Resolution by the Finite Elements of the Equations of Couples Transfers Masses and Energy in a Porous Wood Environment: Case of the Drying of the Iroko

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Abstract: ***In this work, we discuss the transfer phenomena in porous media, applied in a hygroscopic material during drying. The study is undertaken within the framework of initiation to research in master. The coupled equations of heat transfer and mass are written from the Luikov model to which the simplifying assumptions are associated. The space-time equations are solved in 2D by the finite element method, before determination of the average sizes of moisture and temperature for comparison with the experimental results. It is a pattern of behavior contributing to the control of the process to predict possible cases of simulation of influence of drying parameters. The material chosen is the Cameroon Iroko (Chlorofora exelsa) which is a large forest product use for the manufacture of internal or external furniture. Drying is done in an insulated enclosure calm atmosphere. Thermophysical and hygroscopic properties necessary for solving the equations are derived from the literature and previous works. The analysis of theoretical and experimental results leads to an average relative error of 3.238% for different temperatures applied.***

Keywords: mass and heat transfer - Porous medium - finite elements - numerical simulation

1. Introduction

Forest products right from the cut to the state of utilization are the seat of several transformations, following the transfer phenomena that take place there. Cameroon forest is estimated to be 17.5 million hectares [1], that is about 44% of its surface area. In this space, there are more than 300 species of trees in which we find that some suffer from excessive exploitation because of their thermo mechanical properties. Iroko studied here is one and like the other species, they are hygroscopic and natural porous media. Only a few works have been devoted to iroko although some data from the drying and thermophysical properties are available in the literature [2, 3, 4]. This study is required for understanding the transfer phenomena that takes place in this product during drying. The drying process will result in the control of deterioration species by microorganisms, improved mechanical strength and in turn preserve the ecological species. The presence of a thermal gradient in a porous unsaturated medium results in a transfer of mass inside the product, and vice versa. Thus, the study of transfer mechanisms taking place in a porous medium is quite complex because the coupling between mass and heat is essential [5,6]. Numerical analysis is best suited to solve this problem as an experimental study is not only expensive, but is also difficult because of the multidimensional nature of the transfer phenomena [5, 6, 7]. Thus, the objective of this work is to establish a model of diffusive drying which makes it possible to study the kinetics of drying at the scale of the product.

2. Mathematical Formulation of the Problem

2.1 Physical Model

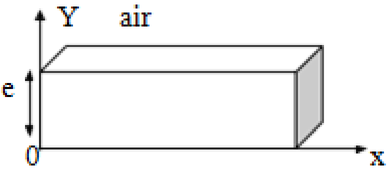

Figure 1: Cross Section of Wood

Figure 2: Photographs of the Thermal Drier

Samples of dimensions 2 cm thick, 5 cm wide and 10 cm in long are placed in the thermal drier set at a specific temperature and humidity of the air. The samples are then weighed regularly at intevals of 15 minutes until the mass varies more. The reduced water content is then obtained using equation (1) (Lopez A. et al, 2000; Lahsasni, 2002.) [8,9]

$$M_R = \frac{M - M_e}{M_0 - M_e} \tag{1}$$

M is the water contents of the sample, M_0 the initial content and Me its content at equilibrium.

Rather than determining the content at equilibrium Me, one can approximate the equation (1) as underlined by Piamente L.M et al..(1993) [10], Togrul.I.T et al. (2002) [11]:

$$M_R = \frac{M}{M_0} \tag{2}$$

This evaluation made it possible to constitute the experimental data for the comparison of the kinetics. The medium is considered homogeneous and continuous when estimating the thermophysical properties. The equations of transfers are only established in the product.

1.2 Mathematical Model

The porous unsaturated timber medium consists of its dry mass, the mass of bound water, water vapor and the mass of free water. The density of the porous medium may then be estimated using equation (3) below:

$$\rho_s(M) = (\varepsilon s \rho_l + X_b \rho_s + \varepsilon(1-s)\,\rho_v c_v) \tag{3}$$

Where ε is the porosity of the wood, S is the fraction of free water in the pores, X_b bound water content in the wood, c_v the fraction of vapor in the gas phase.

