Study and Analysis of Multiwavelet Transform with Threshold in Image Denoising: A Survey

Jyoti Sahu[1,] Abha Choubey[2]

[1, 2]Department of Computer Science and Engineering, SSCET, CSVTU, Bhilai, Chattishgarh, India

Abstract: ***Removing noise from the Medical image is still a challenging problem for researchers. Noise added is not easy to remove from the images. There have been several published algorithms and each approach has its assumptions, advantages, and limitations. This paper summarizes the major techniques to denoise the medical images and finds the one is better for image denoising. We can conclude that the Multiwavelet technique with Soft threshold is the best technique for image denoising.***

Keywords: Wavelet, Multiwavelet, Image denoising, Gaussian noise, Speckle noise linear filters, Wavelet transform.

1. Introduction

Images play an important role in everywhere whether it is in daily life or in applications such as in satellite communication, television, computer tomography etc. data of images are corrupted by noise and removing of noise plays the main role [1]. In this paper the technique used for image denoising is Multiwavelet Soft-Thresholding. Multiwavelets are a new addition to the body of wavelet theory. Realizable as matrix-valued filter banks leading to wavelet bases, multiwavelets offer simultaneous orthogonality, symmetry, and short support [1]. The noise is characterized by its pattern and by its probabilistic characteristics. There is a wide variety of noise types they are; Gaussian noise, speckle noise, poison noise, impulse noise, salt and pepper noise etc. [2]. Multiwavelet Soft Thresholding is nonlinear. Multiwavelet transformation is directly applicable only to one dimensional signals, but images are two-dimensional signals, so there must be a way to process them with a one-dimensional transform [3]. Image denoising is used when we obtain a noisy image and we want to remove as many of the speckles from the image as possible, without removing or distorting any feature in it [4].

First part of paper describes introduction , 2nd part describe problem statement, 3rd part describes method used for image denoising .In the 4th part we compare between other thresholding techniques and the last part is conclusion of this study.

2. Problem Identification

In the image denoising process, information about the type of noise present in the original image plays a significant role. Noise is present in an image either in an additive or multiplicative form.

An additive noise follows the rule

$$w(x, y)=s(x, y)+n(x, y),$$

while the multiplicative noise satisfies

$$w(x, y)=s(x, y)\times n(x, y),$$

where $s(x,y)$ is the original signal, $n(x,y)$ denotes the noise introduced into the signal to produce the corrupted image $w(x,y)$, and (x,y) represents the pixel location. The above image algebra is done at pixel level. Image addition also finds applications in image morphing. By image multiplication, we mean the brightness of the image is varied. There are various types of noise in image processing some of them are-

2.1 Gaussian noise - Gaussian noise is statistical noise that has a probability density function of the normal distribution (also known as Gaussian distribution). In other words, the values that the noise can take on are Gaussian-distributed. It is most commonly used as additive white noise to yield additive white Gaussian noise (AWGN).

2.2 Poisson noise - Poisson noise has a probability density function of a Poisson distribution.

2.3 Salt & pepper noise - It represents itself as randomly occurring white and black pixels. An effective noise reduction method for this type of noise involves the usage of a median filter. Salt and pepper noise creeps into images in situations where quick transients, such as faulty switching, take place. The image after distortion from salt and pepper noise looks like the image attached.

2.4 Speckle noise –Speckle noise is a granular noise that inherently exists in and degrades the quality of images. Speckle noise is a multiplicative noise, i.e. it is in direct proportion to the local grey level in any area. The signal and the noise are statistically independent of each other.

3. Methods For Image Denoising

Various denoising techniques have been proposed so far and their application depends upon the type of image and noise present in the image. Image denoising is classified in two categories:

3.1 Wavelet Denoising

This approach focuses on exploiting the multi resolution properties of Wavelet Transform. This technique identifies close correlation of signal at different resolutions by observing the signal across multiple resolutions. This method produces excellent output but is computationally much more complex and expensive. The modeling of the wavelet coefficients can either be deterministic or statistical.

3.1.1 Deterministic

The Deterministic method of modeling involves creating tree structure of wavelet coefficients with every level in the tree representing each scale of transformation and nodes representing the wavelet coefficients. This approach is adopted in. The optimal tree approximation displays a hierarchical interpretation of wavelet decomposition. Wavelet coefficients of singularities have large wavelet coefficients that persist along the branches of tree. Thus if a wavelet coefficient has strong presence at particular node then in case of it being signal, its presence should be more pronounced at its parent nodes. If it is noisy coefficient, for instance spurious blip, then such consistent presence will be missing. Lu et al., tracked wavelet local maxima in scale space, by using a tree structure. Other denoising method based on wavelet coefficient trees is proposed by Donoho.

