International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Mathematical Model for Court Expertise of a Class Economic Problems

Samoil Malcheski

Centre for research and development of the education, St. ASNOM No. 160/1-16 1000 Skopje, Macedonia

Abstract: ***Economic expertise is of great importance in solving a number of court cases, therefore it is important when the expertise is done by using a valid procedures. For that purpose in the expertise is necessary to take into account all parameters, i.e. if there is a statistical characteristics the same one should be analyzed because in many cases the qualitative characteristics may contain hidden quantitative characteristics. In this paper is considered exactly such class of problems that occur in practice and for the same is offered a model for expert examination, model who takes into account all the variables and gives an objective solution to the posed problem.***

Keywords: court expertise, mathematical model, rebate

JEL classification: C020, C810, C830

1. Introduction

In practice we often meets with a court expertise class of problems, which with minor changes can be described with the following example, a real problem of court practice, where by obvious reasons, we will omitted the names of companies and products that are subject of analysis.

Example. Two companies A and B over several years selling the same product with different quality, classified in 9 classes. In the considered period there are sold various quantities of product with different prices. The company A sells a product with rebate of 24%, and the company B with 36% rebate. Data for individual sales by class, years of sales, quantities and achieved prices expressed in den/kg are given in Tables 1 and 2.

Table 1: Product that has company A

Year[1)]	Class[1)]	Price den/kg[1)]	Sold kg[1)]	Price with rebate den/kg[2)]	Value[2)]
2000	Class 2	105,00	1006,00	79,80	80278,80
2001	Class 2	209,24	7566,50	159,02	1203224,83
2002	Class 2	132,59	483,50	100,77	48722,30
1999	Class 3	88,50	15490,68	67,26	1041903,14
2000	Class 3	88,50	4630,00	67,26	311413,80
2001	Class 3	165,00	35489,00	125,40	4450320,60
2002	Class 3	64,04	72260,00	48,67	3516894,20
2001	Class 4	100,00	7652,00	76,00	581552,00
2002	Class 4	64,04	52929,00	48,67	2576054,43
2002	Class 5	64,04	41747,00	48,67	2031826,49
2001	Class 6	40,00	705,00	30,40	21432,00
2001	Class 7	40,34	842,00	30,66	25815,72
2002	Class 8	6,73	3005,00	5,11	15355,55
2001	Class 9	17,00	503,00	12,92	6498,76
2002	Class 9	2,92	527,00	2,22	1169,94
Total			244835,68	-	15912462,56

1) The data are taken from real court cases expertise
2) Price with rebate is obtained by reduction of 24% in a given price
den/kg, and the value is obtained by multiplying sold kg with a price with rebate

During the procedure the court requires from the expert to compare the average prices at which companies A and B sell the products, or which company and what percentage on average sell the product more expensive. ♦

Table 2: Product that has company B

Year[1)]	Class[1)]	Price den/kg[1)]	Sold kg[1)]	Price with rebate den/kg[2)]	Value[2)]
2000	Class 1	235,00	114579	150,40	17232681,60
2000	Class 2	182,00	1462	116,48	170293,76
2000	Class 3	140,00	497	89,60	44531,20
2002	Class 2	182,00	246	116,48	28654,08
2002	Class 3	140,00	27225	89,60	2439360,00
2002	Class 4	80,00	50499	51,20	2585548,80
2002	Class 5	45,00	42638	28,80	1227974,40
Total			237146	-	23729043,84

1) The data are taken from real court cases expertise
2) Price with rebate is obtained by reduction of 36% in a given price den/kg, and the value is obtained by multiplying sold kg with a price with rebate

In most cases, but also in the case from the example the experts analysis the data with calculation of the arithmetic mean of grouped data $(x_i, f_i), i = 1,2,...,n$, where x_i denote the price with rebate, and f_i quantity, i.e. sold kilos and thus to calculate the average cost they use the following formula

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + ... + x_n f_n}{f_1 + f_2 + ... + f_n}. \quad (1)$$

