Demonstrating Chaos on Financial Markets through a Discrete Logistic Price Dynamics

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Abstract: ***The paper highlights the role that speculation plays in making stock price fluctuation chaotic. The positive feedback produce by speculative behavior determines the general dynamics of stock prices. The price dynamics is described by a logistic equation. This logistic equation is also known as Verhulst equation. This equation was originally developed to describe the dynamic behavior of population of an organism. A discrete form of the Verhulst equation called as Ricker model is done to simulate the price dynamics. The simulation of the iterative process in the Ricker model demonstrates that speculation can produce chaos. By varying the value of the parameter describing speculation, the price dynamics becomes chaotic for sufficiently high degree of speculation. The extreme sensitivity to initial condition of a chaotic system produced the so-called "butterfly effect". A simulation of the butterfly effect is done using two exactly identical discrete logistic equations. The equations differed only in their initial values by a very minute amount. It shows how two exactly identical dynamical systems quickly behave very differently even if the difference in their initial conditions is so infinitesimally small. The implication of the butterfly effect in doing experiments in the physical world is analyzed. The presence of butterfly effect in a chaotic system raises the issue of measurement errors in the conduct of physical experiments. No matter how accurate the scientific device used in the experiment, it is still subject to measurement errors. Butterfly effect tremendously magnifies the measurement errors over a short span of time. This implies that long-term prediction in a chaotic system is impossible***

Keywords: chaos, logistic equation, price dynamics, discrete dynamical systems

1. Introduction

This paper demonstrates how speculative behavior can create chaotic price fluctuations in stock market. The activity of buying more stocks as its price increases in the hope that it will further increase, provides a positive feedback mechanism in price dynamics [1]. This positive feedback then produces a looping sequence of price increases. However, the series of price increases is being limited by the fundamental value of the stock. It is the expected present value of future income flows of the stock's underlying assets. The price of stock cannot continually grow if it is already way above its fundamental value. In this case, there is a downward adjustment of the stock price towards its fundamental value.

A model of price dynamics is formally outlined in Section 2. The model is in the form of a discrete logistic equation. This equation was originally developed to describe the behavior of population of biological specie.

Section 3 shows the result of a simulation of the discrete logistic equation. The section illustrates how the movement of stock prices becomes chaotic as the degree of speculation is increased. A chaotic behavior can be thought to be random, but unlike randomness, chaotic movement follows a deterministic rule. Thus, stock price fluctuations though appearing to be random, may just be a product of chaos.

The so-called butterfly effect, which is a consequence of a chaotic system's extreme sensitivity to initial condition, is simulated in Section 4. It shows how two exactly identical dynamical systems quickly behave very differently even if the difference in their initial conditions is so infinitesimally small.

Section 5 discusses the impact of butterfly effect in doing scientific experiments especially in the issue of measurement errors. This leads to the conclusion that making long-term prediction in a chaotic system is impossible [2].

2. The Model

The paper proposes that the stock price dynamics is governed by the following differential equation

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{v}\right) \quad (1)$$

where p is the price, v is the fundamental value of the stock and r is a positive-valued parameter denoting the degree of speculation.

The fundamental value of a stock is derived from the expected present value of future income flows of its underlying assets. Speculative behavior may push the price above its fundamental value but will exert a downward pressure to price to adjust towards its fundamental value. The parameter r is the mechanism by which speculation creates positive feedback in the price dynamics. This can result into a looping system of price increases. If p is below its fundamental value, then both r and the term in the parenthesis reinforce each other in producing positive feedback.

Eq. (1) is a form of logistic equation also known as Verhulst equation [3] originally developed to model population growth limited by the carrying capacity of the available resources. A discrete form of this dynamical equation can be written as

$$p_{t+1} = p_t e^{r\left(1 - \frac{p_t}{v}\right)} \quad (2)$$

Eqn. (2) is the Ricker model [4] developed to study the population dynamics of fisheries. The iterative process in Eq. (2) can produce chaotic behavior in the price movement [5] as we will see later.

3. Simulating Price Dynamics

Normalizing to unity, let v = 1 and assume initial price is p_0 = 0.5. Figures 1 to 4 below shows the price dynamics for the cases r = 0.5, r = 1, r = 2 and r = 3 after 200 iterations. For the case r = 0.5 and r = 1, price adjust smoothly towards the fundamental value [Fig. 1 and 2]. For the case r = 2, price adjust towards the fundamental value in a regular fluctuating pattern [Fig. 3]. Finally, for the case r = 3, price behaves erratically [Fig. 4]. Although there is no discernible pattern in its fluctuations, its behavior is not actually random. This is because the behavior is ruled by the relatively simple dynamics in Eq. (2). This is an example of chaos.