Mass Conservation

The general equation of mass transfer is written as:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{q} + K = 0 \tag{4}$$

Where $\vec{q}$ represents the resultant flow of the various phases of water present in the timber. Explicitly, equation (4) will be written [12,13].

$$\frac{\partial(\rho_s M)}{\partial t} + \vec{\nabla}.(\overrightarrow{q_l} + \overrightarrow{q_b} + \overrightarrow{q_v}) = 0 \tag{5}$$

According to T.SIMO [12] S. MERAKEB and [14], Gustavo E. [15] the vapor flux given by:

$$\overrightarrow{q_v} = \rho_g \overrightarrow{V_g} + -\rho_g \overset{=}{D_v} \vec{\nabla} C_v \tag{6}$$

$\overset{=}{D_v}$: Tensor diffusion coefficient of the vapor

The liquid water flow is given by Darcy's law generalized by:

$$\overrightarrow{q_l} = \rho_l \overrightarrow{V_l} = \rho_l \frac{\overset{=}{K_l}}{\mu_l} (\overrightarrow{\nabla P_l} - \rho_l \vec{g}) \tag{7}$$

μ_l is the dynamic viscosity of liquid water, P_l the pressure of the liquid phase at the pore ($\overset{=}{K_l}$ is the permeability of the medium in the liquid phase.

The bound water flow

$$\overrightarrow{q_b} = -\rho_s \overset{=}{D_H} \vec{\nabla}.H - \overset{=}{D_T} \vec{\nabla}.T \tag{8}$$

By defining the capillary pressure gradients and weight fraction as a result of temperature gradients of water content M, the equation of conservation becomes [12]:

$$\frac{\partial M}{\partial t} - \vec{\nabla}.(\overset{=}{D_{HH}} \vec{\nabla}.M + \overset{=}{D_{HT}} \vec{\nabla}.T) = 0 \tag{9}$$

with

$$\overset{=}{D_{HH}} = \overset{=}{D_H} - \frac{\rho_l}{\rho_s} \frac{\overset{=}{K_l}}{\mu_l} \frac{\partial P_c}{\partial M} + \frac{\rho_g \overset{=}{D_v}}{\rho_s(1-C)} \frac{\partial C}{\partial M} \tag{10}$$

$$\overset{=}{D_{HT}} = \overset{=}{D_T} - \frac{\rho_l}{\rho_s} \frac{\overset{=}{K_l}}{\mu_l} \frac{\partial P_c}{\partial T} + \frac{\rho_g \overset{=}{D_v}}{\rho_s(1-C)} \frac{\partial C}{\partial T} \tag{11}$$

$\overset{=}{D_{HH}}$: Tensor diffusion coefficient of water, $\overset{=}{D_{HT}}$: Tensor coefficient of thermal diffusion of water in the wood.

Energy Conservation

The general heat equation is given by:

$$\rho c_p \frac{\partial T}{\partial t} = q_s - \vec{\nabla}.\overrightarrow{q_c} \tag{12}$$

According to Fourier's law, we have

$$\overrightarrow{q_c} = -\overset{=}{\lambda} \vec{\nabla} T \tag{13}$$

The source term q_s takes into account the energy developed for the movement and transformations undergone by the water in medium. So the insertion of all the terms in the energy equation (12) gives:

$$\rho c_p \frac{\partial T}{\partial t} + (c_{pl}(\rho_l \overrightarrow{V_l} + \rho_b \overrightarrow{V_b}) + c_{pg} \rho_v \overrightarrow{V_v}).\vec{\nabla} T + K_l L_v + K_b(L_v + L_s) = \vec{\nabla}.\overset{=}{\lambda} \vec{\nabla} T \tag{14}$$

Assuming that the heat capacity of the timber is the sum of the heat capacities of its constituents, we have:

$$(\rho c_p)_g + (\rho c_p)_l + (\rho c_p)_s = \rho_s c_p \tag{15}$$

Assuming also that the water vapor is retained in the process, we have [12]:

$$\vec{\nabla}.\overrightarrow{q_v} = K_l + K_b \tag{16}$$

By neglecting the heat transfer by convection in the timber and assuming that the variation rate of the bound water content is close to that of the whole water content, we obtain the following heat transfer relationship according to Luikov's model found in the works of L. Aissani et al [16]:

$$\rho_s c_p \frac{\partial \mathrm{T}}{\partial t} - \vec{\nabla}.\left(D_{\overline{\overline{TT}}}\, \vec{\nabla}.T + D_{\overline{\overline{TH}}}\, \vec{\nabla}.M\right) = \vec{\nabla}.(\bar{\bar{\lambda}}\vec{\nabla}\mathrm{T}) \quad (17)$$

with

$$D_{\overline{\overline{TT}}} = \frac{(L_v+L_s)\rho_g D_{\overline{\overline{v}}}}{(1-C)}\frac{\partial C}{\partial T} - L_v \frac{\rho_l K_{\overline{\overline{l}}}}{\mu_l}\frac{\partial P_c}{\partial T} \quad (18)$$

