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Derivation and Analysis of Wave Excitation Force on A Rectangular Floating Barge in Three-Dimension for a Fluid of Constant Density

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Abstract: ***Surface waves may have significant effects on the hydrodynamics of offshore bodies or structures on a fluid of finite depth. Their influences are very crucial in Engineering Analysis, Design, and Optimization. It is therefore paramount to understand how offshore bodies are affected by the hydrodynamic waves. Due to this curiosity, this paper aims at investigating and analyzing the wave exciting force resulting from the presence of a rectangular floating barge on the wave terrain. The Green functions developed by Weahausen and Laitone will be used together with the method of boundary integral equations to obtain the radiation potential on the wetted body surface for a rectangular floating barge. The advantage of these method is that the pressure forces and the velocity potential will gotten directly by integration. The present study will be of great importance to Kenya after so much discovery of oil on the Southern Coast, the equipment to be used in the exploitations of these resources will be subject to the principles advanced in this study. The results obtained will help in analyzing the heave motion in which the drillers and designer of drilling equipment can rely on and help designer to design structure with low heave motion to ensure that it is possible to drill in a high percentage at time as possible. Furthermore, in case of ships the information can be used to choose optimum ship routes based on relevant criteria like minimum fuel consumption or the shortest time of voyage***

Keywords: Incidence Wave Velocity Potential, Diffraction Velocity Potential, Froude Krylov force, Diffraction Force, Wave Excitation Forces.

1. Introduction

Various studies have established that surface waves cause periodic loads on all man-made structures in the sea regardless of whether these structures are rigid or floating, or whether they are deep in the ocean or on the surface. These periodic loads are caused by the interaction between water waves and floating bodies. The study of these periodic loads has received considerable attention from designers. This attention is attributable to the fact that accurate predictions for the hydrodynamics' loads are crucial in designing any offshore structure (Wehausen and Laitone, 1960; Zakaria, 2009; Manyanga et al., 2012). The hydrodynamic interaction of a floating body and surface waves, which causes periodic loads, can be decomposed into the radiation problem (where the body undergoes oscillatory and translatory motion), and the diffraction problem (where the body is fixed and restrained from oscillating). Much of the available body of literature focuses on radiation and diffraction problems from the perspective that all offshore structures are cylindrically shaped (Ursel, 1949; Havelock, 1955; Garrett, 1971; Yeung, 1981; Bhatta and Rahman, 2003; Hassan and Bora, 2012; Finnegan et al, 2013). From these previous studies, it is evident that there is a deliberate effort to understand the hydrodynamic forces on cylindrical bodies, omitting the reaction on rectangular bodies. However, Gou, (2012) advice that it is not realistic to presuppose that all structures are and will always have a cylindrical cross-section. The proposed study focuses on the assessment of the hydrodynamic loads in particular the wave excitation forces acting on a rectangular barge in a fluid of constant and finite depth.
Nonetheless, many researcher who have worked on the wave excitation problem on a rectangular floating barge have done so for two-layer fluid (Nguyen and Yeung, 2010; Manyanga et al. 2014). This paper shows the derivation of the wave excitation force in a rectangular floating barge at zero forward speed using panel methods developed by Hess and Smith (1964). Although, Manyanga et al., 2014, explored the said method in the analysis of the wave excitation forces, the method is yet to be explored by past studies in solving wave excitation force especially for a fluid of constant density.