For unknown reasons they overlooked the fact that the number f_i, $i = 1,2,...,n$ denotes the frequency of occurrence of data x_i, $i = 1,2,...,n$, which means that f_i, $i = 1,2,...,n$ must be natural numbers, but that is not the case with the data for firm A. Clearly, this procedure is wrong, the average prices obtained in the expertise of examples of companies A and B are amounted to

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + ... + x_{15} f_{15}}{f_1 + f_2 + ... + f_{15}} = \frac{15912462,56}{244835,68} = 64,992 \text{ denars, and}$$

$$\bar{y} = \frac{y_1 f_1 + y_2 f_2 + ... + y_7 f_7}{f_1 + f_2 + ... + f_7} = \frac{23729043,84}{237146} = 100,061 \text{ denars}$$

and they are not correctly calculated. However, in this case were made a few more gaps, concerning the calculation of

Licensed Under Creative Commons Attribution CC BY

average prices. This gaps are owed to the use of nominal prices achieved at different times periods, as well as calculating, i.e. comparing the average prices obtained from different classes to product, which gives the character of finding average prices of various products.

2. Model of Court Expertise for Considerations Class of Problems

Previously we have seen that during the expertise of the problem from the example, despite the incorporation of rabat, the experts only use the arithmetic mean for grouped data, which is inapplicable for the data concerning the company A. However, in this case, this is the smaller problem, because the expert described procedure does not take into account several crucial elements that has influence on the outcome of the expertise. Namely, considering the fact that we are talking about multidimensional characteristics we must first do one-dimensional analysis of individual characteristics and to determine the impact of each of them on the final result.

From data given in table 1 and 2 we can see that we have two data sets that have four characteristics, i.e.:

- ***Class of the product***, which is qualitative characteristic and for which the company A has eight characteristics: class 2, class 3, class 4, class 5, class 6, class 7, class 8 and class 9, and the company B has five characteristics: class 1, class 2, class 3, class 4 and class 5;
- ***Year,*** which is qualitative characteristic and for which the company A has four characteristics which is expressed in a numerical designation: 1999, 2000, 2001 and 2002, and company B has two characteristics: 2000 and 2002;
- ***The unit price,*** which is a quantitative characteristic and for which the firm A receives 12 characteristics, and the firm B receives 5 characteristics, and
- ***Quantity,*** which is a quantitative characteristic and for which the firm A receives 15 characteristics, and the firm B receives 7 characteristics.

Considering that companies A and B sold the product with rebates of 24% and 36%, respectively, and the properties of one dimensional characteristics, we will suggest the expertise that will compare the average prices at which companies A and B sell the product, based on following model:

1) rebate should be deducted from the nominal prices at which are sold certain classes of product in all years of the considered period, as it is done in Tables 1 and 2, thus we will obtain the rebate prices expressed in the den/kg,
2) because it is required to compare the average prices at which the analyzed product is sold, from further considerations we should exclud the classes who are not represented in the portfolios of both companies A and B, so we get m class of product represented in the portfolios of both companies, (in this particular case the determination of the average prices at which companies A and B sell the product should be done only on the basis of classes 2, 3, 4 and 5),
3) obtained, from different years, nominal prices with rebates should be reduced to the realistic prices with rebate, whereas each of the prices in a given year is divided by the index price increase of industrial (agricultural) products in terms of the initial year of review depending on the type of products, thus qualitative characteristic actually gets the quantitative meaning cumulative inflation rate,
4) for selected classes in each of the firms we sholud define average prices under which they are sold, whereby for class j from which firm A (B) sold quantities (weights) $\alpha_i, i=1,2,...,k$ at prices x_i, $i=1,2,...,k$ average price $\overline{x}_j$ is calculated as middle weight by formula

$$\overline{x}_j = \frac{\sum_{i=1}^{k} \alpha_i x_i}{\sum_{i=1}^{k} \alpha_i}, \quad (1)$$

and in further elaboration it is use specified average price $\overline{x}_j$ at which is sold quantity of goods $\sum_{i=1}^{k} \alpha_i$,

