

Study and Analysis of a Noisy Signal by Viterbi Decoding

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Abstract: *In this work, we are primarily interested in digital transmissions as these are mainly used in communication standards. In this paper we propose to study the performance of a multi-carrier data transmission in the presence of Gaussian white noise and to introduce issues related to the transmission of information from the point of view of the information's representation and adaptation to the constraints of the communication channel and present the impact of disturbances that may affect the transmission of a signal between a transmitter and a receiver to determine under what conditions a channel will ensure the transmission correctly, since it is not limited only to the physical medium of information transfer. It also includes features that allows to adapt the signal to be transmitted to the channel and to minimize errors reception.*

Keywords: Communication Channel, AWGN, convolutional encoder, VITERBI decoding, OFDM.

1. Introduction

The role of a communication system is to transmit information from a remote transmitter to one or more receivers through a channel in a reliable manner and at low cost. In a digital transmission system, a finite sequence of symbols represents information. This last is transmitted on the transmission channel by a "real" or analog signal. This signal can take an infinite number of different values and is thus subjected to various forms of disruption and interference, which could lead to misinterpretations of the signal collected by the receiver [1]. The role of any communication system is to ensure that the receiver understands all the messages transmitted by the transmitter, regardless of the compression format or type of data, but also induced disturbances on the transmission channel and its parasitic effect.

The information transfer requires a source of data, translated into an understandable system by the transmitter and receiver (encoding, format, compression previously defined). The channel itself is the link or the carrier information transport of information between two communicating entities, but it also includes devices for input and output of the support of transmission that will help the issue, receipt and correct extraction of the digital data [2]. Once the transmitted signal, the receiver gets to the other end of the canal disturbed, distorted and weakened signal. This signal, it must extract the original digital information without error. Filter compensates for the adverse effects of the transmission medium.

The rest of the paper is organized as follows. In section 2, we present the architecture of the OFDM operating. The principle of modulation and demodulation is explained and we detail the method of construction of an OFDM waveform. The explanation of the principle of convolutional coding and description of these different representations are dedicated in section 3. The simulation and the analysis results are shown in Section 4. The last section concludes the paper and provides possible directions for the future work.

2. Architecture of the functioning of OFDM

If the first studies on the multi-carrier date from the late 1950s, the division multiplex orthogonal frequency, more known as OFDM (Orthogonal Frequency Division Multiplexing) has emerged a decade later. Since then, OFDM has remained a predominant technique, since it is used for many applications such as digital television DVB (Digital Video Broadcasting) or standard ADSL (Asymmetric Digital Subscriber Line) for internet broadband connections. Finally adapts perfectly to mobile communications, and seems unavoidable for future standards of third and fourth generations. OFDM divides the channel in accordance with the cell axes of time and frequency. The channel is subdivided into a resulting sequence of sub frequency bands and another segment of time. For each time-frequency is assigned a carrier. It will thus distribute the information to be carried on all of these carriers, each one is modulated by a low-speed modulation of BPSK (binary phase shift keying), QPSK (quadrature phase shift keying) or QAM (quadrature amplitude modulation). The techniques called multicarrier works on transmitting digital data by modulating them on a large number of carriers simultaneously [3].

2.1 Principle of Modulation

The principle of frequency multiplexing is to transmit at the same time, more analog signals, on a single channel. These signals can be multiplexed together by modulating a carrier signal with each other and different occupying a defined portion of the bandwidth of communication channel [4]. Consider a sequence of N data. c_0, c_1, \dots, c_{N-1} Call T_s the symbol period that is to say, the time between two sequences of N data. Each data unit c_k modulate a signal to an f_k frequency. Individual signal can be written in complex form: $c_k e^{2j\pi f_k t}$ (1).