2. Mathematical Formulation

2.1 Incident Wave Potential

Incident wave potential is very important in the analysis of the wave excitation force. With the assumption of the fluid being inviscid, incompressible, and irrotational, the incident wave velocity potential satisfies the Laplace equation;

$$\nabla^2 \phi_I = 0 \tag{1}$$

The linearized boundary conditions on the free surface that is the dynamic free surface condition and the kinematic free surface condition together with the sea bottom conditions must also be satisfied.

$$\frac{\partial^2 \phi_I}{\partial t^2} + g\frac{\partial \phi_I}{\partial y} = 0, y = 0 \;, \left(\frac{\partial \phi}{\partial t}\right)_{y=0} + g\eta = 0 \;,$$

$$\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{2}$$

The velocity potential that satisfy equations (2) is defined as follow (Ngina *et al.*, 2014)

$$\phi_I = -i\frac{a\omega}{k}\frac{\cosh k\left(h+y\right)}{\sinh kh}e^{i\left(kx\cos\theta + kz\sin\theta - \omega t\right)} \tag{3}$$

But from the dispersion relation,

$$\frac{\omega^2}{g} = k \tan kh \tag{4}$$

$$= \frac{k \sinh kh}{\cosh kh}$$

$$\sinh kh = \frac{\omega^2}{kg} \cosh kh \tag{5}$$

Hence substituting equation (5) on (3) we get,

$$\phi_I = -i \frac{ag}{\omega} \frac{\cosh k(y+h)}{\cosh kh} e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{6}$$

Taking the real part of equation (6) we have.

$$\phi_I = \frac{ag}{\omega} \frac{\cosh k(y+h)}{\cosh kh} e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{7}$$

And the wave elevation from equation (6) is given by;

$$\frac{\partial \phi}{\partial t} = \eta = agk \frac{\cosh k(y+h)}{\cosh kh} e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{8}$$

2.2 Diffraction Potential

From the assumptions of linear water wave theory, the total velocity potential can be divided into a known incident potential and an unknown diffractive potential (Manyanga et al., 2014)

$$\phi = \phi_I + \phi_D \tag{9}$$

$$G(P,Q) = \frac{1}{\sqrt{R^2 + (y-\eta)^2}} + \frac{1}{(R^2 + 2h + y + \eta)^2} + 2\int_0^\infty \left(k + \frac{\omega^2}{g} \right) \frac{\cosh k(y+h) \cosh k(\eta+h)}{k \sinh(kh) - \frac{\omega^2}{g} \cosh ky} e^{-kh} J_o(kR)\, dk \tag{18}$$

Where

$$R = \sqrt{(x-\xi)^2 + (z-\zeta)^2}$$

Applying the Green's second identity below

$$\iiint_V \left(\phi_D \nabla^2 G - G \nabla^2 \phi_D \right) dv = \iint_s \left(\phi_D \frac{\partial G}{\partial n} - G \frac{\partial \phi_D}{\partial n} \right) ds = 0 \tag{19}$$

From equation (19) we get the integral equations as

$$-2\pi\phi_D(P) + \iint_S \phi_D \frac{\partial G(P,Q)}{\partial n_Q} dS_Q = \iint_S G(P,Q) \frac{\partial \phi_D}{\partial n_Q}(Q)\, dS_Q \tag{20}$$

Equation (20) is the Fredholm equation of the second kind for the values of the potential on the body surface, the entire surface of integration will be described by the sum of all its parts;

The diffraction potential must satisfy the following boundary conditions and governing equations.

$$\nabla^2 \phi = 0 \tag{10}$$

$$\frac{\partial \phi_D}{\partial y} = -\frac{\partial \phi_I}{\partial y}, Seabed \tag{11}$$

$$\frac{\partial \eta_D}{\partial t} = \frac{\partial \phi_D}{\partial y}, y = 0 \quad (12) \qquad \frac{\partial \phi_D}{\partial t} = -g\eta_D, y = 0 \tag{13}$$

$$\nabla \phi \to 0, t \to \infty, s_\infty \tag{14}$$

It is worth noting that both the ϕ_D and $\frac{\partial \phi_D}{\partial t}$ are bounded uniformly on s_∞

Furthermore, since the free surface boundary conditions is second order in time, then

$$\phi_D \to 0, t \to -\infty$$

$$\frac{\partial \phi_D}{\partial t} \to 0, t \to -\infty \tag{15}$$

In the derivation of the integral equations for the diffraction potential, the Wehausen and Laitone (1960) Green's function will be used

$$\phi(x,y,z) = \frac{1}{4\pi} \iint_{s_B} f_{S_B}(\xi,\eta,\zeta) G(x,y,z,\xi,\eta,\zeta) \tag{16}$$