3.1.2 Statistical Modeling of Wavelet Coefficients

This approach focuses on some more interesting and appealing properties of the Wavelet Transform such as multiscale correlation between the wavelet coefficients, local correlation between neighborhood coefficients etc This approach has an inherent goal of perfecting the exact modeling of image data with use of Wavelet Transform. A good review of statistical properties of wavelet coefficients can be found. The following two techniques exploit the statistical properties of the wavelet coefficients based on a probabilistic model. Wavelets are mathematical functions that analyze data according to scale or resolution. They aid in studying a signal in different windows or at different resolutions. For instance, if the signal is viewed in a large window, gross features can be noticed, but if viewed in a small window, only small features can be noticed. Wavelets provide some advantages over Fourier transforms. For example, they do a good job in approximating signals with sharp spikes or signals having discontinuities. Wavelets can also model speech, music, video and non-stationary stochastic signals. Wavelets can be used in applications such as image compression, turbulence, human vision, radar, earthquake prediction, etc.. The term “wavelets” is used to refer to a set of orthonormal basis functions generated by dilation and translation of scaling function φ and a mother wavelet ψ. The finite scale multi-resolution representation of a discrete function can be called as a discrete wavelet transforms. DWT is a fast linear operation on a data vector, whose length is an integer power of 2. This transform is invertible and orthogonal, where the inverse transform expressed as a matrix is the transpose of the transform matrix. The wavelet basis or function, unlike sines and cosines as in Fourier transform, is quite localized in space. But similar to sines and cosines, individual wavelet functions are localized in frequency. Some of the properties of discrete wavelet transforms are listed below.

(a) DWT is a fast linear operation, which can be applied on data vectors having length as integer power of 2.

(b) DWT is invertible and orthogonal. Note that the scaling function φ and the wavelet function ψ are orthogonal to each other in L2 (0, 1), i.e., <φ,ψ>=0.
(c) The wavelet basis is quite localized in space and frequency.
(d) The coefficients satisfy some constraints
(e) The wavelet coefficients of a fractional Brownian motion (fBm) supports Stationarity, i.e., *g j* (*k*) = *g j* (0), ∀ *k*.
(f) Wavelet coefficients exhibit Gaussianity:

where σψ is a constant depending on ψ and *H*, the Hurst parameter for fBm. This property aids wavelets in the removal of Gaussian noise from images.
(g) The wavelet coefficients are almost decorrelated, where *N* refers to the number of vanishing moments.

3.2 Multiwavelet Denoising Technique

To compute the redundant wavelet transform with two detailed images, a smoothing function Ø(*x*,*y*) and two wavelets $\Psi^i(x,y)$ are needed. The dilation of these functions are denoted by

$$\phi_s(x,y) = \frac{1}{s^2}\phi,\left(\frac{x}{s},\frac{y}{s}\right),$$
$$\psi_s^i(x,y) = \frac{1}{s^2}\psi,$$
$$\left(\frac{x}{s},\frac{y}{s}\right), = I = 1,2 \qquad \textbf{(1)}$$

and the dyadic wavelet transform *f* (*x*,*y*) at a scale s =2j, has two detail components, given by

$$W^1_{2\,j\,\mathrm{f}(x,y)} = (\mathrm{f}*^{\psi^1_{2j}})(x,y), i = 1,2 \qquad \textbf{(2)}$$

and one low-pass component, given by

$$S_{2j_{\mathrm{f}(x,y)}} = (f*^{\phi 2j})(x,y) \qquad \textbf{(3)}$$

There coefficients W^1_{2j} *f* (*x*,*y*) and W^2_{2j} *f* (*x*,*y*) represent the details in the *x* and *y* directors, respectively. Thus,the image gradient at the resolution 2j can be approximated by

$$W_{2j} f(x,y) = \begin{pmatrix} W^1_{2j} f(x,y) \\ W^2_{2j} f(x,y) \end{pmatrix} \qquad \textbf{(4)}$$

The Multi-Wavelet Transform of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared to other multi-scale representations such as Gaussian and Laplacian pyramid. Recently, Multi-Wavelet Transform is preferred for image de-noising. Multi-wavelet iterates on the low-frequency components generated by the first decomposition. After scalar wavelet decomposition, the low-frequency components have only one sub-band, but after multiwavelet decomposition, the low-frequency components have four small sub-bands, one low-pass sub band and three band-pass sub bands.