The total signals $s(t)$ corresponding to all the data of an OFDM symbol, is the sum of the individual signals

$$s(t) = \sum_{k=0}^{N-1} c_k e^{2j\pi f_k t} \tag{2}$$

Multiplexing is orthogonal if the space between the frequencies is $1/T_s$. Then

$$f_k = f_0 + \frac{k}{T_s} \tag{3}$$

and
$$S(t) = e^{2j\pi f_0 t} \sum_{k=0}^{N-1} c_k e^{2j\pi \frac{kt}{T_s}} \tag{4}$$

The modulation principle is illustrated in Figure .1

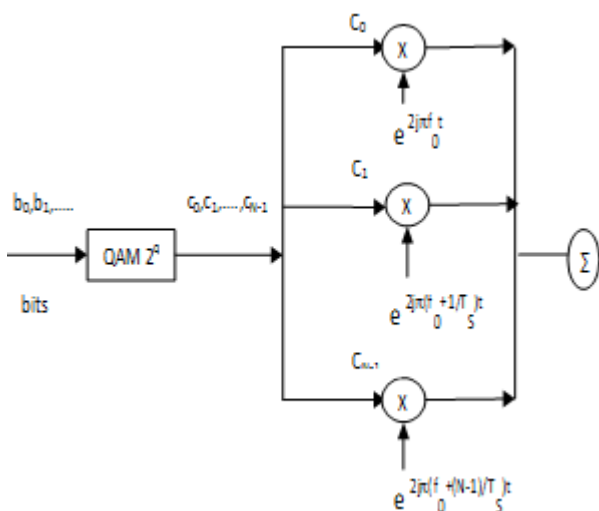


Figure 1: Diagram of the principle of modulation

More specifically, the digital data C_k are complex numbers defined from bits by a constellation of amplitude modulation QAM quadrature multi-state (4, 16, 64, generally 2^q states). These data are symbols q-ary formed by group of q bits. They are called numeric symbols.

From equation (1) we can deduce the real expression of the signal: If $c_k = a_k + jb_k$, then

$$s(t) = \sum_{k=0}^{N-1} (a_k + jb_k) e^{2j\pi(f_0 + \frac{k}{T_s})t} \tag{5}$$

The multicarrier modulations are particularly useful for channels with multiple echoes [4]. The main disturbance suffered by the transmitted signal is the sum of a Gaussian noise. Even limiting the frequency band of the received signal, the receiver must operate with a useful signal vitiated by an interfering signal whose amplitude at a given time follows a Gaussian probability. Intuitively, we understand that the work of the receiver becomes extremely difficult when a large noise spike. The principle of the demodulation depends disturbances suffered by the signal after passing through the channel.

2.2 Principle of demodulation

The signal arriving at the receiver is written to a symbol

period T_s :
$$y(t) = \sum_{k=0}^{N-1} c_k e^{2j\pi f_k t} H_k(t) \tag{6}$$

$H_k(t)$ Is the channel transfer function around the frequency f_k and time t. This function varies slowly and can be assumed constant over the period T_s . Demodulation is to demodulate the signal according to the N subcarriers [4]. The demodulation principle is illustrated in figure .2

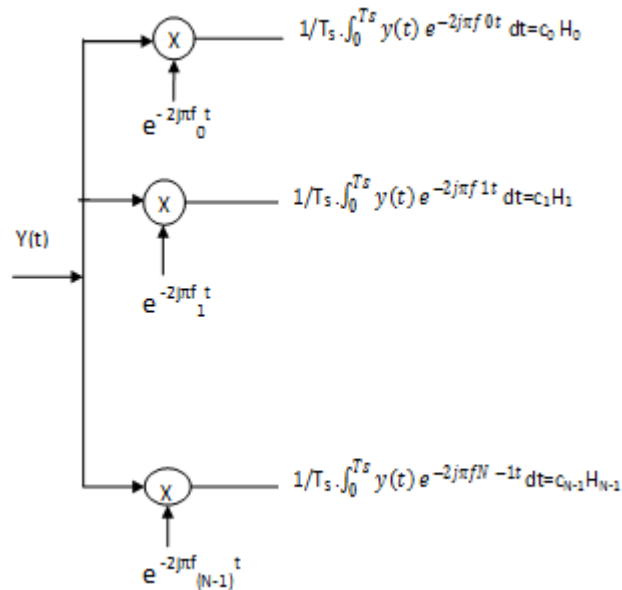


Figure 2: Diagram of the principle of demodulation

2.3 OFDM waveform

2.3.1 The transmitter

In this work we briefly recall the method of construction of an OFDM waveform. It is considered that it has a set of communications symbol

$(s_i)_{i \in [1, N]}$ that we wish to transmit. These symbols often belong to a constellation like QAM (quadrature amplitude modulation) [4]. Where the vector comprising the set of symbols is:

$$s = (s_1, s_2, \dots, s_N)^T \tag{7}$$

The exponent T represents transposition.

