Location and Stability of Equilibrium Points in a PRTBP under Perturbations, Bigger Primary is an Oblate Spheroid

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Abstract: ***We have examined the location and linear stability of triangular points in photogravitational restricted three body problems, bigger primary is an oblate spheroid. We observe that the triangular points L_4, L_5 form equilateral triangle with the primaries and are linearly stable if the mass ratio μ is less than the critical mass value $\mu_c = 0.0385209$ … … …***

Keywords: PRTBP, Equilateral Triangle, Triangular Points, Critical mass

1. Introduction

The present work "Location and stability of equilibrium points in a Photogravitational restricted three body problem, bigger primary is an oblate spheroid" deals with the equation of motion and the triangular stability. Here, both the primaries are taken as sources of radiation and the bigger primary as an oblate spheroid. The perturbations in coriolis and centrifugal forces are taken into consideration.

(A) Equations of motion and Location of triangular points:-

The equations of motion of a perturbed photogravitational restricted three body problem in bigger primary is an oblate spheroid is obtained as:

$$\left.\begin{aligned} \ddot{x} - 2\alpha\dot{y} &= \frac{\partial\Omega}{\partial x} \\ \ddot{y} + 2\alpha\dot{x} &= \frac{\partial\Omega}{\partial y} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(1)$$

where

$$\Omega = \frac{1}{2}(x^2+y^2)\beta + \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)q_1 A}{2r_1^3}\right\} \ldots\ldots \qquad (2)$$

and

$$\left.\begin{aligned} q_1 &= 1-\delta_1;\ 0<\delta_1<1 \\ q_2 &= 1-\delta_2;\ 0<\delta_2<1 \end{aligned}\right\} \ldots\ldots\ldots \qquad (3)$$

$$\alpha = 1+\epsilon_1;\ |\epsilon_1| \ll 1$$
$$\beta = 1+\epsilon_2;\ |\epsilon_2| \ll 1$$

and

$$\left.\begin{aligned} r_1^2 &= (x+\mu)^2 + y^2 \\ r_2^2 &= (x-1+\mu)^2 + y^2 \end{aligned}\right\} \qquad \ldots\ldots \qquad (4)$$

The coordinates of the triangular equilibrium points, upto the first order terms in the parameters ϵ_2, q_1, q_2 and A are:

$$x = \frac{1}{2} - \mu - \frac{1}{3}\delta_1 + \frac{1}{3}\delta_2 + \frac{1}{2}A + \frac{1}{3}A\delta_1 - \frac{7}{6}A\delta_2$$
$$y = \pm\frac{\sqrt{3}}{2}\left(1 - \frac{2}{9}\delta_1 - \frac{2}{9}\delta_2 - \frac{4}{9}\epsilon_2 - \frac{1}{3}A + \frac{4}{9}A\delta_1 + \frac{5}{9}A\delta_2\right. \ldots\ldots \qquad (5)$$

(B) Stability of triangular points :-

Differentiating (4), we have

$$\frac{\partial r_1}{\partial x} = \frac{x+\mu}{r_1}; \qquad \frac{\partial r_1}{\partial y} = \frac{y}{r_1}$$
$$\frac{\partial r_2}{\partial x} = \frac{x-1+\mu}{r_2}; \qquad \frac{\partial r_2}{\partial y} = \frac{y}{r_2}$$

Now differentiating (2) partially with respect to x, we get

$$\Omega_x = \beta x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(x+\mu)q_1}{r_1^3} + \frac{\mu(x-1+\mu)q_2}{r_2^3} + \frac{3(1-\mu)(x+\mu)Aq_1}{2r_1^5}\right\} \ldots\ldots \qquad (6)$$

$$\Omega_{xx} = \beta - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)Aq_1}{2r_1^5} - \frac{3(1-\mu)(x+\mu)^2 q_1}{r_1^5} - \frac{3\mu(x-1+\mu)^2 q_2}{r_2^5} - \frac{15(1-\mu)(x+\mu)^2 A q_1}{2r_1^7}\right\}$$

$$\Omega_{xy} = \frac{1}{1+\frac{3A}{2}}\left\{\frac{3(1-\mu)(x+\mu)q_1}{r_1^5} + \frac{3\mu(x-1+\mu)q_2}{r_2^5} + \frac{15(1-\mu)(x+\mu)Aq_1}{2r_1^7}\right\}y \qquad \ldots\ldots \qquad (8)$$

$$\Omega_y = \left[\beta - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)q_1 A}{2r_1^5}\right\}\right]y$$

$$\Omega_{yy} = \beta - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)q_1 A}{2r_1^5}\right\} + \frac{1}{1+\frac{3A}{2}}\left\{\frac{3(1-\mu)q_1}{r_1^5} + \frac{3\mu q_2}{r_2^5} + \frac{15(1-\mu)q_1 A}{2r_1^7}\right\} y^2 \qquad \ldots\ldots\ldots. \quad (9)$$

