Research on Kinematics Modeling of a Welding Robot

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Abstract: ***According to the problem of industrial robot controlling, it uses mechanical theory as guide and makes some mechanical analyses and calculation on the position of end effectors and position of each joint to meet the target off end effectors path. It is important to formulate the suitable kinematics models for a robot mechanisms and it is very crucial for analyzing the behavior of industrial manipulator. So this paper analyzes optimum direct and inverse kinematics of an industrial robot.***

Keywords: Welding robot, modeling, kinematics

1. Introduction

This paper covers the current practical methodologies for kinematics modeling and computations. The kinematics model represents the motion of the robot without considering the forces that cause the motion. Kinematics modeling is a prerequisite for the dynamics model and fundamental for practical aspects like motion planning, Singularity and workspace analysis, and manufacturing cell graphical simulation. For example, the majority of the robot manufacturers and many independent software vendors offer graphical environments where users, namely developers and System integrators can design and simulate their own manufacturing cell projects. The objective this paper is to formulate the position analysis of Reis RV60-60 industrial robot.

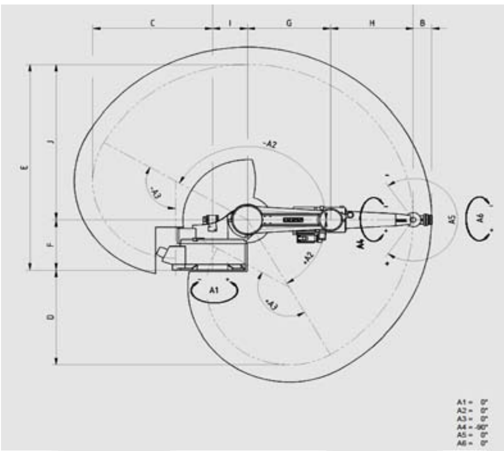

Figure 1: Workspace of ReisRV60-60[1]

Table 1: Denavit-Hartenberg Parameters for the ReisRV60-60

Link	$\theta_i(^\circ)$	$\alpha_{i-1}(^\circ)$	a_{i-1}(mm)	d_{i-1}(mm)
1	$\theta_1(0^\circ)$	0°	0	670
2	$\theta_2(90^\circ)$	90°	435	0
3	$\theta_3(0^\circ)$	0°	1035	0
4	$\theta_4(0^\circ)$	90°	0	780
5	$\theta_5(0^\circ)$	-90°	0	0
6	$\theta_6(0^\circ)$	90°	0	230+d

Where **d** is an extra length associated with the end effecter

2. Direct Kinematics

By simple inspection of Fig.2, it is easy to conclude that the last three axes. Form a set of ZFZ *Euler* angles [1,2] with respect to frame 4. In fact, the overall rotation produced by those axes is obtained from:

1. Rotation about Z4 by θ4
2. Rotation about Y\=*Z* '5 byθ5
3. Rotation about Z' '4=Z"5 byθ63

This gives as the following rotation matrix3 Y'4 corresponds to axis Y4 after rotation about Z4 by $\theta_4$and Z"4 corresponds to Z4 after rotation about Y'4=Z'5 byθ5\

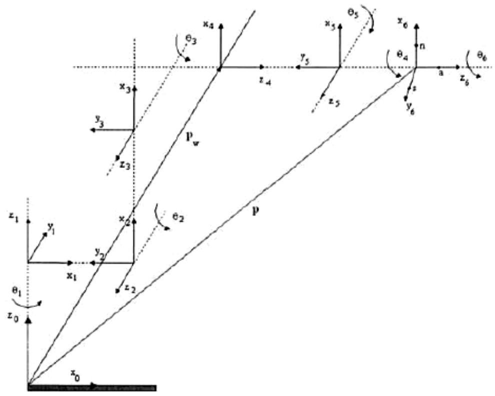

Figure 2: Link frame assignment

The RV60-60 is an anthropomorphic manipulator with spherical wrist. The anthropomorphic structure of the first three joints is the one that offers better dexterity to the robot manipulator. The first three joints are used to position the wrist. The orientation of the wrist is managed by the wrist spherical structure, which is also the one that gives higher dexterity. Using the definition of link transformation matrix

$$
\text{Tii-1}=\begin{bmatrix} c_i & -s_i & 0 & a_{i-1} \\ s_i c\alpha_{i-1} & c_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s_i c\alpha_{i-1} & c_i c\alpha_{i-1} & -c\alpha_{i-1} & -c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} (1)
$$