Thermal diffusion tensor

$$D_{\overline{\overline{TH}}} = \frac{(L_v+L_s)\rho_g D_{\overline{\overline{v}}}}{(1-C)}\frac{\partial C}{\partial M} - L_v \frac{\rho_l K_{\overline{\overline{l}}}}{\mu_l}\frac{\partial P_c}{\partial M} \quad (19)$$

thermal diffusion Tensor coefficient of heat. We thus obtain the equations of heat transfer and mass strongly coupled system:

$$\{\frac{\partial H}{\partial t} = D_{\overline{\overline{HH}}}\Delta H + D_{\overline{\overline{HT}}}\Delta T \; \rho_s c_p \frac{\partial T}{\partial t} = D_{\overline{\overline{TT}}}\Delta T + D_{\overline{\overline{TH}}}\Delta H + \bar{\bar{\lambda}}\Delta T \; (20)$$

Migrating thermo model coefficients are not easy to determine experimentally [15]. The following additional assumptions are made:

- The overall diffusion of moisture is equal to that of free water, bound water and water vapor. Thus, the capillary pressure gradient must be low. This is verified when the wood drying takes place at low temperature.
- The partial diffusion of moisture due to the temperature gradient is proportional to the diffusion of bound water, and negligible effects, the coefficient of proportionality being the coefficient of thermal migration. We will not take into account the diffusion coefficients due to moisture gradient; which will be formulated by:

$$\{\frac{\partial M}{\partial t} - \vec{\nabla}.\left(D_{\overline{\overline{HH}}}\, \vec{\nabla}.M\right) = 0 \; \rho_s c_p \frac{\partial \mathrm{T}}{\partial t} - \vec{\nabla}.\left(D_{\overline{\overline{TH}}}\, \vec{\nabla}M\right) = \vec{\nabla}.(\bar{\bar{\lambda}}\vec{\nabla}\mathrm{T}) \; (21)$$

$$D_{HH} = D_H \quad (22)$$

$$D_{TH} = \rho_s(E + Lv)D_H \quad (23)$$

E is the differential heat of sorption.

3. Numerical Resolution

Obtaining and using an analytical solution of the equations developed is complicated if not impossible. The numerical resolution is the only alternative that we can make use of. In addition, we gain flexibility. This approach allows introducing the source terms, modifying the boundary conditions and representing exchanges.

3.1 Initial conditions and limitations

Transfers at boundaries are characterized by the mass transfer coefficients $h_{\overline{\overline{m}}}$ and the heat transfer $h_{\overline{\overline{c}}}$, evaporation of water is included in the equation of the internal heat transfer. The transfer is also symmetrical to the exposed faces, where the nodes of the medians are extrema.

$$\{at\ the\ large\ exposed\ surface, -D_{\overline{\overline{HH}}}\nabla M = h_{\overline{\overline{m}}}(M - M_{air})\vec{n}\ \lambda\vec{\nabla}T = h_{\overline{\overline{c}}}(T_{air} - T)\vec{n}\ at\ the\ level\ of\ the\ median\ plane\ e/2\ \vec{\nabla}H\cdot\vec{n} = 0\ \vec{\nabla}T\cdot\vec{n} = 0 \; (24)$$

Initially, the medium is assumed to be uniform in moisture and in thermal equilibrium with the surrounding air.