Substituting these values in (9), (10) and (11), we get

$$\Omega_{xx}^0 = \frac{3}{4} - \frac{1}{2}\delta_1 + \delta_2 + \frac{5}{4}\epsilon_2 + \frac{9}{4}A + \mu\left(\frac{3}{2}\delta_1 - \frac{3}{2}\delta_2 - 3A\right)$$

$$\Omega_{yy}^0 = \frac{9}{4} + \frac{1}{2}\delta_1 - \delta_2 + \frac{7}{4}\epsilon_2 + \frac{3}{4}A - \mu\left(\frac{3}{2}\delta_1 - \frac{3}{2}\delta_2\right)$$

$$\Omega_{xy}^0 = \frac{3\sqrt{3}}{4} - \frac{1}{2\sqrt{3}}\delta_1 + \frac{1}{\sqrt{3}}\delta_2 + \frac{11}{4\sqrt{3}}\epsilon_2 + \frac{5\sqrt{3}}{4}A - \mu\left(\frac{3\sqrt{3}}{2} + \frac{1}{2\sqrt{3}}\delta_1 + \frac{1}{2\sqrt{3}}\delta_2 + \frac{11}{2\sqrt{3}}\epsilon_2 + \sqrt{3}A\right)$$

Using these values, we obtain

$$\Omega_{xx}^0 + \Omega_{yy}^0 - 4\alpha^2 = -1 - 8\epsilon_1 + 3\epsilon_2 + 3\mu A$$

and $\Omega_{xx}^0 . \Omega_{yy}^0 - (\Omega_{yy}^0)^2$

$$= \frac{27}{4}\mu(1-\mu)\left(1 + \frac{2}{9}\delta_1 + \frac{2}{9}\delta_2 + \frac{4}{3}A + \frac{22}{9}\epsilon_2\right)$$

The characteristic equation for the triangular equilibrium points, therefore, takes the form

$$\lambda^4 + (1 + 8\epsilon_1 - 3\epsilon_2 - 3\mu A)\lambda^2 + \frac{27}{4}\mu(1-\mu)\left(1 + \frac{2}{9}\delta_1 + \frac{2}{9}\delta_2 + \frac{4}{3}A + \frac{22}{9}\epsilon_2\right) = 0 \qquad \ldots\ldots (10)$$

This is a quadratic equation in λ^2. The discriminant D of the quadratic equation (10) is given by

$D = b^2 - 4ac$

where $b = 1 + 8\epsilon_1 - 3\epsilon_2 - 3\mu A$

$a = 1$

$$c = \frac{27}{4}\mu(1-\mu)\left(1 + \frac{2}{9}\delta_1 + \frac{2}{9}\delta_2 + \frac{4}{3}A + \frac{22}{9}\epsilon_2\right)$$

In order that λ should be pure imaginary in conjugate pair, λ^2 must be negative quantity.

Here, $$\lambda^2 = \frac{-b \pm \sqrt{D}}{2a} = -\frac{b}{2} \pm \frac{1}{2}\sqrt{D}$$

$$= -\frac{1}{2}(1 + 8\epsilon_1 - 3\epsilon_2 - 3\mu A) \pm \frac{1}{2}\sqrt{D}$$

The roots are pure imaginary if $D \geq 0$.

This inequality may be written as

$$1 + 16\epsilon_1 - 6\epsilon_2 - 6\mu A - 27\mu(1-\mu)\left(1 + \frac{2}{9}\delta_1 + \frac{2}{9}\delta_2 + \frac{4}{3}A + \frac{22}{9}\epsilon_2\right) - \epsilon \geq 0 \qquad \ldots (11)$$

where ϵ is a positive quantity whose limit is zero.

The equation (11) may be written as

$$\mu^2 - \left(1 + \frac{2}{9}A\right)\mu + \frac{1}{27}\left(1 - \frac{2}{9}\delta_1 - \frac{2}{9}\delta_2 - \frac{4}{3}A + 16\epsilon_1 - \frac{76}{9}\epsilon_2 - \epsilon\right) = 0$$

Since μ represents the mass which is less than one half, the negative sign must be taken. At the limit $\epsilon = 0$

$$\mu = \frac{1 + \frac{2}{9}A - \sqrt{1 + \frac{4}{9}A - \frac{4}{27}\left(1 - \frac{2}{9}\delta_1 - \frac{2}{9}\delta_2 - \frac{4}{3}A + 16\epsilon_1 - \frac{76}{9}\epsilon_2\right)}}{2}$$

$$= \frac{1}{2}\left\{1 + \frac{2}{9}A - \sqrt{\frac{23}{27}}\left(1 + \frac{4}{207}\delta_1 + \frac{4}{207}\delta_2 + \frac{26}{69}A - \frac{32}{23}\epsilon_1 + \frac{152}{207}\epsilon_2\right)\right\}$$

The critical value of μ is easily obtained as

$$\mu_c = 0.1371742 - 0.0070111\delta_1 - 0.0070111\delta_2 - 0.02560585A - 0.500480105\epsilon_1 - 0.2664228\epsilon_2$$

Thus, when $0 \leq \mu < \mu_c$, $D \geq 0$, the values of λ^2 given by the characteristic equation are negative and all the four roots of the characteristic equation are purely imaginary. Thus the triangular equilibrium points are stable. Moreover the range of stability of triangular equilibrium points decreases on account of oblateness and photo gravitational effect of the primaries. When oblateness and radiating effects are ignored, the critical value reduces to $\mu_c = 0.0385209$……

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