Given two dictionaries $D(y_1)$ and $D(y_2)$ whose column space is a point on $\mathbb{G}_{N,d}$, we determine the velocity matrix A such that travelling in this direction from $\mathcal{Y}_1$ leads to $\mathcal{Y}_2$ in unit-time. Let $\bar{D}(y_1)$ and $\bar{D}(y_2)$ denote the $d \times N$ matrices obtained by orthonormalizing the columns of $D(y_1)$ and $D(y_2)$ respectively.

- Compute the $d \times d$ orthogonal completion Q of $\bar{D}(y_1)$.
- Compute the thin CS decomposition of $Q^T \bar{D}(y_2)$, $Q^T \bar{D}(y_2) = \begin{pmatrix} X_C \\ Y_C \end{pmatrix} = \begin{pmatrix} U_1 & 0 \\ 0 & \tilde{U}_2 \end{pmatrix} \begin{pmatrix} \Gamma(1) \\ -\Sigma(1) \end{pmatrix} V_1^T$
- Compute $\{\bar{\phi}_i\}$ which are given by the arcsine and arccos of the diagonal elements of Γ and Σ respectively, i.e. $\gamma_i = \cos(\bar{\phi}_i)$, $\sigma_i = \sin(\bar{\phi}_i)$. Form the diagonal matrix $\bar{\Phi}$ containing $\bar{\phi}_i$'s as diagonal elements.
- Compute $A = \tilde{U}_2 \bar{\Phi} U_1$.

Algorithm 1: Numerical computation of the velocity matrix: The inverse exponential map [14].

From the gallery faces y_i's constituting M classes, and probe faces $\tilde{y}_i$, compute the respective dictionaries $D(y_i)$ and $D(\tilde{y}_i)$. Orthonormalize their columns to obtain $\bar{D}(y_i)$ and $\bar{D}(\tilde{y}_i)$.

Training:

- Compute the matrix $[K_{train}]_{ij} = k_P(\bar{D}(y_i), \bar{D}(y_j))$ for all $\bar{D}(y_i), \bar{D}(y_j)$ in the training set, where k_P is the projection kernel defined earlier.
- Solve $\max_\gamma L(\gamma)$ by eigen-decomposition (6), with $K^\star = K_{train}$.
- Compute the $(M$-1)-dimensional coefficients, $F_{train} = \gamma^T K_{train}$

Testing:

- Compute the matrix $[K_{test}]_{ij} = k_P(\bar{D}(y_i), \bar{D}(\tilde{y}_j))$ for all $\bar{D}(y_i)$ in training, and $\bar{D}(\tilde{y}_j)$ in testing.
- Compute $(M$-1)-dimensional coefficients, $F_{test} = \gamma^T K_{test}$ by solving for (6) with $K^\star = K_{test}$.
- Perform 1-NN classification from the Euclidean distance between F_{train} and F_{test}

The Rayleigh quotient $L(\gamma)$ is given by,

$$L(\gamma) = \max_{\gamma} \frac{\gamma^T K^\star (\bar{V} - 1_B 1_B^T / B) K^\star \gamma}{\gamma^T (K^\star (I_B - \bar{V}) K^\star + \sigma^2 I_B) \gamma} \qquad (6)$$

where $K^\star$ is the Gram matrix (K_{train} or K_{test}), 1_B is a uniform vector $[1...1]^T$ of length B corresponding to the number of gallery images, $\bar{V}$ is the block-diagonal matrix whose m^{th} block $(m = 1 \text{ to } M)$ is the uniform matrix $1_{B_m} 1_{B_m}^T / B_m$, B_m is the number of gallery images in m^{th} class, and $\sigma^2 I_B$ is a regularizer to make computations stable.

Algorithm 2: Kernel Linear Discriminant Analysis (KLDA) [15].

4.1.2 Learning from data on $\mathbb{G}_{N,d}$

In cases where there is more data available for each person in the gallery portraying other intra-class facial variations, it paves the way for performing statistics on the point cloud on $\mathbb{G}_{N,d}$.Since the blur-invariants have a resultant dimension of $(d-N) \times N$[8], with d significantly higher than N, it would require large number of samples to learn class-specific distributions. We hence pursued the method of Hamm and Lee [15] that performs kernel linear discriminant analysis on the blur-invariants using the projection kernel $k_P(\bar{D}(y_1), \bar{D}(y_2)) = \|\bar{D}(y_1)^T \bar{D}(y_2)\|_F^2 = trace[(\bar{D}(y_1)\bar{D}(y_1)^T)(\bar{D}(y_2)\bar{D}(y_2)^T)]$,which is a Mercer kernel that implicitly computes the inner product between $\bar{D}(y_i)'s$ in the space obtained using the following embedding; $\omega_P : \mathbb{G}_{N,d} \rightarrow \mathbb{R}^{d \times d}$, $span(\bar{D}(y_i)) \rightarrow \bar{D}(y_i)\bar{D}(y_i)^T$. To make the paper self-contained, we present the details of this method in Algorithm 2.