From a purely algebraic standpoint, the OFDM approach amounts to be transmitted, not the original vector s, but a vector x obtained from s by a simple multiplication by a matrix of the transconjugate F of what it's called normalized Fourier Transform. F is a matrix of N rows and N columns, numbered 0 to N - 1. The term $f_{n, m}$ located on the line n and

column m is defined:
$$f_{n, m} = \frac{1}{\sqrt{N}} e^{-j \frac{2\pi n m}{N}} \tag{8}$$

$n = 0, \dots, N - 1$

$m = 0, \dots, N - 1$

The vector x can be written simply:

$$x = F^H s \tag{9}$$

The exponent H is the transconjugation.

The vector x is actually constituted by the terms of the inverse Fourier Transform (IFT) of the vector s .

Before being issued a cyclic prefix is added to the vector x . This prefix is nothing other than copying [5], head of the vector, the last component of the latter. Reception, once synchronized, the cyclic prefix is removed. His presence is primarily intended to reduce the effect of the propagation channel to a cyclic convolution by the terms of the channel impulse response. Thus, one can simply formalize the effect of the propagation channel on the communications symbol, by multiplying the vector x by a circulant matrix H [6].

The first column of the matrix is constituted by the different terms of the channel impulse response, then the column is filled with zeros [7].

$(S) \in [1, N]$ Noting the terms of the impulse response of the channel matrix can be written as follows:

$$H = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ h_{L-1} & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots \\ \vdots & \dots & \dots & 0 \\ 0 & \dots & 0 & h_{L-1} \dots h_0 \end{bmatrix}$$

After deletion of the cyclic prefix, the received signal is simply formalized by:

$$r = Hx + n \tag{10}$$

The vector n is a vector of N terms of additive white Gaussian noise [3].

The transmitted signal is subject to many external and internal disturbances in the transmission channel. Firstly, the ambient noise may disrupt digital communication, degrading the amplitude of the received symbols which increases the risk of these identification symbols errors. Techniques for signal processing, coding and modulation have been developed in recent years to improve the robustness of the bonds vis-à-vis the noise. However, the noise is not the only source of disturbance, the transfer function of the channel distorts the signal during propagation.

2.3.2 The transmission channel

The channel in the sense of digital communications includes the support of transmission that represents the physical link between the transmitter and the receiver. The channel model most frequently used for the simulation of digital transmission, which is also one of the easiest to generate and analyze the channel, is additive white Gaussian noise (AWGN). This noise model both internally noise (electronic equipment) and noise from external sources (antenna noise due to interference from other users of the transmission medium) [8].

The easiest and the most appropriate channel is additive white Gaussian noise. At the output of this channel, the received signal results from the addition of the signal and the white noise. If we exclude this noise, the signal does not

change: we say that the channel is without distortion. The additive noise is independent of signal. It is modeled by a random stationary white Gaussian process and center. His bilateral spectral power density is constant.

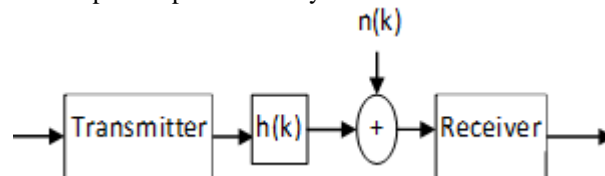


Figure 3: Diagram of a noisy signal

2.3.3 The Receiver

In reception, the first step is to perform the Fourier Transform of the received signal, thus amounts to multiplying the vector r by the matrix F .

Is then obtained: $y = Fr$ (11).