The direct kinematics of the ReisRV60-60 robot manipulator can be easily obtained as shown below.

$$T10=\begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T21=\begin{bmatrix} -s_2 & -c_2 & 0 & a_1 \\ 0 & 0 & -1 & 0 \\ c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T32=\begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T43=\begin{bmatrix} c_4 & -s_4 & 0 & a_4 \\ 0 & 0 & -1 & -d_4 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T54=\begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -s_5 0 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T65=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & -d_6 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T20=\begin{bmatrix} -c_1s_2 & -c_1c_2 & s_1 & a_1c_1 \\ -s_1s_2 & -s_1c_2 & -c_1 & a_1s_1 \\ c_2 & -s_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T30=\begin{bmatrix} -c_1s_{23} & -c_1c_{23} & s_1 & a_2c_1s_2+a_1c_1 \\ -s_1s_{23} & -s_1c_{23} & -c_1 & -s_1s_2+a_1s_1 \\ c_{23} & -s_{23} & 0 & a_2c_2 \quad +d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T40=\begin{bmatrix} -c_1s_{23}c_4+s_1s_4 & c_1s_{23}s_4+s_1c_4 & c_1c_{23} & d_4c_1c_{23}-a_3c_1s_{23}-a_2c_1s_2+a_1c_1 \\ -s_1s_{23}c_4-c_1s_4 & s_1s_{23}s_4-c_1c_4 & s_1c_{23} & d_4s_1c_{23}-a_3s_1s_{23}-a_2s_1s_2+a_1s_1 \\ c_{23}c_4 & c_{23}s_4 & s_{23} & d_4s_{23}+a_3c_{23}+a_2c_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T63=\begin{bmatrix} c_4c_5c_6-s_4s_6 & -c_4c_5s_6-s_4c_6 & c_4s_5 & d_6c_4s_5+a_3 \\ s_5c_6 & -s_5c_6 & -c_5 & d_6c_5+d_4 \\ s_4c_5c_6+c_4s_6 & -s_4c_5s_6+c_4c_6 & s_4s_5 & d_6s_4s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T64=\begin{bmatrix} -c_5c_6 & -c_5s_6 & s_5 & d_6s_5 \\ s_6 & c_6 & 0 & 0 \\ -s_5c_6 & s_5s_6 & c_5 & d_6c_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And

$$T10=\begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x^0 \\ r_{21} & r_{22} & r_{23} & p_y^0 \\ r_{31} & r_{32} & r_{33} & p_z^0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ with}$$

r11= ((s1s4-c1s23c4) c5-c1c23s5) c6+ (c1s23s4 + s1c4) s6
r12= ((-s1s4+ c1s23c4)c5+ c1c23s5)s6+ (c1s23s4 + s1c4)c6
r13= (-c1s23c4 + s1s4)s5+ c1c23c5
r21= ((-s1s23c4 - c1s4)c5-s1c23s5)c6+ (s1s23s4-c1c4)s6
r22= ((s1s23c4 + c1s4)c5+s1c23s5)s6+ (s1s23s4 - c1c4)c6
r23= (-s1s23c4 - c1s4)s5+s1c23c5
r31= (c23c4c5-s23s5)c6-c23s4s6
r32= (-c23c4c5+ s23s5)s6-c23s4c6
r33=c23c4c5+ s23c5
p0x = ((-c1s23c4 + s1s4)s5 + c1c23c5)d6 + d4c1c23– a3c1s23 - a2c1s2 + a1c1
p0y = ((-s1s23c4-c1s4)s5 + s1c23c5)d6 + d4s1c23– a3s1s23 - a2s1s2 + a1s1
p0x =d6 (c23c4s5 + s23c5) + d4s23+ a3c1s23–a2c2 +d1

Having derived the direct kinematics of the ReisRV60-60 robot, it's now possible to obtain the *end-effecter* position and orientation from the individual joint angles$(\theta_{1,}\theta_{2,}\theta_{3,}\theta_{4,}\theta_{5,}\theta_{6,})$.

3. Inverse Kinematics

Inverse kinematics deals with the problem of finding the required joint angles to produce a certain desired position and orientation of the *end-effector*. Finding the inverse kinematics solution for a general manipulator can be a very tricky task. Generally they are non-linear equations. Close-form solutions may not be possible and multiple, infinity, or impossible solutions can arise. Nevertheless, special cases have a closed-form solution and can be solved. The sufficient condition for solving a six-axis manipulator is that it must have three consecutive revolute axes that intersect at a common point: *Pieper* condition [5].Three consecutive revolute parallel axes is a special case of the above condition, since parallel lines can be considered to intersect at infinity. ReisRV60-60 robot meets the *Pieper* condition due to the spherical wrist. For these types of manipulators, i.e. manipulators that meet the *Pieper* condition, it is possible to decouple the inverse kinematics problem into two sub-problems: position and orientation. A simple strategy [1, 2] can then be used to solve the inverse kinematics, by separating the position problem from the orientation problem. Consider Figure2 where the position and orientation of the *end effect*

r is defined in terms of p and R60 = [n s a]. The wrist position (pw) can be found using

$$Pw = p - d_6 . a (2)$$

It is now possible to find the inverse kinematics for θ_1, θ_2 and θ_3 and solve the first inverse kinematics sub-problem, i.e., the position sub-problem. Considering Figure 4 it is easy to see that

$$\theta_1 = \mathrm{Atan2}\left(p_{wy}, p_{wx}\right)^4 (3)$$

Once θ_1 is known the problem reduces to solving a planar structure. Looking to Figure 4 it is possible to successively write

$$Pwx_1 = \sqrt{P_{wx}^2 + P_{wy}^2} (4)$$

$$P_{wz1} = P_{wz} - d_1 \ (5)$$

$$P_{wx1'} = P_{wx1} - a_1 \qquad (6)$$

$$P_{wy1'} = P_{wy1} \ (7)$$

$$P_{wz1'} = P_{wz1} (8)$$

And

$$P_{wx1'} = a_2 s_2 + a_x c_{23}' \qquad (9)$$

$$P_{wz1'} = a_2 c_2 + a_x s_{23}' \qquad (10)$$

Another possibility would be
$\theta_1 = \pi + \mathrm{Atan2}\left(p_{wy,} p_{wy,}\right)$if we set$\theta_2 -> \pi - \theta_2$

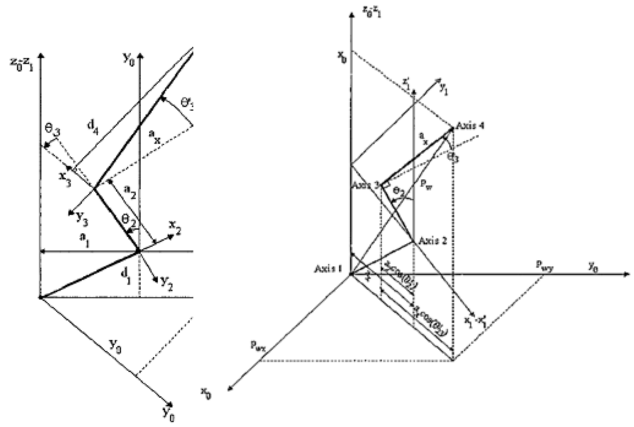

Figure 4: Anthropomorphic structure ReisRV60-60 robot [1]

Squaring and summing equations (9) and (10) results in

$$p^2_{wx1'} + p^2_{wz1'} = a_2^2 + a_x^2 + a_{wz1}(11)$$

which gives

$$s_{3'} = \frac{P^2_{wx1'} + P^2_{wz1'} - a_2^2 - a_x^2}{2a_{2'}\, a_{x'}}(12)$$

Setting $s_{3'} = \pm\sqrt{1 - s_{3'}^2}$ the solution for θ_3' will be

$$\theta_3' = \mathrm{Atan}\ 2(s_3, c_{3'})$$
$$\theta_3 = \theta_3' - \mathrm{A}\tan 2(a_3/d_4)(13)$$

Now, using θ_3' in (9)-(10) results in a system with two equations with S2 andC2 unknowns:

$$P_{wxr} = a_2 s_2 + a_x(c_2 c_{3'} - s_2 s_{3'})(14)$$
$$P_{wzr} = a_2 s_2 + a_x(s_2 c_{3'} + s_{3'} c_2)$$

Solving for s_2 and c_2 gives

$$s_2 = \frac{-(a_2 + a_x s_{3'})P_{wx1'} + a_x c_{3'} P_{wz1'}}{a_2^2 + a_x^2 + 2a_2 a_x s_{3'}}(15)$$

$$c_2 = \frac{(a_2 + a_x s_{3'})P_{wz1'} + a_x c_{3'} P_{wx1'}}{a_2^2 + a_x^2 + 2a_2 a_x s_{3'}}(16)$$

and the solution for θ_2 will be

$$\theta_2 = \mathrm{Atan}\ 2(s_2, c_2)(17)$$

To solve the second inverse kinematics sub-problem (orientation), i.e., to find the required joint angles θ_2,$\theta_5$and θ_6 corresponding to a given *end-effecter* orientationR^3_6 , we simply take advantage of the special configuration of the last three joints. Because the orientation of the *end-effecter*s defined byR^0_6, it's simple to get R^3_6 from,

$$R^3_6 = (R^0_3)^{-1}.R^0_6 = (R^0_3)^T.R^0_6(18)$$

This gives

$$R63=\begin{bmatrix} -c_1 s_{23} & -s_1 s_{23} & c_{23} \\ -c_1 c_{32} & -s_1 c_{23} & -s_{23} \\ s_1 & -c_1 & 0 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$
$$=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}(19)$$

with

r11= -c1s23a11 –s1s23a21+c23a31
r12= -c1s23a12 –s1s23a22+c23a32
r13=-c1s23a13–s1s23a22+c23a33
r21= -c1c23a11 –s1c23a21+s23a31
r22=-c1c23a12–s1c23a22+s23a32
r23= -c1c23a13 –s1c23a23+s23a33
r31=s1a11 –c1a21 r32=s1a12 –c1a22
r33=s1a13 –c1a23

It is now possible to use the previous result for the ZYZ Euler angles to obtain the solutions for θ_4,θ_5 and θ_6

For $\theta_5 \in [0, \pi]$the solution is

$$\theta_4 = \mathrm{Atan}\ 2(r_{33,} r_{13,})$$
$$\theta_5 = \mathrm{A}\tan 2\,(\sqrt{r_{13}^2 + r_{33,}^2} - r_{23,})(20)$$
$$\theta_6 = \mathrm{Atan}\ 2(-r_{22,} r_{21,})$$

For $\theta_5 \in [-0, \pi]$the solution is

$$\theta_4 = \mathrm{Atan}\ 2(-r_{33,} - r_{13,})$$
$$\theta_5 = \mathrm{A}\tan 2\,(-\sqrt{r_{13}^2 + r_{33,}^2} - r_{23,})$$
$$\theta_6 = \mathrm{Atan}\ 2(r_{22,} - r_{21,})(21)$$

4. Future Work

In the future, the orientation of the robot end effector can be planed through analysis of end effector motion plan which lead as to fully dynamic analysis of Raise RV 60-60 robot. Dynamics analysis of Raise robot RV 60-60 is another issue to be studied

5. Conclusions

This paper present forward and inverse position analysis to Reis RV60-60 industrial robot manipulator .the result of the analysis have the following property: optimum capacity of finding the position of end effector and each joint, flexible in implementation of dynamic analysis of Reis RV60-60 industrial robot manipulator.

6. Acknowledgement

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