4.2 Performing Recognition across Blur

4.2.1 Spatially uniform blur

In the case when k remains unchanged over all pixels $(n_1, n_2)$of a $d_1 \times d_2$image y (1), recognition is performed by a nearest neighbor classifier on the two distances (say, SD) discussed before namely, (i) the Riemannian distance d_G (5), and (ii) the Euclidean distance in the lower-dimensional space obtained from KLDA (Algorithm 2). The identity of probe $\tilde{y}$ is therefore obtained by,

$$i^\star = \arg\min_i SD(D(\tilde{y}), D(y_i)) \qquad (7)$$

4.2.2 Spatially varying blur

We now study the more difficult problem, where the blur kernel k is spatially varying. This occurs when different parts of the scene are affected differently by blur, with some common examples being; outof- focus blur in objects with depth discontinuities, and motion blur when there is a sudden change in intensity values of a region due to object movements. The image formation equation for this case can be written as,

$$\tilde{y}_n = y_n * k_n \qquad (8)$$

where the subscript n indicates the pixel location. Since a blur kernel acts on a local spatial neighborhood, allowing it to change at every pixel location makes the problem severely under-constrained. A common assumption made to overcome this condition is to assume the blur to be locally uniform [6], which is valid in most practical cases. Along these lines, if the blur is assumed to be uniform over a region of size $d_1' \times d_2'$ (with $d_1' > b_1$, and $d_2' > b_2$), we can perform recognition by dividing the image into T overlapping patches of $d_1' \times d_2'$ each, and rewriting (7) as,

$$i^\star = \arg\min_i \sum_{t=1}^{T} SD(D(\tilde{y})_t, D(y_i)_t) \qquad (9)$$

where the subscript t denotes the patch at which the quantities in (9) are computed, and the span of $D(.)_t$'sare points on $\mathbb{G}_{N,d'}$, $d' = d_1' \times d_2'$. The inherent assumption while matching patches is that the faces are aligned. However for those patches where there is a transition between blur kernels, the column space of $D(.)_t$will not be invariant to blur. The percentage of such instances depends on the nature of spatially varying blur.

5. Conclusion

We showed that the subspace resulting from convolutions of an image with a complete set of orthonormal basis functions that could represent the blur kernel is invariant to blur under some assumptions, and it can account for more general classes of blur unlike other invariants. We then studied the utility of this invariant for the problem of direct recognition of faces, using techniques that account for their underlying non-Euclidean geometry, and observed an improved performance over other existing deblurring-based and invariant-based approaches. From the point of view of performing robust face recognition under unconstrained settings, it is interesting to study the integration of explicit formulations of other facial variations such as lighting and pose, with this blur-invariant.