3.2 Method of resolution

The finite element method is used in this work to solve the equations in 2D. Although complex, this method is suitable for the control problems involving geometries and irregular boundaries. There was also the need for us to adapt this method for solving differential equations. In general, the equations are discretized using a variational formulation or the Galerkin method which is a special case of the method of weighted residues. This method has some advantages over the finite differences; in particular the flexibility in the choice of mesh size. It is easy to take into account the conditions to any spatiotemporal limitations. Many codes of structural calculation are based on the finite element method (ANSYS, CADDS, CATIA ...). The development of the method leads to the following system:

$$\{\dot{T}_i m_{ij} + T_i A_1 G_{ij} + H_i A_2 G_{ij} + T_i A_1 \frac{h_c}{\lambda} f_{ij} + H_i\left(A_2 \frac{h_m}{D_H} + A_1 \frac{\rho L h_m}{\lambda}\right) f_{ij} = A_1 \frac{h_c}{\lambda} T_{air} g_j + \left(A_2 \frac{h_m}{D_H} H_{eq} + A_1 \frac{\rho L h_m}{\lambda} H_{eq}\right) g_j \; \dot{H}_i m_{ij} + +H_i B_2 G_{ij} + +H_i\left(B_1 \frac{h_m}{D_H}\right) f_{ij} = +\left(B_1 \frac{h_m}{D_H} H_{eq}\right) g_j \; (25)$$

and the matrix formulation:

$$\{[\mathrm{M}]\dot{\mathrm{T}}^{\mathrm{n}} + ([\mathrm{G}^{11}] + [\mathrm{F}^{11}])\mathrm{T}^{\mathrm{n}} + ([\mathrm{G}^{12}] + [\mathrm{F}^{12}])\mathrm{H}^{\mathrm{n}} = [\mathrm{g}^1] \; [\mathrm{M}]\dot{\mathrm{H}}^{\mathrm{n}} + ([\mathrm{G}^{22}] + [\mathrm{F}^{22}])\mathrm{H}^{\mathrm{n}} = [\mathrm{g}^2] \; (26)$$

[M] is the hydro-thermal capacity matrix taking into account the coefficients terms of water content and temperature. Matrices water and heat conductivities are composed of two terms [G] and [F] which explain a transfer resistance inside the area and surface.

The vectors of hydrous and thermal loading $[g]$ are composed of a loading of imposed flow type to which one adds the condition of convection. The system is placed in the form AX = b, with A being a non-symmetrical square matrix, where the solution is obtained by the method GMRES (Generalized Minimum Residual). MATLAB software is used for its simplicity as an interpreted programming language and its qualities of preset libraries. The material used is Iroko (Chlorofora Excelsa) due to the availability of some of its thermo-physical and mechanical properties, or determined in our previous work [2,3] or in the works of other authors [7,17] .

4. Results and Interpretation

We performed sensitivity tests of drying parameters at first to persuade the program's ability and mastery of the finite element method applied in the resolution. Subsequently, the theoretical result is compared with that of the experiment by adjusting the mass transfer coefficients and surface energy. Other thermo physical parameters are taken from the literature and our previous work as noted in the previous paragraph. The initial water content is 80%, the relative humidity of 65%. Figure 3 shows the evolution of the average water content of the sample over time according to the temperature of the drier enclosure. It is observed that the high temperature increases the evaporation power of the air, consistent with the findings from the literature [17,18,19].