By replacing r by its expression after equation (6), we obtain:

$$y = FHx + Fn \tag{12}$$

As the matrix H is circulating, is known to be diagonalized by the matrix of the Fourier transform, we can write [7]:

$$C = FHF^H \tag{13}$$

In this equation, the matrix C is a diagonal matrix whose values are different terms of the discrete Fourier transform on N values of the impulse response of the propagation channel. Substituting equation (9) as $H = F^H C F$ in the equation (8) and using equation (3) we obtain:

$$y = FF^H C FF^H s + Fn \tag{14}$$

Or, by using the property $F^H F = FF^H = I$,

$$y = C F x + F n \tag{15}$$

Therefore it only remains to divide each component of the vector y by the value of the Fourier transform of the impulse response of the channel at that frequency. This division is called frequency equalization. This gives: $s = C^{-1} y$ (16).

Whence $s = s + C^{-1} F n$ (17)

It then recognizes the vector s tainted issued a noise sample vector $C^{-1} F n$. It is quite easy to show that these noise samples are uncorrelated and that the optimal decision in the sense of maximum likelihood can be done component by component [9].

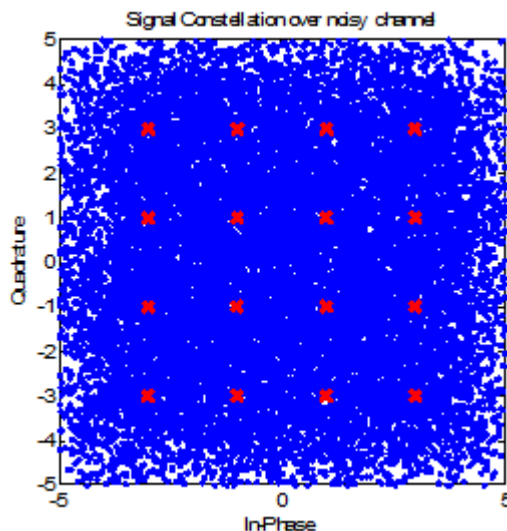


Figure 4: The signal constellation over AWGN channel

The noise and interference channel, which degrade the communication signals transmitted, causing detection errors in reception. For a given level of noise and interference given, the probability of error can be reduced by increasing the transmission power. However, this increase in power is not always desirable. Another solution is the channel coding, which comprises adding to the binary message redundancy bits, so that the encoded message has a particular structure. In reception, the channel decoder checks whether this structure is well respected. Otherwise, an error is detected and corrected if necessary. We present in the following the principle of convolutional coding.

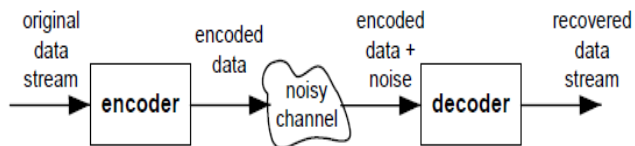


Figure 5: Diagram of an encoder and decoder.

3. Convolutional coding

Coding techniques for error control play an important role in digital communications systems. Convolutional codes or convolutional constitute a large family of error correcting codes [10]. The principle of convolutional coding is the combination of the input bit to transmit more bits previously transmitted by a logical operation, in order to regain its value Incident transmission. So this way to introduce redundancy in the information to be transmitted code gives the ability to detect and correct errors.

The convolutional codes add some redundancy to the bits of the information sequence to be transmitted through a logical operation (exclusive OR). Adding this redundancy enables the decoder to the reception, to correct any errors in the transmission channel. Different algorithms have been developed for decoding. The best known is undoubtedly the Viterbi algorithm, published in 1967 and has the distinction of being optimal [11]. Since convolutional codes with Viterbi algorithm are increasingly used in digital communication systems such as error correction code FEC category ("Forward Error Correction"). Using such an error correction code introduces a gain, relative to non- encoded systems, which is very attractive especially for wireless systems.

3.1 Principle of convolution coding

A convolutional coder comprises a shift register consisting of K cells, V modulo-2 adder, a set of adders and the connections between the cells of the shift register and the iV switch positions [12]. Each convolutional encoder is characterized by the following parameters:

- Coding rate: $R=b/V$.