This satisfies the equation;

$$\nabla^2 G(x,y,z,\xi,\eta,\zeta) = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \tag{17}$$

$$S = S_B \cup S_F \cup S_\varepsilon \cup S_\infty \tag{21}$$

The equation (20) is solved numerically by the Hess and Smith (1964) panel methods. Flat Quadrilaterals panels are used to approximate the body surface and the velocity potential in each panel is taken to be constant. Panel method helps in reducing the dimensionality of the problem by one and also in transforming an infinite domain of interest to finite boundaries in which the far field condition is automatically satisfied.

2.3 Wave Exciting Force

The wave excitation force is a combination of the Froude Krylov and the diffraction force.

These force are heavily related to the velocity potential given by equation (7) and (16)

$$\phi = \phi_I + \phi_D \tag{22}$$

The incident potential satisfies the free surface conditions; kinematic and dynamic boundary conditions (Ngina et al., 2014) and also the sea bed conditions while the diffracted potentials satisfies the radiation condition (Manyanga et al., 2014);

$$\lim_{R\to\infty} \phi_D = 0 \tag{23}$$

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Suppose that the diffraction potential is known and from equation (7) the incident velocity potential is given by:

$$\phi_I = \frac{ag}{\omega}\frac{\cosh k(y+h)}{\cosh kh}e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{24}$$

This implies that the total velocity potential given by equation (22) below is known

$$\phi = \phi_I + \phi_D \tag{25}$$

Consequently, the dynamic pressure can be derived from the linearized Bernoulli equation.

$$p = -\rho\frac{\partial\phi}{\partial t} = -\rho\frac{\partial\phi_I}{\partial t} - \rho\frac{\partial\phi_D}{\partial t} \tag{27}$$

Therefore, the dynamic force is given by;

$$\vec{F} = \iint_S P\vec{n}_j dS, \; j = 1,2,3 \tag{28a}$$

$$\vec{M}_j = \iint P(r\times n)_{j-3}, \; j = 4,5,6$$

Where P in equation (28a) is the fluid pressure and can be derived through Bernoulli's principle

$$\vec{F} = \iint_S \rho\frac{\partial\phi_I}{\partial t}\vec{n}\,dS + \iint_S \rho\frac{\partial\phi_D}{\partial t}\vec{n}\,dS \tag{28b}$$

$$= \iint_S \rho\frac{\partial\phi_D}{\partial t}\vec{n}\,dS + \iint_S \rho\frac{\partial\phi_D}{\partial t}\vec{n}\,dS$$

The equation (28) is the wave exciting force which is a combination of the Froude krylov force and the diffraction force.

The diffraction force is given by;

$$\vec{F}_D = \iint_S \rho\frac{\partial\phi_D}{\partial t}\vec{n}\,dS \tag{29a}$$

But

$$\vec{n} = \frac{\partial\phi_k}{\partial n}$$

The equation (29a) becomes

$$= \iint_S \rho\frac{\partial\phi_D}{\partial t}\frac{\partial\phi_k}{\partial n}dS \tag{29b}$$

$$= i\rho\omega\iint_S \phi_D\frac{\partial\phi_k}{\partial n}dS$$

Equation (29b) can also be written us

$$\vec{F}_D = i\rho\omega\iint_S \phi_k\frac{\partial\phi_D}{\partial n}dS \tag{29c}$$

but the incident velocity potential and the diffraction potential are related by the following potential.

$$\frac{\partial\phi_D}{\partial n} = -\frac{\partial\phi_I}{\partial n} \tag{30}$$

Therefore, from equation (30) then the diffraction potential from equation (29c) becomes;

$$\vec{F}_D = -i\rho\omega\iint_S \phi_k\frac{\partial\phi_I}{\partial n}dS, \; k = 1,2,\ldots,6 \tag{31a}$$