5) in the previous four steps we opted that the firm A sold m classes of the considered product at prices $\overline{x}_i, i=1,2,...,m$ and sold quantities $q_i, i=1,2,...,m$, respectively, while firm B the same m classes have sold at prices $\overline{y}_i, i=1,2,...,m$ and sold quantities p_i, $i=1,2,...,m$, so to find for what percentage company B has sold the product more expensive from the company A, we need to find weight aggregate index and the same one we can calculate by the Laspeyres formula by which the weights are taken from the base firm, in this case firm A, given with

$$I_q = \frac{\sum_{i=1}^{m} \overline{y}_i q_i}{\sum_{i=1}^{m} \overline{x}_i q_i} \quad (2)$$

or by Paasche formula in which the weights are taken from the index company, in this case the company B, given with

$$I_p = \frac{\sum_{i=1}^{m} \overline{y}_i p_i}{\sum_{i=1}^{m} \overline{x}_i p_i}. \quad (3)$$

Remark. The choice between the two formulas is reduced to the choice of system of weights, i.e. whether the index will be calculated on the structure of the base firm (period) or the structure of the index firm (period). In doing so, can not be said that one formula gives correct and the other incorrect result. However, in practice the formula of Paasche is applied more often, though it has no mathematical justification. It is recommended not to deviates from the practice, but if there is a big difference between the indexes calculated according to the formula (2) and (3), then for index of change of prices can be taken arithmetic mean of indexes of Laspeyres and Paasche, i.e. it should be calculated by formula

$$I = \frac{I_p + I_q}{2}. \quad (4)$$

The difference between the methodology used in the introductory part of this work and the proposed model will show on previous example given in introductory part, where we will implement each of the five steps separately. We have:

Licensed Under Creative Commons Attribution CC BY

Steps 1 and 2. The finding of prices with rabat expressed in den/kg is previously done, and they are shown in the fifth column in Tables 3 and 4. While for both companies are taken only data for classes 2, 3, 4 and 5.

Table 3: Product that has company A

Year	Class	Price den/kg	Sold kg	Price with rebate den/kg	Price on the level of 1999 [1)]
2000	Class 2	105,00	1006,00	79,80	82,35
2001	Class 2	209,24	7566,50	159,02	153,79
2002	Class 2	132,59	483,50	100,77	91,28
1999	Class 3	88,50	15490,68	67,26	67,26
2000	Class 3	88,50	4630,00	67,26	69,41
2001	Class 3	165,00	35489,00	125,40	121,28
2002	Class 3	64,04	72260,00	48,67	44,09
2001	Class 4	100,00	7652,00	76,00	73,50
2002	Class 4	64,04	52929,00	48,67	44,09
2002	Class 5	64,04	41747,00	48,67	44,09

1) Price at the level of 1999 are reduced so that every price that applies to any of the following years is divided by the index of producer prices of agricultural products (see Table 5)

Table 4: Product that has company B

Year	Class	Price den/kg	Sold kg	Price with rebate den/kg	Price on the level of 1999[1)]
2000	Class 2	182,00	1462	116,48	120,21
2000	Class 3	140,00	497	89,60	92,47
2002	Class 2	182,00	246	116,48	105,51
2002	Class 3	140,00	27225	89,60	81,16
2002	Class 4	80,00	50499	51,20	46,38
2002	Class 5	45,00	42638	28,80	26,09

1) Price at the level of 1999 are reduced so that every price that applies to any of the following years is divided by the index of producer prices of agricultural products (see Table 5)

Step 3. In this case we are talking about agricultural products, so the prices with rebate expressed in den/kg in the fifth columns in Tables 3 and 4 are divided with the indexes of producer prices of agricultural products (Table 5), thus all the prices are reduced at the level of 1999 and they are given in the sixth column of Tables 3 and 4.

Table 5: Index of producer prices of agricultural products

	Year			
	1999	2000	2001	2002
Index	100	96,9	103,4	110,4

Source. Statistical Yearbook of Macedonia 2003, State Statistical Office

Step 4. The average price at the level of 1999 under which is sold each of the classes by the companies A and B, and sold kilograms by these average prices are given in Table 6 and they are calculated according to the formula (1).

