediate region of constant state resulted in mild dispersion. Therein lays the possibility of the differences in the concentration profiles predicted by the numerical techniques. Again, the use of the orthogonal collocation and finite difference solution provide easier solution to future possible problems that may arise as the simulator is being used. The future scope of this study is extending to experimental investigation and application to unconventional reservoirs.

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Table 1: Reservoir characteristics from the previous work [16]

Parameter	*Value*
Rock density	2.65 g/cm^3
Porosity	0.2
Oil viscosity	5.0 cp
Water viscosity	1.0 cp
Injection pressure gradient (maintained constant)	1.5 psi/ft
Fluid densities	1.0 g/cm^3
Width of injection face	50 ft
Width of central high permeability streak	10 ft
Length of reservoir	100 or 5000 ft
Residual oil saturation	0.2
Connate water saturation	0.1
First injected surfactant	SDS
Second injected surfactant	DPC
Henry's law constant SDS DPC	 2.71×10^{-4} l/g 8.30×10^{-5} l/g
CMC Values SDS DPC	 800 µmol/l 4000 µmoll/l
Injected concentration SDS DPC	 10 CMC 10 CMC
Brine spacer (typical)	≈ 0.05 pore volumes
Slug volumes	≈ 0.10 pore volumes

Table 2: Reservoir Characteristics used for the Simulation work [17]

Parameter	*Value*
Rock density	2.65 g/cm^3
Porosity	0.2
Oil viscosity	0.40 cp
Water viscosity	0.30 cp
Injection pressure gradient (maintained constant)	1.5 psi/ft
Fluid densities	1.0 g/cm^3
Width of injection face	50 ft
Width of central high permeability streak	10 ft
Length of reservoir	100 or 5000 ft
Residual oil saturation	0.2
Connate water saturation	0.2
First injected surfactant	SDS
Second injected surfactant	DPC
Henry's law constant SDS DPC	 2.71×10^{-4} l/g 8.30×10^{-5} l/g
CMC Values SDS DPC	 800 µmol/l 4000 µmoll/l
Injected concentration SDS DPC	 10 CMC 10 CMC
Brine spacer (typical)	≈ 0.05 pore volumes
Slug volumes	≈ 0.10 pore volumes

Table 3: Parameter values used in Trogus adsorption model for verification runs

Parameter	*Value*
Pure component CMCs	C_1*=1.0 mol/m^3 C_2*=0.35 mol/m^3
Phase separation model parameter	θ=1.8
Henry's law constants for adsorption	$\bar{C}_i = k_i C_{i,w}$ ($C_{i,w}$ = aqueous monomer concentration) k_1 =0.21×10^{-3} m^3/kg k_2= 0.80×10^{-3} m^3/kg
Henry's law constant for oleic partitioning	$C_{i,o} = q_i C_{i,w}$ ($C_{i,w}$ = aqueous monomer concentration) q_1=7.1 q_2=1.3
Adsorbent properties	ρ_s =2.1× 10^{+3} m^3/kg Ø =0.2

Table 4: Additional Reservoir Parameters for the coherence work [16]

Model designation	A	B
Grid points in the horizontal direction (m+1)	21	21
Grid points in the vertical direction (n+1)	11	21
Coherent waves of water saturation	28	28
Initial number of points per coherent wave Water Surfactant	 41 81	 41 81
Maximum number of points required per coherent wave	≈ 300	≈300
Average time step size (days) Short reservoir (100 ft) 200 mD streak 1000 mD streak Long reservoir (5000ft) 200 mD streak 1000 mDsreak	 3.47 0.69 174.0 34.7	 3.47 0.69 174.0 34.7
Typical number of time steps required to inject first pore volume Short reservoir Long reservoir	 33 75	 33 75

Table 5: Conditions for case studies of surfactant chromatography [16]

Case	*Injected composition: C1(mol/m^3 bed)*	*Injected composition: C2(mol/m^{3}bed)*	*Initial composition: C1(mol/m^{3}bed)*	*Initial composition: C2(mol/m^{3}bed)*
1	0.17	0.013	0.21	0.181
2	0.042	0.115	0	0
3	0.66	0.875	0.35	0.15

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