We now derive the Froude Krylov force which is related to the incoming wave potential.

$$F_I = -\rho\iint_S \frac{\partial\phi_I}{\partial t}\left(-\vec{n}\right)dS \tag{31b}$$

$$\frac{\partial\phi_I}{\partial t} = \frac{\omega^2\phi_I}{g} = i\omega\phi_I \tag{32}$$

Consequently, substituting equation (32) in equation (31b) the wave diffraction force becomes

$$F_I = \iint_S i\rho\omega\phi_I\vec{n}dS \tag{33}$$

$$= i\omega\rho\iint_S \phi_I\frac{\partial\phi_k}{\partial n}dS$$

Then, since the wave exciting force acting on a body is a combination of the Froude Krylov force and the diffraction force, then combining equation (31a) and (33) we get the wave excitation force as.

$$F_{iI} + F_{iD} = -i\omega\rho\iint_S (\phi_I + \phi_D)n_i dS \tag{34}$$

$$= i\rho\omega\iint_S\left(\phi_I\frac{\partial\phi_k}{\partial n} - \phi_k\frac{\partial\phi_I}{\partial n}\right)dS$$

3. Mathematical Scheme

$$\phi_I = \frac{g}{\omega^2}\frac{\cosh k(y+h)}{\cosh kh}e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{35}$$

$$F_E = i\rho\omega\iint_S\left(\phi_I\frac{\partial\phi_k}{\partial n} - \phi_k\frac{\partial\phi_I}{\partial n}\right)ds \tag{36}$$

Note:

(A)

$$\frac{\partial\phi_k}{\partial n} = n_k, k = 1,2,\ldots,6 \tag{37}$$

$$n_1 = n_x, n_2 = n_z, n_3 = n_y \tag{38}$$

$$D = \begin{bmatrix} i & j & k \\ x & z & y \\ n_x & n_z & n_y \end{bmatrix} \tag{39}$$

$$= \left(zn_y - yn_z\right)i - \left(xn_y - yn_x\right)j + \left(xn_z - zn_x\right)k$$

Then from equation (39)

$$n_4 = zn_y - yn_z, n_5 = yn_x - xn_y, n_6 = xn_z - zn_x \tag{40}$$

So

$$\iint_S \phi_I\frac{\partial\phi_k}{\partial n}ds = \iint_S \phi_I n_k ds \tag{41}$$

In case for heave motion then from equation (41)

$$\iint_S \phi_I \frac{\partial \phi_3}{\partial n} ds = \iint_S \phi_I n_y ds \tag{42}$$

$$\frac{\partial \phi_I}{\partial n} = \vec{n} \bullet \vec{\nabla} \phi_I \tag{43}$$

Where,

$$\vec{n} = n_1 i + n_2 j + n_3 k \tag{44}$$

$$= n_x i + n_z j + n_y k$$

$$\vec{\nabla} \phi_I = \frac{\partial \phi_I}{\partial x} i + \frac{\partial \phi_I}{\partial z} j + \frac{\partial \phi_I}{\partial y} k \tag{45}$$

(B)

$$\frac{\partial \phi_I}{\partial n} = \left(n_x i + n_z j + n_y k \right) \bullet \left(\frac{\partial \phi_I}{\partial x} i + \frac{\partial \phi_I}{\partial z} j + \frac{\partial \phi_I}{\partial y} k \right) \tag{46}$$

$$= n_x \frac{\partial \phi_I}{\partial x} + n_z \frac{\partial \phi_I}{\partial z} + n_y \frac{\partial \phi_I}{\partial y}$$

$$\iint \phi_k \frac{\partial \phi_I}{\partial n} ds = \phi_k \iint \left(n_x \frac{\partial \phi_I}{\partial x} + n_z \frac{\partial \phi_I}{\partial z} + n_y \frac{\partial \phi_I}{\partial y} \right) ds \tag{47}$$

For the analysis, this scheme should be coded using software and graphs drawn.

