Relation between Energy and Time Period of a Rotating a Black Hole

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Abstract: This article is about the relation between time period and mass or energy of a rotating black hole (surface area reduces very slowly) with the help of concept of hawking radiation. Rotating black hole evaporates energy and its size decreases as the radiation takes place. In this article I shall show that the ratio of energy or mass and time period of a rotating black hole remains constant. 

\[ W = E/T, \text{ Where } W \text{ is wasif's constant.} \]

If at a time rotating black hole has time period \( T_1 \) and energy \( E_1 \) and some other time it has time period \( T_2 \) and energy \( E_2 \) then following relation will hold true (only for rotating black hole). We can write it as \( E_1/E_2 = T_1/T_2 \)

Keywords: hawking radiation

1. Introduction

Scientists have discovered that rotating black holes have some entropy so it must have some temperature and must emit radiation and the theory of living legend Stephen hawking has proved the black hole radiation (both rotation and non rotating). We know that when pair production takes place just near the event horizon of a rotating black hole negative particles fall in to the black hole and reduce its mass and positive particles emit as radiation. The reduction in mass of a rotating black hole depends on the surface area of the rotating black hole, larger the surface area means larger the area to fall the negative particles into the black hole. Let at some point rotating black hole has surface area \( S \)

\[ S = 4 \pi r^2 \]

Where ‘r’ represents the radius of the rotating black hole.

Dividing and multiplying the above equation by the mass of the rotating black hole ‘m’.

\[ S = 4 \pi (m^*r^2)/m \]  (1)

Inertia \( I = m^*r^2 \) (at that time) putting this value of \( m^*r^2 \) in (1). We get

\[ S = 4 \pi (I/m) \]  (2)

For rotating black hole let angular velocity is 

\[ \omega = 2 \pi/T \]

Or \[ 2 \pi = \omega * T \]

Putting this value of \( 2 \pi \) in (2). We get

\[ S = 2 \pi (I/\omega)/T/m \]  (3)

We know that angular momentum 

\[ J = I^*\omega \]

Putting the value of \( I^*\omega \) in (3). We get

\[ S = 2 \pi J^*T/m \]  (4)

This is the relation between surface area of the rotating black hole and its time period. Surface area is the function of mass and time period.

Negative particles that originate in pair production just near the event horizon falls into the black hole (on its surface). larger the surface area that means larger the area for negative particle to fall into the rotating black hole and larger amount of negative particles will fall in, and larger amount of radiation will take place as well as larger amount the mass of black hole will decrease. Since no external force in being applied that means angular momentum \( J \) will remain constant. That means as mass of the rotating black hole decreases its angular velocity increases (\( J = I^*\omega \) as I decreases \( \omega \) must increases in order to keep the \( J \) constant) so time period decreases.

Differentiating equation (4) with respect to the mass.

\[ dS/dm = 2J \{ m^*dT/dm - T \}/m^2 \] or this equation may be written as

\[ dS = 2J \{ m^*dT - Tdm/m^2 \} \]

\[ dS = 2J \{ dT/m - TDm/m^2 \} \]  (5)

d\( S \) is the change in surface area, d\( T \) is the change in time period and dm the change in the mass of the rotating black hole.

Energy of the rotating black hole will be given by

\[ E = mc^2 \]

Differentiating above equation with respect to mass (it varies as negative particles falls in). We get

\[ dE = c^2 dm \]  (6)

d\( E \) is change in energy, that energy will emit as hawking radiation.

Putting the value of \( m \) and dm in (5) from (6). We get

\[ dS = 2J \{ c^2 *dT/(E/c^2) + T(dE/c^2)/(E^2/c^4) \} \] after rearranging the equation we get.

\[ dS = 2J \{ c^2 *dT/E - T^2*c^2*dE/E^2 \} \]  (7)

We know that it takes billion billions year for a rotating
black hole to evaporate completely, surface area of the black hole decreases very very slowly with respect to the mass of the black hole so taking it as \( dS = 0 \)

for that condition to hold true we must have

\[
c^2 \frac{dT}{E} - T \frac{c^2}{E^2} dE = 0
\]

or \( dT - T \frac{dE}{E} = 0 \)

or \( \frac{dT}{T} = \frac{dE}{E} \) ..........................(8)

Integrating equation (8)

\[
\int \frac{dE}{E} = \int \frac{dT}{T}
\]

\( \text{Log } E = \text{Log } T + \text{Log } W \)  
(\( \text{log } W = \text{constant of integration} \))

Or \( \text{Log } E = \text{Log } T \times W \)

Taking anti Log

\( E = T \times W \)

\( \frac{E}{T} = W \) .............proved

putting \( E = mc^2 \) we get

\( mc^2 / T = W \)

Or

\( m / T = \text{constant} \) .............proved

That is the result we were looking for, ratio of energy and time period of remains constant.

If at some time energy of the rotation black hole is \( E_1 \) and time period \( T_1 \) and another time energy is \( E_2 \) and time period is \( T_2 \).

Then \( \frac{E_1}{T_1} = W \)

And \( \frac{E_2}{T_2} = W \)

Or \( \frac{E_1}{E_2} = \frac{T_1}{T_2} \) .........proved

2. Significance

Once we know the time period of rotating black hole this article can be used to determine the mass or energy it has.(because ratio of both quantities remains constant)

Above all since this whole article based on Hawking radiation if the result of this article practically found true. That means Hawking’s theory is correct (but not vice versa) and It may be treated as particle proof of the theory of Hawking radiation (black holes do radiate).

Reference