Where b is the number of concurrent bits to the encoder input V and the number of the encoded symbols output from the encoder.

- Length of the encoder constraint k :

Where k is the number of cells of the shift register.

The operation of the convolutional encoder [13] and the influence of these parameters on system performance are illustrated in Figure 6:

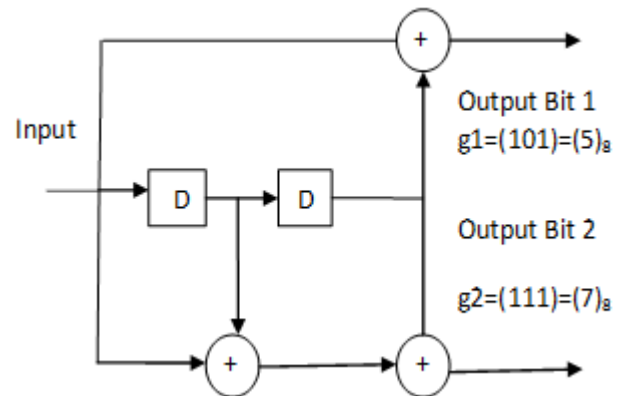


Figure 6: Structure of a convolutional encoder.

The number of adders in this example is $V=2$, the coding rate becomes $R=1/2$ for each input bit to the encoder produces at its output two coded symbols and the constraint length (number of cells of the register offset) is $K=3$ as shown in figure 6. All connections between the shift register and modulo-2 adders is shown by generating vectors. These are the generating vectors of the encoder which determine the rules by which the redundancy bits must be added to the information bits to be transmitted.

In our case, generators worth: $G1 = (101)_2$, $G2 = (111)_2$. Generators are often expressed in octal form, which in this case leads us to the following notation: $G1 = 5$, $G2 = 7$. This last notation we use throughout this work [14]. The encoder operates as follows: prior to the encoding process starts, the content of the shift register is initialized to zero. The information bits arrive at the encoder input continuously. The data bit at the input of the encoder is fed into the shift register and, by using the encoder generating vectors, the corresponding encoded symbols V are calculated [15].

The sequence of encoded symbols is obtained by sampling the modulo-2 adders using switch. Once all the data bits encoded. A sequence of $K-1$ zeros is added (and encoded) to restore the registry to its original state. Each encoded symbol does not only depend on the input bit to the encoder, but also the content of $K-1$ cells of the register before the current bit.

The output sequence is a linear combination of present and past inputs. This sequence can be expressed as the convolution of the input sequence and the impulse response of the encoder, hence the name of convolutional codes.

3.2 Representations of convolutional codes

The idea of a graphical representation of a convolutional code is made from Markov characteristics of the encoder output [13]. In effect, the output of the encoder depends on its input and its states. The equivalent graphs for the polynomial representation are often easier to handle and allows to derive more powerful results. All convolutional code is represented by three equivalent but different graphs: tree code, the code trellis and state diagram [16].

3.2.1 State diagram

A state diagram of the convolutional code represents the relationship between the coded symbols, information and statements encoder bits. This performance is made possible by the fact that the encoder can take only a finite number of states. Figure 7 represent state diagram of figure 6.

Table 1: Table state transitions.

Input	Current state	Next state	Output
0	00	00	00
0	01	00	11
0	10	01	01
0	11	01	10
1	00	10	11
1	01	10	00
1	10	11	10
1	11	11	01

Nodes in the state diagram represent the states of the encoder and the result of transition between two states represents v coded symbols that match the bit of information to the encoder input. Each branch of Figure 7 shows the corresponding encoded bit information corresponding to the symbols.