Table 6: Average prices and sold kilograms

Company A			Company B		
Class	Sold kg	Average prices den/kg	Class	Sold kg	Average prices den/kg
Class 2	9056,00	142,52	Class 2	1708,00	118,08
Class 3	127896,68	69,23	Class 3	27722,00	81,36
Class 4	60581,00	47,80	Class 4	50499,00	46,38
Class 5	41747,00	44,09	Class 5	42638,00	26,08

Source. Own calculations using the middle weight based on the data in tables 3 and 4

Step 5. If we use formula (2), then according to the data in table 6, for the index of Laspeyres we get

$$I_q = \frac{\sum_{i=1}^{m} \bar{y}_i q_i}{\sum_{i=1}^{m} \bar{x}_i q_i} = \frac{15373514,9}{14881345,3} = 1,033073\,,$$

which means that company B sold the product at an average price that is higher for $(1,033073-1)\cdot 100 = 3,3073\%$ from the price by which company A sold the same product in the considered period.

If we use the formula (3), then according to the data in table 6, for the index of Paasche we get

$$I_p = \frac{\sum_{i=1}^{m} \bar{y}_i p_i}{\sum_{i=1}^{m} \bar{x}_i p_i} = \frac{6456379,84}{5911285,22} = 0,915573\,,$$

which means that company B sold the product at an average price that is lower for $(1-0,915573)\cdot 100 = 8,4427\%$ from the price by which company A sold the same product in the considered period.

We can notice that in this case the the indexes of Laspeyres and Paasche give essentially different results, which primarily is result because of the dominant quantity of product in Class 3 in the portfolio of company A. As we said, in cases like this one we need to take that price index is equal to the arithmetic mean of the indexes calculated according Laspeyres and Paasche, so we have

$$I = \frac{I_p + I_q}{2} = \frac{1,033073+0,915573}{2} = 0,974323\,,$$

than finally we can conclude that company B sold the considered product at an average price that is lower for $(1-0,974323)\cdot 100 = 2,5677\%$ from the price by which company A sold it the same product.

3. Conclusion

In previous considerations we referred on an example for us of incomplete, and therefore improper judicial expertise. Based on this example, we established a mathematical model for analysis of a class economic problem, model which takes into account all the important elements that affect the final result. From the foregoing for every expertise:

- when analyzing the data based on which the expertise is done, to make clear distinguish between qualitative and quantitative characteristics that contain the data,
- to be done a qualitative analysis of each characteristic, in order to determine whether it contains a hidden

Licensed Under Creative Commons Attribution CC BY

characteristic which is quantitatively important for expertise,

- if the procedure requires the expertise to perform certain comparison, it is necessary from available data to separate those who are only comparable and they should be further processed, so the comparison is appropriate, logical and comprehensive, and
- to make a variety of methods for data processing, which choice is consistent with existing scientific knowledge, and if for the given problem they do not exist in a form, then the choice of methods must be such that it will fully correspond to the appropriate applicable scientifically verified knowledge, not to apply the methods and procedures because we seem to be adequate.

References

[1] Anderson, D. R.; Sweeney, D. J.; Williams, T. A.: Statistics for Business and Ekonomics, West Publishing Company, San Francisco, 1993
[2] Newbold, P.; Carlson, W.; Thorne, B.: Statistics for Business and Economics, Pearson education, New Jersey, 2007
[3] Malcheski, R., Malcheski, S.: Statistic for business, FON University, Skopje, 2010
[4] Malcheski, R., Malcheski, S.: Financial Mathematics, FON University, Skopje, 2010

Author Profile

Samoil Malcheski received the Ph.D. degree in Economy science from The Institute of Economics at University Ss Cyril and Methodius – Macedonia in 2013. His research interests are in applied mathematics and entrepreneurship, i.e. family businesses. He is the author of five university books, four handbooks, nine professional and eleven scientific papers.

Licensed Under Creative Commons Attribution CC BY