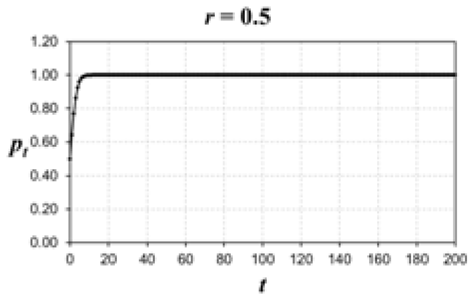

Figure 1: Price dynamics from t = 0 to t = 200 for r = 0.5

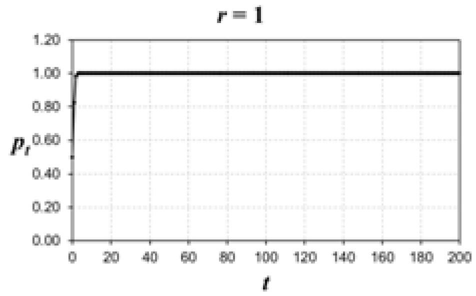

Figure 2: Price dynamics from t = 0 to t = 200 for r = 1

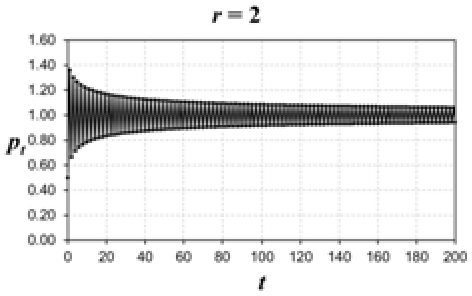

Figure 3: Price dynamics from t = 0 to t = 200 for r = 2

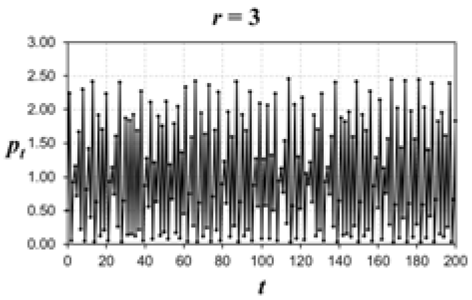

Figure 5: Price dynamics from t = 0 to t = 200 for r = 3

This shows that the irregular fluctuation of stock prices may not be random but just chaotic. Furthermore, this demonstrates that speculative activity can create chaos.

4. Simulating Butterfly Effect

One central feature of a chaotic system is its extreme sensitivity to initial condition. This implies that even with an infinitesimally small difference in their initial conditions, two exactly identical systems will end up having very different dynamics after some relatively short period. This is dubbed as the butterfly effect because it has the implication that a small flap of butterfly wings in Brazil may produce a tornado in Texas several months after.

To illustrate the butterfly effect, let and follow the dynamics specified in Eq. (2). This dynamical system can be written as

$$p^i_{t+1} = p^i_t e^{r\left(1-\frac{p^i_t}{v}\right)} \quad (3)$$

where i = 1, 2.

Letting $\delta_t = p^1_t - p^2_t$ implies that, $\delta_0 = p^1_0 - p^2_0$. In other words, δ_0 is the difference between the initial condition of p^1_t and p^2_t. This also further implies that $p^1_0 = p^2_0 + \delta_0$.

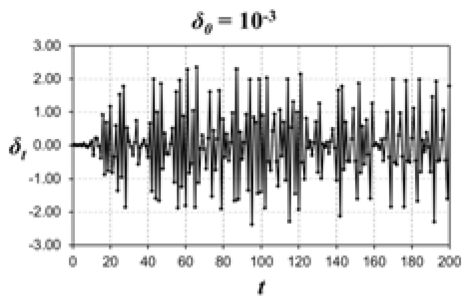

Figure 6: The butterfly effect for $\delta_0 = 10^{-3}$ given $p^1_0 = 0.5$, v = 1, and r = 3

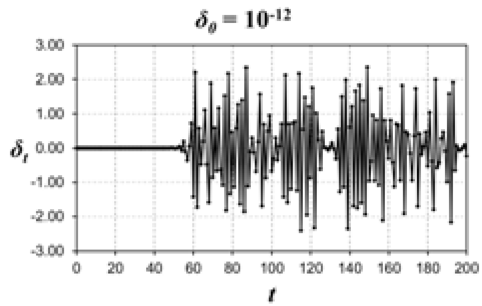

Figure 7: The butterfly effect for $\delta_0 = 10^{-12}$ given $p^1_0 =$ 0.5, v = 1, and r = 3

Assuming p^1_0 = 0.5, v = 1, and r = 3, Figure 5 and 6 show the dynamics of δ_t for the initial conditions, $\delta_0 = 10^{-3}$ and $\delta_0 = 10^{-12}$ respectively. For $\delta_0 = 10^{-3}$ [Fig. 5], δ_t begins moving irregularly almost right at the start. This means that the dynamics of p^1_t and p^2_t quickly behave very differently even though they initially differ by only one-tenth of 1%. Thus, the system is extremely sensitive to initial condition.

For some very close initial position like the case of $\delta_0 = 10^{-12}$ [Fig. 6], initially, δ_t remains very close to zero with no noticeable fluctuation. After some period however, δ_t starts to behave erratically. This shows that even for p_t^1 and p_t^2 which have almost exactly the same initial values, after moving together for some periods, they will end up moving very differently. The extreme sensitivity to initial condition is again demonstrated.

5. Conclusion

The existence of chaos even in a relatively simple dynamic equation like Eq. (2) has enormous implication in the conduct of experiment and data gathering. The extreme sensitivity of a chaotic system to initial condition confounded the problem of measurement error. It raises the issue of the accuracy of apparatus or measuring device used in experiments.

Consider modeling a system which behavior is described by p_t^1 in the previous section and let δ_0 be the measurement error. Even if the exact model is known, and the measuring device has a very high degree of accuracy in such a way that its measurement error is to the magnitude of $\delta_0 = 10^{-12}$, the butterfly effect tremendously magnifies this error over a short span of time. This implies that long-term prediction in a chaotic system is impossible.

For a system to be chaotic it must be nonlinear. Hence, chaos poses no problem for most of the well-studied systems in physics so far, which are linear. However, it is believed that far more numerous hidden mysteries in the natural world can be unlocked with nonlinear systems.

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Author Profile

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