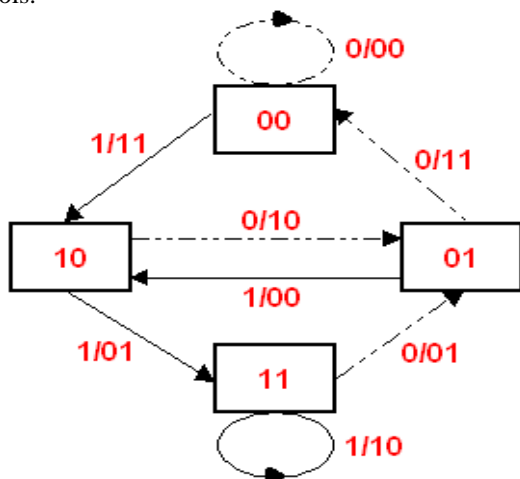


Figure 7: State diagram of Figure 6 (K = 3, R = 1/2)

3.2.2 Tree diagram

The tree is a graph of infinite height and width. A node in the tree represents a possible state of the encoder. A branch symbolizes a transition from one state to another. Virtually tree starts at the top by the state 0 (shift register is initialized to 0). Any path in the tree code is a possible sequence (a code word) at the output of the convolutional encoder [17]. The Trellis is obtained by bending the shaft about its width.

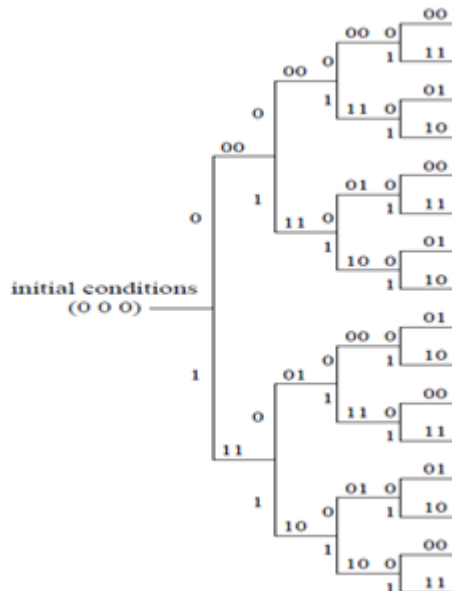


Figure 8: Tree representation of the convolutional code in Figure 6

3.2.3 Trellis diagram

A trellis encoding is a representation of the convolutional encoder that takes into account the fact that the number of states of the encoder is finished and the convergence property [16]. This characteristic reflects the fact that when two information are identical sequences for at least (K-1) consecutive bits, then the coder is in the same state for both sequences. The mesh consists of nodes representing the states of the encoder in question and branches connecting the nodes of the mesh. As shown in Figure 9.

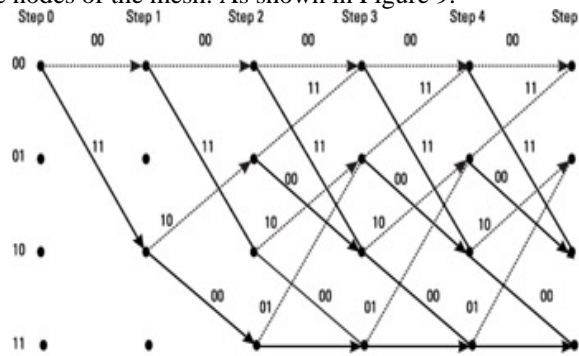


Figure 9: Trellis representation of the convolutional encoder of Figure 6.

4. Simulation and results

The message consisted of a binary sequence is composed of a sequence of 800 bits "0" and "1" (0110100 1001), the transmitter" must adapt the message to the physical channel , the signal takes the form of a sinusoid, the amplitude, or the frequency code information. The receiver interprets the physical signals received binary messages. This interpretation is disturbed in that the channel affects the transmitted signals.