The next iteration continued to decompose the low frequency components L={$L1L1$, $L1L2$, $L2L1$, $L2L2$}. In this situation, a structure of 5(4*J+1) sub-bands can be generated after J times decomposition, as shown in figure 1. The hierarchical relationship between every sub-band is shown in figure 2. Similar to single wavelet, multi-wavelet can be decomposed to 3 to 5 layers. The Gaussian noise will near be averaged out in low frequency Wavelet coefficients. Therefore only the Multi-Wavelet coefficients in the high frequency level need to hard are threshold.

L_1L_1	L_1L_2	L_1H_1	L_1H_2
L_2L_1	L_2L_2	L_2H_1	L_2H_2
H_1L_1	H_1L_2	H_1H_1	H_1H_2
H_2L_1	H_2L_2	H_2H_1	H_2H_2

Figure 1: The structure of sub-band distribution

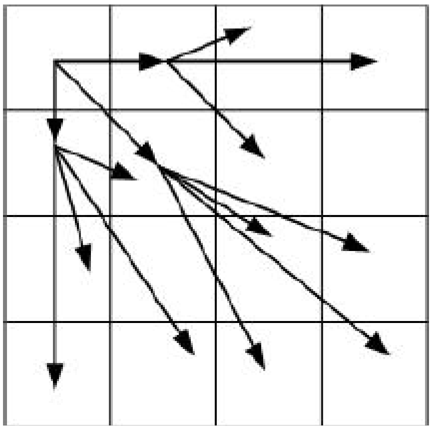

Figure 2: The hierarchical relationship between every sub-band

4. Types Of Thresholding

Thresholding is non-linear operation performed on the M ultiwavelet coefficients of noisy signal. It is widely used in noise reduction, signal and image compression or recognition. This can be done by comparing the absolute value of the empirical Multiwavelet coefficients with a value called Threshold Value (***Thv***). It is clear that if the Multiwavelet coefficient is equal to or less than the threshold value, then one cannot separate the signal from the noise. In this case, a good estimation for that Multiwavelet coefficient is zero. In the case of an empirical Multiwavelet coefficient is greater than the threshold value, then a natural estimation for this Multiwavelet coefficient is empirical Multiwavelet coefficient itself. This idea is called *Thresholding*.

4.1 Method 1: Visushrink

Threshold T can be calculated using the formulae,

T= σ√2logn2

This method performs well under a number of applications because wavelet transform has the compaction property of having only a small number of large coefficients. All the remaining wavelet coefficients are very small. This algorithm offers the advantages of smoothness and adaptation. However, it exhibits visual artifacts.

4.2 Method 2: Neighshrink

Let d (i,j) denote the wavelet coefficients of interest and B(i,j) is a neighborhood window around d(i,j). Also let S2=Σd2 (i,j) over the window B(i,j). Then the wavelet coefficient to be thresholded shrinks according to the formulae,

d(i,j)= d(i,j)* B(i,j) (5)

where the shrinkage factor can be defined as B(i,j) = (1- T2/ S2 (i,j))+, and the sign + at the end of the formulae means to keep the positive value while set it to zero when it is negative.

4.3 Method3: Modineighshrink

During experimentation, it was observed that when the noise content was high, the reconstructed image using Neighshrink contained mat like aberrations. These aberrations could be removed by wiener filtering the reconstructed image at the last stage of IDWT. The cost of additional filtering was slight reduction in sharpness of the reconstructed image. However, there was a slight improvement in the PSNR of the reconstructed image using wiener filtering. The denoised image using Neighshrink sometimes unacceptably gets blurred and lost some details. The reason could be the suppression of too many detail wavelet coefficients. This problem will be avoided by reducing the value of threshold itself. So, the shrinkage factor is given by

B(i,j) = (1- (3/4)*T2/ S2 (i,j))+ **(6)**

Method 4. *Hard-Thresholding* - Which can be computed by:-

$$A_k^j = T_h(G_k^j, Thv) = \begin{cases} G_k^j \ldots\ldots\ldots\ldots & |G_k^j| > Thv \\ 0 \ldots\ldots\ldots\ldots & |G_k^j| \le Thv \end{cases}$$

(7)