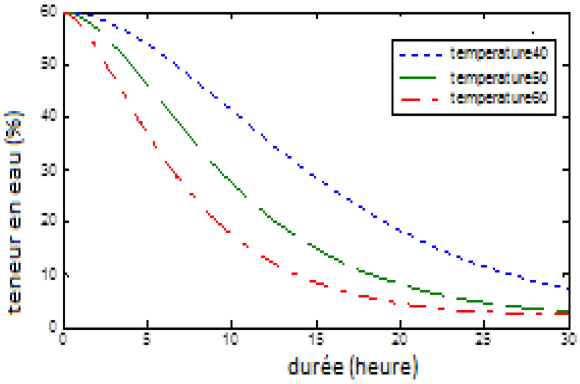

Figure 3: influence de la température de l'air de séchage sur la durée

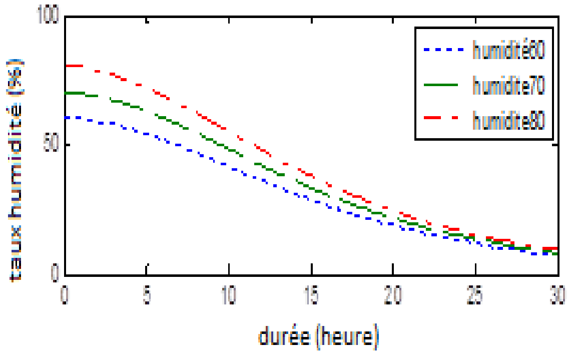

Figure 4: influence de l'humidité initiale du produit sur la durée du séchage

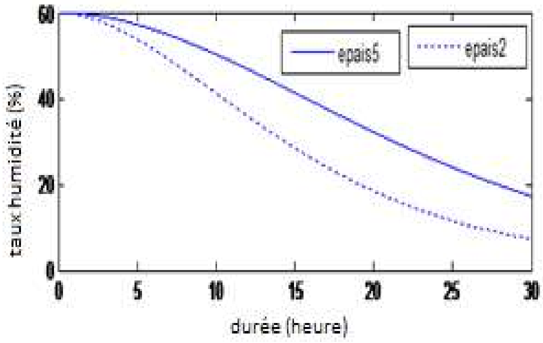

Figure 5: influence de l'épaisseur sur la durée du séchage.

Figures 4 and 5 show for their respective influences of the initial water content of the dried product and its thickness: the smaller the thickness, the greater the product takes time to dry. Although initial moisture contents are different, the same drying air condition, the balance point is the same.

Figures 6 to 9 show the curves of experimental measurement results compared to theoretical where the drying conditions were possible in case the reproduced program. The evaluation of the average relative error gives the following values: 0.8899%; 3.0449%; 1.1150%; 7.90479% of the drying air temperatures of 40 °C respectively; 45 °C; 50 °C; 60 °C. This level of uncertainty is satisfactory and we will exploit code developed in the analysis of drying processes across the product and give other interpretations outside the sensitivity of drying parameters.

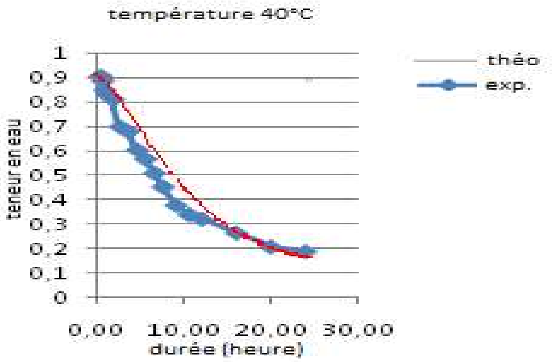

Figure 6: résultat essai expérimental et comparaison au résultat théorique à 40°C

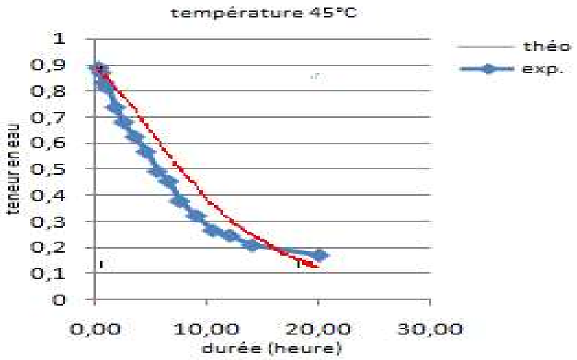

Figure 7: résultat essai expérimental et comparaison au résultat théorique à 45°C

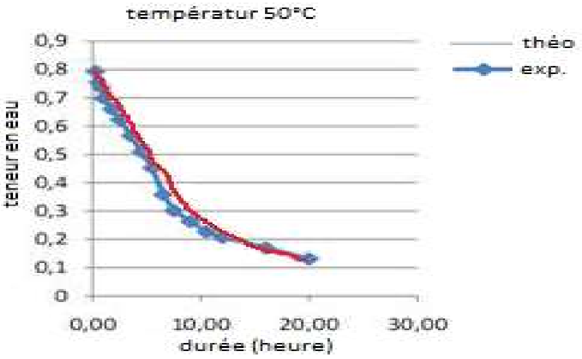

Figure 8: résultat essai expérimental et comparaison au résultat théorique à 50°C