Consider the simplest case of an ideal transmission channel, so noise is not present, Figure 10. (1a) shows the results of simulation of the sequence of 800 bits transmitted on 200

sub-carriers. This representation was preferred to the temporal representation for visibility. The additive channel noise is white noise type Gaussian with zero mean value. The entered values of the noise have been generated using Gaussian random values. The simulation result of the noisy received signal is shown in Figure 10. (2a), for a SNR = 10dB

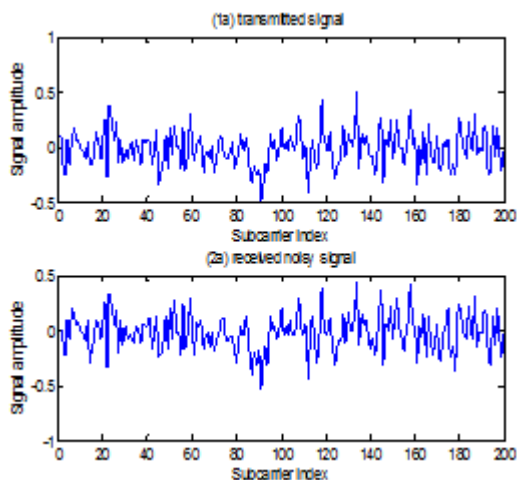


Figure 10: Transmitted and received noisy signal

The received signal during the transmission of a symbol $S_i(t)$ is not $S_i(t)$, but a noisy signal, as shown in the figure, in the case of a disturbance of the white Gaussian noise, this disturbance signal communication can lead the receiver to misinterpret, and could detect different from those issued binary messages.

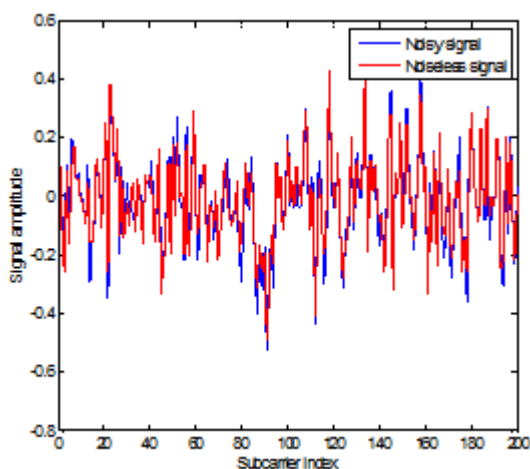


Figure 11: comparison of original signal and noisy signal

The two signals, the first issued but not noisy (red curve) and the second received noisy is accompanied by a white Gaussian noise with zero mean (blue curve) are plotted on the same Figure 11, to guide the eye, we see clearly the effect of the added noise on the shape of the original signal.

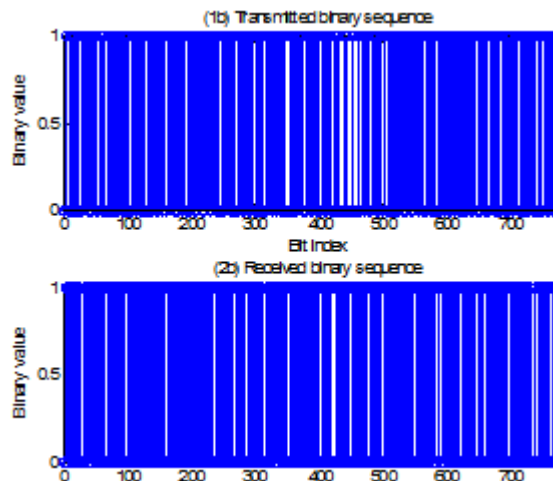


Figure 12: transmission and reception of a binary sequence.

Finally, the transmitted bit sequence that represents the original message is illustrated in FIG 12 (1b) and the received bit sequence that represents the recovered message is illustrated in FIG 12 (2b).

Table 2: Simulation parameter

Simulation parameter	Value
channel	AWGN
Subcarrier number	200
modulation	16 QAM
Guard type	Cyclic prefix
SNR	10 dB
Number of bits	800

5. Conclusion and Prospects

The objective of a communication system is to provide for all users, the maximum flow with minimum bit error probability (probability of being wrong on the value of a bit). We know these two objectives are contradictory, so it is a tradeoff between the quality (speed and error rate) desired for the intended application and the constraints imposed by the channel.