Method 5. Soft-Thresholding - Can be computed as

$$A_k^j = T_h(\mathrm{G}_k^j, Thv) = \begin{cases} \mathrm{G}_k^j \ldots\ldots\ldots\ldots & \left|\mathrm{G}_k^j\right| > Thv \\ 0 \ldots\ldots\ldots\ldots\ldots & \left|\mathrm{G}_k^j\right| \le Thv \end{cases}$$

Where,

$$\sin(\mathrm{G}_k^j, Thv) = \begin{cases} +1 \ldots\ldots\ldots & \mathrm{G}_k^j > 0 \\ 0 \ldots\ldots\ldots\ldots & \mathrm{G}_k^j = 0 \\ -1 \ldots\ldots\ldots & \mathrm{G}_k^j < 0 \end{cases} \quad \textbf{(8)}$$

5. Evaluation Criteria for Wavelet and Multi-Wavelet

The above said methods are evaluated using the quality measure Peak Signal to Noise ratio which is calculated using the formulae,

PSNR = 10log 10 (255) 2 / MSE (db)

where MSE is the mean squared error between the original image and the reconstructed de-noised image. It is used to evaluate the different de-noising scheme like Wiener filter, Visushrink, Neighshrink, Modified Neighshrink, wavelet and multi-wavelet for all medical images.

6. Conclusion

A variety of survey has been done in this paper. We have discussed various denoising algorithms. A literature survey for various images denoising process was done. Proposed method can provide better results in terms of image quality and similarity measures. Future scope is to calculate the amount of noise added to the pixel, removal of noise and evaluating the signal to noise ratio. Multi-Wavelet transform is best suited for performance because of its properties like sparsity, multiresolution and multiscale nature. Thresholding techniques used with Multi - wavelet are simplest to implement.

References

[1] Strela, V.; Heller, P.N.; Strang, G.; Topiwala, P.; Heil, C., "The application of multiwavelet filterbanks to image processing," Image Processing, IEEE Transactions on , vol.8, no.4, pp.548,563, Apr 1999 doi: 10.1109/83.753742

[2] S. Grace Chang, Student Member, IEEE, Bin Yu, Senior Member, IEEE, and Martin Vetterli, Fellow, IEEE “Adaptive Wavelet Thresholding for Image Denoising And Compression”, IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 9, NO. 9, SEPTEMBER 2000.

[3] S. A. Fahdel “Retrieving Image from Noisy Version depending on Multiwavelet Soft-Thresholding with Smoothing Filter”, IBN AL- HAITHAM J. FOR PURE & APPL. SCI. VOL.23 (2) 2010.

[4] Kother Mohideen, Arumuga Perumal, Krishnan and Mohamed Sathik,” Image Denoising And Enhancement Using Multiwavelet With Hard Threshold In Digital Mammographic Images”, International Arab Journal of e-Technology, Vol. 2, No. 1, January 2011.

[5] Pankaj Hedaoo and Swati S Godbole “WAVELET THRESHOLDING APPROACH FOR IMAGE DENOISING”, International Journal of Network Security & Its Applications (IJNSA), Vol.3, No.4, July 2011.

[6] Shruti Bhargava, Dr. Ajay Somkuwar, Mini Guptanumber “A survey Paper on Image Denoising Techniques”. International Journal of Electronics Communication and Computer Engineering Volume 3, Issue 3, ISSN 2249 –071X.

[7] Usha Rani, Charu Narula & Pardeep “Image Denoising Techniques A comparative study”, International Journal of Electronics , Communication & Instrumentation Engineering Research and Development (IJECIERD) ISSN 2249-684X Vol.2, Issue 3 Sep 2012 64-74

[8] N. K. Ragesh1, A. R. Anil2, Dr. R. Rajesh3, “Digital Image Denoising in Medical Ultrasound Images: A Survey”. ICGST AIML-11 Conference, Dubai, UAE, 12-14 April 2011

[9] S.Satheesh1, Dr. KVSVR Prasad,”Medical Image Denoising Using Adaptive Threshold Based on Contourlet Transform”, Advanced Computing: An International Journal (ACIJ), Vol.2, No.2, March 2011.

Author Profile

Jyoti Sahu has completed Master in Computer Application in 2010 from CSVTU, Bhilai. She is pursuing her Master of Engineering in Computer Technology and Application from CSVTU, Bhilai (C.G.).

Abha Choubey, received the M.E. degree in Computer Technology and Application from CSVTU, Bhilai (C.G.) and pursuing PhD. She is a Senior Associate Professor of Computer Science Department in SSCET, Bhilai (C.G.)