References

[1] P.-A. Absil, R. Mahony, and R. Sepulchre. Riemannian geometry of Grassmann manifolds with a view on algorithmic computation. Acta Appl. Math., 80:199–220, Feb. 2004.
[2] A. Agrawal and Y. Xu. Coded exposure deblurring: Optimized codes for PSF estimation and invertibility. In CVPR, pages 2066–2073, June 2009.
[3] T. Ahonen, E. Rahtu, V. Ojansivu, and J. Heikkila. Recognition of blurred faces using local phase quantization. In ICPR, pages 1–4, Dec. 2008.
[4] H. Andrews and B. Hunt. Digital Image Restoration. Prentice Hall Signal Processing Series, 1977.
[5] E. Begelfor and M. Werman. Affine invariance revisited. In CVPR, pages 2087–2094, June 2006.
[6] A. Chakrabarti, T. Zickler, and W. Freeman. Analyzing spatially-varying blur. In CVPR, pages 2512–1519, June 2010.
[7] Y. Chikuse. Statistics on special manifolds. Springer Verlag, 2003.
[8] A. Edelman, T. Arias, and S. Smith. The geometry of algorithms with orthogonality constraints. SIAM Journal of Matrix Analysis and Application, 20:303–353, Apr. 1999.
[9] R. Fergus, B. Singh, A. Hertzmann, S. T. Roweis, and W. T. Freeman. Removing camera shake from a single photograph. ACM Transactions on Graphics, 25:787–794, Mar. 2006.
[10] J. Flusser, J. Boldys, and B. Zitov´a. Moment forms invariant to rotation and blur in arbitrary number of dimensions. IEEE TPAMI, 25:234–246, Feb. 2003.
[11] J. Flusser, J. Boldyˇs, and B. Zitov´a. Invariants to convolution in arbitrary dimensions. Journal of Mathematical Imaging and Vision, 13:101–113, Feb. 2000.
[12] J. Flusser and T. Suk. Degraded image analysis: an invariant approach. IEEE TPAMI, 20:590 –603, June 1998.
[13] J. Flusser, T. Suk, and S. Saic. Recognition of images degraded by linear motion blur without restoration. In Proc. Theoretical Foundations of Computer Vision, pages 37–51, Sep. 1996.
[14] K. Gallivan, A. Srivastava, X. Liu, and P. Van Dooren. Efficient algorithms for inferences on grassmann manifolds. In Workshop on Statistical Signal Processing, pages 315–318, Feb 2003.
[15] J. Hamm and D. D. Lee. Grassmann discriminant analysis: a unifying view on subspace-based learning. In ICML, pages 376–383, July 2008.
[16] P. C. Hansen, J. G. Nagy, and D. P. O'Leary. Deblurring Images: Matrices, Spectra, and Filtering. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2006.
[17] H. Hu and G. de Haan. Low cost robust blur estimator. In ICIP, pages 617–620, Oct. 2006.
[18] K. Lee, J. Ho, and D. Kriegman. Acquiring linear subspaces for face recognition under variable lighting. IEEE TPAMI, 27:684– 698, May 2005.
[19] A. Levin, D. Lischinski, and Y. Weiss. A closed-form solution to natural image matting. IEEE TPAMI, 30:228–242, Feb. 2008.
[20] A. Levin, Y. Weiss, F. Durand, and W. Freeman. Efficient marginal likelihood optimization in blind deconvolution. In CVPR, pages 2657 –2664, June 2011.
[21] Y. M. Lui and J. R. Beveridge. Grassmann registration manifolds for face recognition. In ECCV, pages 44–57, Oct. 2008.
[22] M. Nishiyama, A. Hadid, H. Takeshima, J. Shotton, T. Kozakaya, and O. Yamaguchi. Facial deblur inference using subspace analysis for recognition of blurred faces. IEEE TPAMI, 33:838 –845, Apr. 2011.
[23] M. Nishiyama, H. Takeshima, J. Shotton, T. Kozakaya, and O. Yamaguchi. Facial deblur inference to improve recognition of blurred faces. In CVPR, pages 1115–1122, June 2009.
[24] V. Ojansivu and J. Heikkil¨a. A method for blur and affine invariant object recognition using phase-only bispectrum. In ICIAR, pages 527–536, June 2008.
[25] P. Phillips, P. Flynn, T. Scruggs, K. Bowyer, J. Chang, K. Hoffman, J. Marques, J. Min, and W. Worek. Overview of the face recognition grand challenge. In CVPR, pages 947–954, June 2005.
[26] P. Phillips, H. Moon, S. Rizvi, and P. Rauss. The FERET evaluation methodology for face-recognition algorithms. IEEE TPAMI, 22:1090–1104, Oct. 2000.
[27] L. I. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. Physica D, 60:259–268, Nov. 1992.
[28] T. Sim, S. Baker, and M. Bsat. The CMU pose, illumination, and expression database. IEEE TPAMI, 25:1615–1618, Dec. 2003.
[29] I. Stainvas and N. Intrator. Blurred face recognition via hybrid network architecture. In ICPR, pages 805–808, Sep 2000.
[30] T. Suk and J. Flusser. Combined blur and affine moment invariants and their use in pattern recognition. Pattern Recognition, 36:2895 – 2907, Dec. 2003
[31] A. N. Tikhonov and V. Y. Arsenin. Solutions of Ill-Posed Problems. V. H. Winston & Sons, Washington, D.C.: John Wiley & Sons, New York,, 1977.
[32] P. Turaga, A. Veeraraghavan, A. Srivastava, and R. Chellappa. Statistical computations on grassmann and

stiefel manifolds for image and video-based recognition. IEEE TPAMI, 33:2273–2286, Nov. 2011.
[33] Y. Wong. Differential Geometry of Grassmann Manifolds. Proc. of the National Academy of Science, 57:589–594, 1967.
[34] L. Yuan, J. Sun, L. Quan, and H.-Y. Shum. Progressive interscale and intra-scale non-blind image deconvolution. ACM Transactions on Graphics, 27:1–10, Mar. 2008.

Author Profile

Mr. Ankush Jain [M.Tech] working as assistant professor in Computer Science Engineering from CVSR COLLEGE OF ENGINEERING from ANURAG GROUP OF INSTITUTIONS Venkatapur(V), Ghatkesar(M), Ranga Reddy District, Hyderabad-500088, Telangana State.

P. Santhosh Kumar received the B.Tech degree in Computer Science and Engineering from JNTU Hyderabad 2011 and pursuing M.Tech Degree in Computer science and Engineering from JNTU Hyderabad.