In general, the information conveyed by the signal is degraded or lost, in the presence of noise superimposed on the useful signal. The outlook for the next job is to present the different sources of noise in a transmission channel to present the variables to characterize (signal to noise ratio) and to link the amount of noise degradation of a digital signal (relationship between the signal to noise ratio and bit error rate). Given that knowledge of the noise power has a value only if it can be compared to that of the signal and infer its impact on signal degradation. This is why we generally use a power ratio called signal -to-noise (Signal Noise Ratio) report. Based on these criteria the minimum amplitude required of the signal to avoid an erroneous transmission, it is possible to dimension the power to be transmitted in the channel, signal characteristics, gains and losses of the different elements of the channel.

References

- [1] Richard van Nee, Ramjee Prasad, "OFDM for Wireless Multimedia Communications", Artech House, London, ISBN 0-89006-530-6, 2000.
- [2] Marius Oltean, "An Introduction to Orthogonal Frequency Division Multiplexing", available on-line: shannon.etc.upt.ro/docs/cercetare/articole/aiofdm2004.pdf.
- [3] Anders Vahlin, "Efficient algorithms for modulation and Demodulation in OFDM-systems", 2003.
- [4] Sara Riahi, Ali El Hore, Jamal El Kafi, "analysis and simulation of ofdm", IJSR, ISSN Online: 2319-7064, volume 3, Issue 3, March 2014.
- [5] Yuping Zhao, "In-band and Out-band Spectrum Analysis of OFDM Communication Systems Using IC1 Cancellation Methods", 0-7803-6394-9/00, IEEE, 2000.
- [6] Robert M. Gray, "Toeplitz and Circulant Matrices: A review", Department of Electrical Engineering Stanford University, Stanford 94305, USA. March 2000.
- [7] IRWIN KRA AND SANTIAGO R. SIMANCA, "On circulant matrices", Notices of the AMS, Volume 59, Number 3, March 2012.
- [8] Baoguo Yang, Member, IEEE, Khaled Ben Letaief, Senior Member, IEEE, Roger S. Cheng, Member, IEEE, and Zhigang Cao, Senior Member, IEEE, "Channel Estimation for OFDM Transmission in Multipath Fading Channels Based on Parametric Channel Modeling", IEEE Transactions on communications, volume . 49, No. 3, March 2001.
- [9] Romain Couillet, Mérouane Debbah, "Outage performance of flexible OFDM schemes in packet-switched transmissions", Journal on Advances in Signal Processing, Special Issue 'Dynamic Spectrum Access for Wireless Networking', Couillet-Eurasip09, hal-00414534, version 1 - 9 Sep 2009, (2009).
- [10] Man Guo. M. Omair Ahmad, Fellow IEEE, M.N.S. Swamy, Fellow IEEE, and Chunyan Wang, "An adaptive VITERBI algorithm based on strongly connected TRELIS decoding", 0-7803-7448-7/02, 2002.
- [11] Varsha P. Patil, Prof. D. G. Chougule, Radhika.R.Naik, "Viterbi Algorithm for error detection and correction", IOSR Journal of Electronics and Communication Engineering (IOSR-JECE), ISSN: 2278-2834-, ISBN: 2278-8735, PP: 60-65.
- [12] Chip Fleming, "A tutorial on convolution coding with viterbi decoding", 2002-07-05.
- [13] S.V.Viraktamath, G.V.Attimarad, Ravish, V.P.Geji, "Error Control Mechanisms using CODEC", 978-0-7695-3522-7/09, International Conference on Communication Software and Networks, IEEE, 2009.
- [14] LECTURE 9, "Viterbi Decoding of Convolutional Codes", MIT 6.02 DRAFT Lecture Notes Fall 2010.
- [15] K. Hueske, J. Geldmacher, and J. Götze, "Adaptive decoding of convolutional codes", Advances in Radio Science, Adv. Radio Sci., 5, 209–214, 2007.
- [16] Rupali Dhobale, Kalyani Ghate, Nikhil Pimpalgaonkar, R. B. Khule, "Implementation of Adaptive Viterbi Decoder for Wireless Communication", IJSR, ISSN Online: 2319-7064, Volume 2 Issue 3, March 2013.
- [17] Rastislav Sramek, "the on-line Viterbi algorithm", Department of Computer Science Faculty of Mathematics, Physics and Informatics Comenius University, Bratislava, 2007.

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