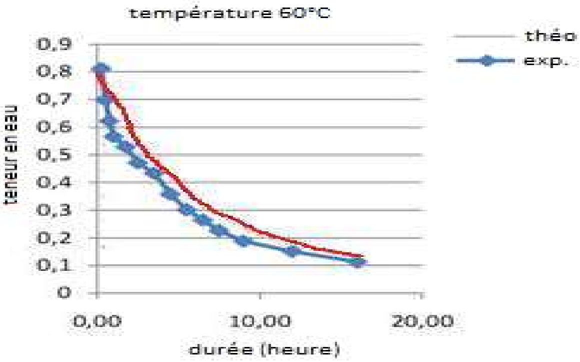

Figure 9: résultat essai expérimental et comparaison au résultat théorique à 60°C

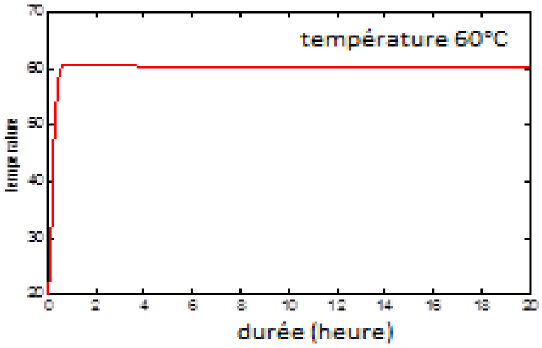

Figure 10: évolution de la température du produit à 60°C

Figure 10 shows the evolution of the average temperature of the product; one can make the observation that the thermal equilibrium between the product and the drying chamber is reached in less than 2 hours after the start of the process. The rapid rise of temperature is favorable to the elimination of the causes produced by the temperature gradient, the effects oppose the phenomenon of desorption (thermomigratoire effect) [12,18] moisture gradient.

5. Conclusion

In this work, it was a question of modeling heat transfer and mass transfer in porous media. For this we based model Luikov wrote a system of equations taking into account the effects thermomigratoires within the product, equations arise strongly coupled. Assisted by simplification assumptions, the transfer material is treated solely in one of the bound water and water vapor. The system obtained is resolution by the finite element method in Matlab, in the ambition of the control method. The results in terms of specific average moisture content of the product over time are shown as test sensitivity including drying parameters: the influence of the thickness of the product, the temperature of the drying air and the initial water content of the product. The results are also compared with those obtained from the experiment for the validation of code that will also allow us to analyze the influence of many variations of the boundary conditions that the evolution of the drying front. The values of the relative error calculated between the experimental and theoretical data are satisfactory order and give an overall average of 3.238% all test temperatures combined.

6. Nomenclature

C : Mass fraction (-)
C_p : Specific heat capacity (J/kg.K)
$D_{\overline{\overline{HH}}}$: Tensor of the coefficients of diffusion of water due to a water gradient (m2 / S)

$D_{\overline{\overline{HT}}}$: Tensor of the coefficients of water diffusion due to a change in temperature (m2 / S)
$D_{\overline{\overline{TT}}}$: Tensor of the coefficients of diffusion of heat due to a change in temperature (m2 / S)
$D_{\overline{\overline{TH}}}$: Tensor of the coefficients of diffusion of heat due to a water gradient (m2 / S)
L_s : desorption heat (J/kg)
L_v : Latent heat of vaporisation (J/kg)
H : wood humidity (%) or (decimal)
T : Temperature (°C)
$\vec{q}_{a,l,b,v}$: mass flow, respectively (of dry air, free water, bound water, water vapor) (kg/(m^2.s))
$\vec{j}$:partial flow of conduction (kg/(m^2.s))
K : mass flow per unit volume (kg/(m^3.s))
$\vec{V}_{l,b,v}$: phase velocity :resp(liquid, bound water, water vapor (m/s)
$\bar{k}$, $\bar{k}_r$:Tensors of permeabilitiees resp. of the medium (m^2) and relative (-)
P : totale Pressure (Pa)
q_s: Energy per unit volume (J/m^3)
h_T : heat exchange coefficient $(W/(m^2.K))$
h_m : mass transfer coefficient (m/s)
x : water content (kg/kg)
S : Fraction of free water per unit volume (décimal)
t : Time (s)

Greek:
ε: Substance's porosity
μ: Dynamic viscosity (kg/(m.s))
λ : Thermal conductivity (W/(m.K))
ρ : Density (kg/m^3)

Indices:
a : Air
v : vapor
b : bound water
l : free water
c : Capillarity
g.l,s : Resp. gaseous, liquid and solid phases (or anhydrous)
o : Initial
e : equilibrium

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