4. Discussion and Conclusion

From this derivation it's clear that for the analysis of the wave excitation force, Froude Krylov and the diffraction forces play a very important role. These two forces are directly related by using equation (30). The study (Ngina *et al*., 2014) shows that the vertical motions has high impact on any offshore structures that is why many researchers have analyzed the wave exciting force in the heave direction (Manyanga *et al.* 2014; Nguyen and Yeung, 2010; Mohaptra and Bora, 2008).

References

[1] Bhatta, D.D.and Rahman, M., (2003). scatterring and radition problem for a cylinder in water of finite depth. International, *International Journal of Engineering Science*, **49(9):** 931-967

[2] Finnegan,W., Meere, M., Goggins,J., (2013).The wave excitation forces on truncated vertical cylinder in water of infinite depth,*Journal of Fluids and Structures.***40**:201-213.

[3] Gou, Y., Chen, X., Teng, B. and Zheng, Y. (2012). Study on Wave Diffraction from a 3D Box in a Two-layer Fluid by Time-Domain Approach, *Proceedings of the Twenty-second (2012) International Offshore and Polar Engineering Conference.*

[4] Garrett, C.J.R., (1971). Wave forces on a circular dock, *Journal of Fluid Mechanics*. **46(01):** 129-139.

[5] Hess, J.L., Smith A.M.O., (1964). Calculation of nonlifting potential flow about arbitrary three-dimension bodies. Journal of Ship Research. 8(2): 22-44

[6] Hassan, M., Bora, S.N., (2012). Exciting forces for a pair of coaxial hollow cylinder and bottom-mounted cylinder in water of finite depth. Ocean Engineering.50(0):38-43

[7] MacCamy, R.C., and Fuchs, R.A., (1957).Wave Forces on Piles. A Diffraction Theory, Technology Memo.69, U.S. Army Corps of Engineers, Beach Erosion Board.

[8] Manyanga, D.O., Duan ,W.Y., (2011). Internal wave propagation from pulsating Sources in two-layer Fluid of finite depth. Journal of applied mechanics and Material. 201-202: 503-507. Trans Tech. Publications Switzerland.

[9] Manyanga, D.O., Duan, W.Y., Han, X., Cheng, P., (2014). Internal wave exciting force on rectangular barge in two-layer fluid of finite depth. Advancement in scientific Engineering Research,2(3): 48-61

[10] Masoud Baghfalaki , Samir K. D., (2013). Mathematical Modeling of Roll Motion for a Floating Body in Regular Waves Using Frequency Based Analysis with Speed. Universal Journal of Engineering Science,1:34 - 39.

[11] Mohapatra, S., Bora, S.N., (2008) Water wave interaction with a sphere in Two-Layer fluid flowing through a channel of finite depth. Arch. Applied mechanics. 79:25-740

[12] Nguyen, T.C., Yeung, R.W., (2010). Unsteady three-dimensional sources for a two-layer fluid of finite depth and their application. Journal of Engineering Mathematics. 70(1-3): 67-91

[13] Ngina, P.M., Manyanga, O.D. and Kaguchwa, J.N., (2014). Analysis of incident surface wave characteristic in 3-dimension for a fluid of finite depth. Advancement of Scientific and Engineering Research journal. 2(8): 62-69

[14] Ursell, F., (1949). On the heaving motion of a circular cylinder on the surface of a fluid, The Quarterly Journal of Mechanics and Applied Mathematics.2(2):218-231

[15] Wehausen, J.V. and Laitone, E.V., (1960).Surface Waves in Fluid Dynamics III, Handbuch Der Physik. 9(3): 446-778

[16] Yeung, R. W., (1981), Added mass and damping of a vertical cylinder in finite-depth waters. Journal of applied ocean and research. 3(3): 119-133

[17] Zakaria, N.M.G., (2009). Effect of ship size, forward speed and wave direction on a relative wave height of container ships in rough seas. Journal-The Institution of Engineers, Malaysia. 72: