

Modified Multiphase Level Set Image Segmentation Search for Energy Formulation with Bias Correction

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Abstract: *Level set methods are a popular way to solve the image segmentation problem. The solution contour is found by solving an optimization problem where a cost functional is minimized. The purpose of this paper is the level set approach to simultaneous tissue segmentation and bias correction of Magnetic Resonance Imaging (MRI) images. A modified level set approach to joint segmentation and bias correction of images with intensity in homogeneity. A sliding window is used to transform the gradient intensity domain to another domain, where the distribution overlap between different tissues is significantly suppressed. Tissue segmentation and bias correction are simultaneously achieved via a multiphase level set evolution process. The proposed methods are very robust to initialization, and are directly compatible with any type of level set implementation. Experiments on images of various modalities demonstrated the superior performance over state-of-the-art methods.*

Keywords: Bias correction, Gradient, Levelset, Contour.

1. Introduction

One popular approach for solving the image segmentation problem is to use the calculus of variations. The objective of the segmentation problem is defined with energy functional, and minimization of this functional represents the resulting segmentation. The solution of the segmentation problem is the contour which extremizes this functional. The standard way of solving this optimization problem is by gradient descent search in the solution space, which typically suffers from many unwanted local optima and poor convergence. Classically, these problems have been circumvented by modifying the energy functional. The functional depends on properties of the image such as gradients, curvatures and intensities, as well as regularization terms, e.g. smoothing constraints. Early variation methods, such as Snakes and Geodesic Active Contours [1]–[4], often have boundary based terms such as an edge map. A parameterized curve, the active contour, is evolved according to the minimization of the cost functional until it converges to an equilibrium state representing the resulting segmentation. Later variation methods often include terms which are more region-based, allowing segmentation of objects without distinctive edges. These methods are often based on the Mumford-Shah segmentation model [5] where the image is partitioned into piecewise smooth regions with short boundaries. Chan and Vese used this model together with implicit contours represented by level sets [6]–[9]. This made the optimization problem easier and it naturally handles segmentation topology changes.

2. Related Work

Level set methods are a popular way to solve the image segmentation problem in computer image analysis. A contour is implicitly represented by the zero level of a

signed distance function, and evolved according to a motion equation in order to minimize a cost function. This function defines the objective of the segmentation problem and also includes regularization constraints. Gradient descent search is the defacto method used to solve this optimization problem. It deforms an initial contour in the steepest (gradient) descent of the energy. The equations of motion for the contour, and the corresponding energy gradients, are derived using the Euler-Lagrange equation [10] and the condition that the first variation of the energy functional should vanish at a (local) optimum. Then, the contour is evolved to convergence using these equations. The use of a gradient descent search commonly leads to problems with convergence to local optima and slow convergence in general. The problems are accentuated with noisy data or with a non-stationary imaging process, which may lead to varying contrasts for example. The problems may also be induced by inaccurate initial conditions for certain applications. Traditionally, the energy functional have been modified to avoid these problems by, for example, adding regularizing terms to handle noise, rather than to analyze the performance of the applied optimization method. This is however discussed in [11], [12], where the metric defining the notion of steepest descent (gradient) has been studied. By changing the metric in the solution space, local optima due to noise are avoided in the search path.

There are many and much more advanced, alternatives to gradient descent. For non-convex functional, there are global optimization techniques such as subdivision schemes with much better performance [13]. Their high complexity, however, makes them applicable for small problems only. Stochastic methods such as the Monte-Carlo (MC) family are another alternative. Simulated annealing implemented with the Metropolis algorithm and local updates have been a popular choice [14], [15]. Unfortunately its convergence to a global solution is logarithmically slow, limiting its

usefulness but spurring the development of more advanced MC methods, such as basin hopping [16].

In spite of these more advanced methods, optimization using gradient descent search is still very common and in active use. This is partly due to its simple implementation, but also to its direct applicability to general non-convex functional. The focus of this paper is to show that the performance of these standard searches can easily be improved upon by very simple modifications with, (i) Momentum Method and (ii) Rprop method. These methods are commonly used in the machine learning community. In a series of 2D/3D experiments using real and synthetic data with ground truth, the modifications are shown to reduce the sensitivity for local optima and to increase the convergence rate.

2.1. Momentum Method

Turning to gradient descent with Momentum, we will adopt some terminology from the machine learning community and choose a search vector according to:

$$s_k = -\eta(1 - \omega)\nabla f_k + \omega s_k - 1 \quad (1)$$

Where η is the learning rate and $\omega \in [0, 1]$ is the momentum. Note that $\omega = 0$ gives standard gradient descent $s_k = -\eta\nabla f_k$, while $\omega = 1$ gives "infinite momentum" $s_k = s_{k-1}$. The intuition behind this strategy is that the current solution x_k has a momentum, which prohibits sudden changes in the velocity. These will effectively filter out high frequency changes in the cost function and allow for larger steps in favorable directions. Using appropriate parameters, the rate of convergence is increased while local optima may be overstepped.

2.2. Rprop Method

In standard implementations of steepest descent search, $\alpha k = \alpha$ is a constant not adapting to the shape of the cost-surface. Therefore if we set it too small, the number of iterations needed to converge to a local optimum may be prohibitive. On the other hand, a large value of α may lead to oscillations causing the search to fail. The optimal α does not only depend on the problem at hand, but vary along the cost-surface. In shallow regions of the surface a large α may be needed to obtain an acceptable convergence rate, but the same value may lead to disastrous oscillations in neighboring regions with larger gradients or in the presence of noise. In regions with very different behaviors along different dimensions it may be hard to find an α that gives acceptable convergence performance.

The Resilient Propagation (Rprop) algorithm [17] was developed to overcome these inherent disadvantages of standard gradient descent using adaptive step-sizes k called update-values. There is one update-value per dimension in x , i.e. $\dim(k) = \dim(x_k)$. However, the defining feature of Rprop is that the size of the gradient is never used. Only the signs of the partial derivatives are considered in the update rule. Another advantage of Rprop, very important in practical use, is the robustness of its parameters; Rprop will work out-of-the-box in many applications using only the standard values of its parameters [18].

Rprop provides a modification which uses individual (one per parameter) adaptive step sizes and considers only the signs of the gradient components. This modification makes Rprop less sensitive to local optima and avoids the harmful influence of the magnitude of the gradient on the step size. Individual adaptive step sizes also allow for cost functional with very different behaviors along different dimensions because there is no longer a single step size that has to be appropriate for all dimensions. The Rprop method succeeds in reaching the global minimum neighborhood within ten iterations. It passes the area with small gradients quickly since it only uses the signs of the gradient components, not the magnitude.

3. Our Contribution

This paper proposes a modified variation level set approach to bias correction and segmentation for images corrupted with intensity inhomogeneities. A unique feature of this method is that the computed bias field is intrinsically ensured to be smooth by the data term in the variation formulation, without any additional effort to maintain the smoothness of the bias field, and it can approximate bias fields of more general profiles, such as those in 3T or 7T MRI images.

Magnetic resonance imaging (MRI), is a medical imaging technique used in radiology to visualize internal structures of the body in detail. MRI makes use of the property of nuclear magnetic resonance (NMR) to image nuclei of atoms inside the body. To produce an image, an MRI machine uses a powerful magnet to generate a magnetic field. MRI provides good contrast between the different soft tissues of the body, which makes it especially useful in imaging the brain, muscles, the heart, and cancers compared with other medical imaging techniques such as computed tomography (CT) or X-Rays. Unlike CT scans or traditional X-Rays, MRI does not use ionizing radiation.

MRI scans require a magnetic field with two properties, uniform field density and strength. The magnetic field cannot vary more than 1/10,000 of 1% and field strength ranges (depending on the scanner) from 0.2 to 3 Teslas in strength in scanners currently used clinically, with research scanners investigating higher field strengths such as 7 Teslas. The Tesla (T) Images unit of magnetic flux density in the International System of Units, equal to the magnitude of the magnetic field vector necessary to produce a force of one Newton on a charge of one coulomb moving perpendicular to the direction of the magnetic field vector with a velocity of one meter per second. The lower field strengths (3T) can be achieved with permanent magnets, which are often used in "open" MRI scanners for claustrophobic patients. Higher field strengths (7T) can be achieved only with superconducting magnets.

3.1. Geometric Model of Images with Intensity in Homogeneity

Intensity inhomogeneities in MRI images can substantially reduce the accuracy of segmentation and registration. Despite advances of correcting spatial intensity in-

homogeneities of modern scanner software, the advent of multichannel phased array coils and 7T+ scanners have increased (again) the importance of this problem for post-scan processing since images may exhibit substantial intensity in-homogeneities across space. (image on left side). In this approach, low-order polynomials are used to model low-frequency variations across 3T image space; these polynomials are fitted to a subset of voxels that have been labeled as belonging to white matter. After estimating the low-frequency intensity fluctuations (producing a "bias field", image on lower right side), they are removed from the data producing voxels with much more homogeneous intensities (image on upper right side) that improve visualization and also are better starting points for subsequent segmentation tools.

The image intensity can be approximated by Gaussian distribution with mean and variance. However, using only a single Gaussian model is not accurate enough to describe the statistical characteristics of the image intensity. In order to accurately model the intensity distribution, the intensity distribution in each object domain should be ascribed to a Gaussian model.

3.2. Bias Correction

A bias field is a low frequency smooth undesirable signal that corrupts MRI images because of the in-homogeneities in the magnetic fields of the MRI machine. Bias field blurs images and thus reduces the high frequency contents of the image such as edges and contours and changes the intensity values of image pixels so that the same tissue has different gray level distribution across the image. A low level variation will not have great impact on clinical diagnosis. However it degrades the performance of image processing algorithms such as segmentation and classification or any algorithm that is based on the assumption of spatial invariance of the processed image. A preprocessing step is needed to correct for the effect of bias field before doing segmentation or classification.

Methods of bias correction can be categorized into two classes: prospective methods and retrospective methods. Prospective methods aim to avoid intensity in-homogeneities in the image acquisition process. These methods, while capable of correcting intensity in-homogeneity induced by the imaging device, are not able to remove subject-induced intensity in-homogeneity. In contrast, retrospective methods only rely on the information in the acquired images. Therefore, they can also remove intensity in-homogeneities regardless of their sources.

The bias field in a brain MRI image can be assumed to be formed of two components, i.e. one observed image and an unknown field, as follows:

$$I = bJ + n \quad (2)$$

Where I : observed image, J : true image to be restored, b : an unknown bias field and n : additive Gaussian noise with zero-mean. The aim is to obtain the value of bias field component from the observed image and correct the intensity in-homogeneity. In general, the bias field b is

assumed to be slowly varying in the entire image domain. Ideally, the intensity J in each tissue should take a specific value, reflecting the physical property being measured. This property, in conjunction with the spatially coherent nature of each tissue type, implies that the true signal J is approximately a piecewise constant map.

While bias field correction is needed for good segmentation, many approaches have exploited the idea that a good segmentation helps to estimate the bias field. Dawant et al. it manually selected some points inside white matter and estimated the bias field as the least-squares spline fit to the intensities of these points [19]. They also presented a slightly different version where the reference points are obtained by an intermediate classification operation, using the estimated bias field for final classification.

3.3. Energy Formulation

The intensity in homogeneity caused by the bias field measure intensities are not separable by using traditional, intensity based classification methods. A new method for joint segmentation and bias correction. An observation that the intensities in a relatively small region are separable, which can be varied by the below assumptions of (i) The bias field is slowly varying in the entire image domain. (ii) The true image intensities approximately a constant within each class of tissue, being a partition of bias field. Directly minimizing the energy with the partition variable is not convenient. The multiple levels set functions to represent a partition. The energy minimization can thus be performed by solving a level set evolution. The gradient descent search flow for an energy functional consisting of texture (proportional to length) and bulk energies (proportional to region), is developed.

The local intensity clustering property explained above exemplifies that the intensities in the neighborhood can be classified into N clusters, with centers $m_i \approx b(x) c_i, i= 1, 2, 3, 4...N$. Hence, standard membership function to classify these local intensities can be applied. In particular, for the intensities Image (X) in the neighborhood NG_y , the membership function includes boundary condition and threshold function is an iterative process to minimize the pixel intensity which can be presented as follows,

$$EF_y = \sum_{k=1}^N \int_{\Omega_i \cap NG_y} |Image(X) - m_i|^2 u_i(x) dx \quad (3)$$

Where EF : Energy formulation; m_i : cluster center of the i -th cluster, u_i : membership function of the region Ω_i i.e. to be determined, $u_i(x) = 1$, for $x \in \Omega_i$ and $u_i(x) = 0$, for $x \notin \Omega_i$.

The kernel function K is chosen as a truncated Gaussian function defined by

$$K(u) = \begin{cases} \frac{1}{a} e^{-|u|^2/2\sigma^2}, & \text{for } |u| \leq \rho \quad (4) \\ 0, & \text{other wise} \end{cases}$$

Where a is a normalization constant such that $\int K(\mu) = 1$, σ is the standard deviation (or the scale parameter) of the

Gaussian function, and ρ is the radius of the neighborhood NG_y .

The radius ρ of the neighborhood NG_y should be selected appropriately according to the degree of the intensity inhomogeneity. For more localized intensity inhomogeneity, the bias field b varies faster, and therefore the approximation is valid only in a smaller neighborhood. A smaller ρ should be used as the radius of the neighborhood NG_y , and for the truncated Gaussian function, the scale parameter σ (variance) should also be smaller.

The objective is integrated over the entire domain and incorporated into a variation level set formulation. The energy minimization is performed via a level set evolution process. By minimizing this energy, this method is able to simultaneously segment the image and estimate the bias field, and the estimated bias field can be used for bias correction. Moreover, it is robust to initialization, and therefore allows automatic applications.

3.4. Multiple Level Set Evolutions

The significant number of problems in science reduces to the study of the evolution of curves, which are usually the boundaries between different media. During the evolution process, the Multiple Level Set Functions (MLSFs) need to be periodically regularized in order to keep numeric stability. The constant kernel values in each iteration, which can be seen as a special case of the method. The MLSF keeps the property of Euclidean distance function, so it can make the evolution stable. The size of the kernel should not be too large. The contours are constants with different signs inside and outside. With this initialization, the convergence speed is much faster than that by distance function, and the tissue segmentation results are also much more accurate.

In the multiple-phase case: the image domain Ω is segmented into two disjoint regions Ω_1 and Ω_2 . The level set function ϕ is used to represent the two regions Ω_1 and Ω_2 . The regions Ω_1 and Ω_2 can be represented with their membership functions defined by $M_1(\phi) = H(\phi)$ and by $M_2(\phi) = 1 - H(\phi)$, respectively, where H is the Heaviside function. Thus, for the case of $N=2$, the energy can be expressed as the following level set formulation:

$$\varepsilon = \int (\sum_{i=1}^N \int K(y-x) |I(x) - b(y) c_i|^2 M_i(\phi(x)) dx) dy \quad (5)$$

By exchanging the order of integrations,

$$\varepsilon = \int (\sum_{i=1}^N (\int K(y-x) |I(x) - b(y) c_i|^2 dy) M_i(\phi(x)) dx) \quad (6)$$

The constants c_1, \dots, c_N are represented with a vector $c = (c_1, \dots, c_N)$. The level set function, the vector c and the bias field b are the variables of the energy ε , which can be written as $\varepsilon(\phi, c, \text{and } b)$. The energy can be rewritten as:

$$\varepsilon(\phi, c, b) = \int \sum_{i=1}^N e_i(x) M_i(\phi(x)) dx \quad (7)$$

By minimizing this energy, the result of image segmentation given by the level set function ϕ and the estimation of the

bias field b will be obtained. The energy minimization is achieved by an iterative process: In each iteration, minimize the energy $\varepsilon(\phi, c, b)$ with respect to each of its variables ϕ, c and b , given the other two updated in previous iteration. The solution to the level set energy minimization with respect to each variable is given the image region.

4. Implementation

The level set algorithm has also been implemented in Matlab: Version 2010a. The implementation steps are:

Step 1: Pre-processing: In this step, the pre-processing stage checks whether the input image contains number of channels. If there are 3 channels then the image is converted into grayscale as indexed image. For regularizing the level set function after the each iteration the kernel value is mentioned along with the channel. Assign the Point Spread Function (PSF) with corresponding pixel intensity variance.

Step 2: Contour Plot: The contour plot is defined as separating the background and foreground region in the MRI image.

Step 3: Regularization: In Regularization process, (i) The typical image is passed from the degradation function (in practice this degradation function) then an additive noise is also added to get an artificial corrupted image. Using these two images an optimal filter mask is found. (ii) To find $m \times n$ coefficients of the filter mask, iteratively. For implementing fitness function must be defined as a criterion for maximization. Absolute difference between the degraded image and the image estimated by filter mask is the natural fitness function.

Step 4: Bias Correction: In bias correction process, the image membership values are convoluted according to the image dimensions. The convolution process is establishes the unfairness pixels present in the instance.

The proposed method is demonstrated in the two phase case (i.e., $N=2$) as well as multiphase case. The parameter σ is set to 4 for the experiments. The multiphase method is applied to 10 medical images with intensity inhomogeneities. As a pre-processing step, a convolution with a Gaussian kernel is applied to smoothen the image. The scale parameters of the Gaussian kernel are chosen as 2. The initial contours are plotted on the original images. The corresponding results of segmentation, bias field estimation are shown in ensuing pages. These results demonstrate desirable performance in segmentation.

5. Results

To evaluate the algorithm on both synthetic data and real datasets in 3T and 7T (Tesla) with the metrics: PSNR, coefficient of variation, segmentation time calculation. The narrow band parameter N controls the size of the region close to the zero level set where calculations are performed. This is a computational optimization which is valuable especially in 3T. There are three classes of tissues: white matter (WM), gray matter (GM), and background. The

histogram of the bias corrected image by MLS method has three well-separated peaks, which correspond to the background, GM, and WM from left to right, respectively, while the peak of the GM is not distinct since its volume is relatively small. It should be noted that it is very flexible to initialize the LSFs in MLS method. The initial contours can be set inside, outside or across the object boundary. The 7T MRI image, whose intensity in-homogeneity is so severe that even the experts are difficult to tell the structures, by applying MLS method; the image quality can be significantly improved, Many regions (e.g. the regions inside the red circles and ellipses) that are difficult to distinguish can be much better viewed after the bias correction.

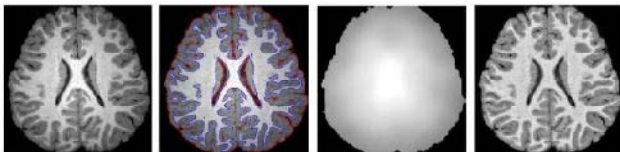


Figure 1: 3T MRI image of brain with MLS Method

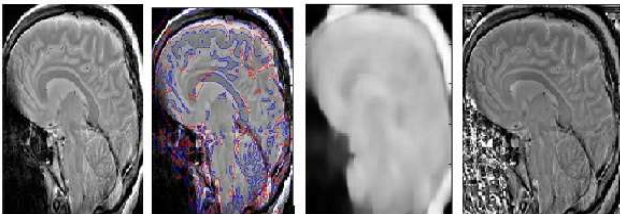


Figure 2: 7T MRI image of brain with MLS Method

MLS method has been tested on 7T MRI images. At 7T, significant gains in image resolution can be obtained due to the increase in signal-to-noise ratio. However, susceptibility-induced gradients scale with main field, while the imaging gradients are currently limited to essentially the same strengths as used at lower field strengths (i.e., 3T). This appears as a localized and stronger bias, which is challenging to traditional methods for bias correction.

The test image is matched with database to identify high frequency regions. The PSNR fraction measure of quality of reconstruction of lossy compression codecs (e.g., for image compression). The signal in this case is the original data, and the noise is the error introduced by compression.

$$PSNR = 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE)$$

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i, j) - K(i, j)]^2$$

Table: 1 MRI images with Comparison of PSNR Values in dB

MRI Images	Momentum	Rprop	MLS method
3T Brain Image	15.32	18.27	46.26
7T Brain Image	20.56	24.53	48.57

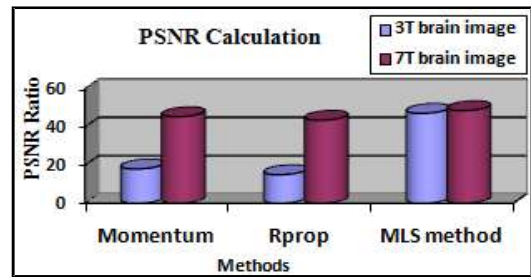


Figure 3: PSNR Calculations

Above Table 1 shows comparison of MRI Image with PSNR Values an approximation to human perception of reconstruction quality. Although a higher PSNR generally indicates that the reconstruction is of higher quality.

The coefficient of variation (CV) is a normalized measure of dispersion of a probability distribution or frequency distribution. The coefficient of variation represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different from each other. Coefficient of Variance is defined as a quotient between standard deviation and mean value of selected tissue class. A good algorithm for bias correction and segmentation should give low CV values for the bias corrected intensities within each segmented region.

Our MLS method is compared with the existing Leemput *et al.* and Wells *et al* [20]-[22] methods. Coefficient of variance (CV) metric is used to evaluate the performance of the algorithms for bias correction and segmentation. Two simulated images obtained from Brain Web in the link <http://www.bic.mni.mcgill.ca/brainweb/>, one corrupted with bias field without noise and the other corrupted with both bias and noise. The CV values for the two images are listed in Table 2 and Table 3. It can be seen that the CV values of MLS method is lower than those of Leemput's and Wells's methods, which indicates that the bias corrected images obtained in this method is more homogeneous than those of the other two methods.

Table 2: Co-efficient of Variation of MRI Image with bias and noise

	Tissue	Wells	Leemput	MLS method
Image with Bias and Noise	White matter	6.97%	7.92%	6.45%
	Grey matter	13.68%	14.68%	12.03%

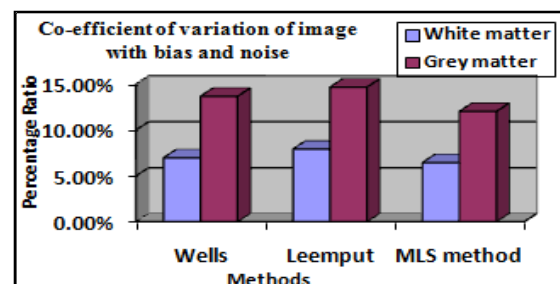


Figure 4: Co-efficient of variation of MRI image with bias and noise

Table 3: Co-efficient of Variation of MRI Image with bias

MRI Images	Momentum	Rprop	MLS method
3TBrianImage	16.24 sec	14.45 sec	12.23 sec
7TBrain Image	18.35 sec	16.54 sec	14.26 sec

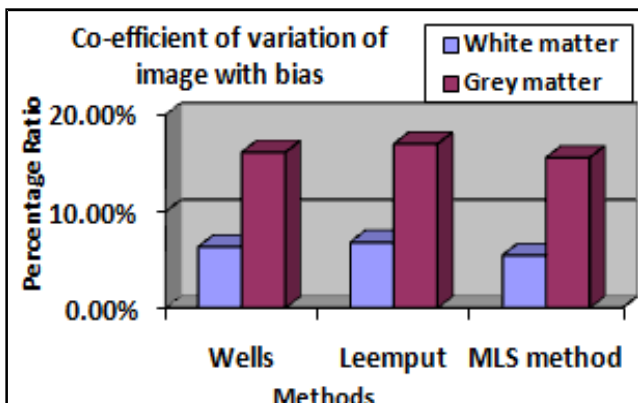


Figure 5: Co-efficient of variation of MRI image with bias
Above Table 2 shows the comparison of the coefficient of variation of MRI Image with bias and noise for two tissue methods and Table 3 with bias for two tissue methods.

Execution speed is another performance metric that is used to analyze the performance of the algorithms. It is defined as the time taken by the algorithm to produce the segmented image. The results obtained are shown in Table 4.

Table 4: Segmentation Time (In seconds)

Image with Bias	Tissue	Wells	Leemput	MLS method
	White matter	6.33%	6.81%	5.45%
Grey matter	16.11%	16.97%	15.55%	

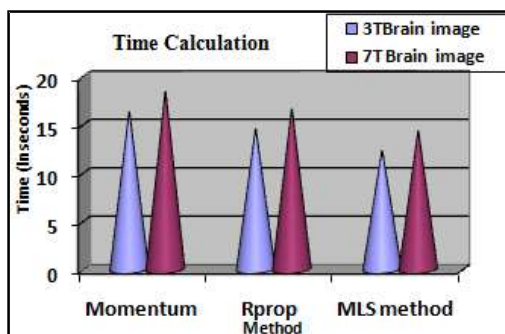


Figure 6: Segmentation Time calculation

From the Table 4, it is evident that the MLS Method produces quick result when compared to the traditional Momentum and Rprop Methods.

6. Conclusion

In this paper we have worked to the multiphase level set with bias correction and energy formulation. We presented a unified framework of bias correction and segmentation. A unique advantage of this method is that the smoothness of the computed bias field is intrinsically ensured by the data term in the distinction formulation. This method is able to capture bias of quite general profiles, and can be used for images of various modalities. Moreover, it is robust to initialization, thereby allowing automatic applications.

Comparisons with two well-known bias correction methods demonstrate the advantages of the proposed method.

7. Future Work

In future we have planned to find the solutions by changing the unfairness object correction method of solving the optimization problem rather than modifying the instance evolution and energy functional using hardware basis.

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