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Risk Measurement Strategy; An Alternative to Merging that Offers A Capital Relief in Risk Management

Godswill. U. Achi[1], Ogwo Obiageri[2], Solomon Okechukwu[3]

[1]Department of Mathematics, Abia State Polytechnic, Aba, PMB 7166 Aba ,Abia State,Nigeria

[2, 3]Department of Mathematics, Abia state University, Uturu, Abia State, Nigeria

Abstract: ***In this paper, we suggest the authenticity of a distortion risk measurement strategy that can be used instead of the risk management slogan 'Avoiding merging increases shortfall' which justifies the well known advice ' don't put all your eggs in one basket'. There are lots of distortion risk measures like conditional value at risk (expected shortfall) or the Wang transform risk measure, in spite of being coherent they do not always provide incentive for risk management because of lack of giving a capital relief in some simple two scenarios situation of reduced risk. To prevent the existence of such pathological counter examples, we introduce a Weibull distortion measure that preserves the higher degree stop loss order and offer a capital relief.***

Keywords: Coherent risk measure, tail free distortion risk measures, distortion risk measures, merging, Weibull distortion measure.

1. Introduction

The use of distortion risk measures to determine capital requirements of a risky business can be found in many papers (e.g. [25,7,23]). To provide incentive for active risk management, it is argued that some coherent distortion risk measure should preserve some higher degree stop loss order. Such risk measures are called tail free risk measures. The axiomatic approach to risk measures is an important and very active subject, which applies to different topics of actuarial and financial interest like premium calculation and capital requirements. Besides the *coherent risk measures* by [1,2], one is interested in the *distortion risk measures* by [5,6,18,19,24]. Under certain circumstances, distortion risk measures are coherent risk measures (e.g.[24]). Consequently, they can be used to determine the capital requirements of a risky business, as suggested by several authors including [25,7,23]. However, in spite of being coherent, a lot of distortion risk measures, like conditional value-at-risk (identical to expected shortfall) or the very Wang transform risk measure, do not always provide incentive for risk management because of its inability to giving a capital relief in some simple two scenarios situations of reduced risk (see Examples 1 and 2). In this paper, we consider the known risk management slogan 'Avoiding merging increases shortfall' which justifies the well known advice ' don't put all your eggs in one basket' which if not adhered to, may lead to higher loss in capital of a risky portfolio. This is a desirable property because increased risk should be penalized with an increased risk measure .To prevent the existence of such pathological counterexamples, we are interested in a weibull distortion risk measures ([17]) that preserve the higher degree stop-loss orders (e.g. [12,26]) and offers a capital relief which can serve as an alternative to the known slogan.

2. Formulation of Problems

2.1 Diversification and Sub-additive Axioms On Capital Requirement

Given two portfolios with respective losses x_1 and x_2. Assume that the solvency capital requirement imposed by the regulator is given by the risk measure P, if each portfolio is not liable for the shortfall of the other one, the capital requirement for each portfolio is given by P(x_i), if the two portfolios are (together) both liable for the eventual shortfall of the aggregate loss $x_1 + x_2$, we will say that the portfolios are merged.

Therefore, the solvency capital requirement imposed by the supervisory authorities will in this case be equal to $(x_1 + x_2$). Merging the two portfolios will lead to a decrease in shortfall given by

$$\sum_{j=1}^{2}(x_j - P(x_j)) - (x_1 + x_2 - P(x_1 + x_2)) \quad (1)$$

where the capital requirement for each portfolio = P(x_j)P = solvency capital for the risk exposure imposed by the regulators. Eventual shortfall or aggregate loss. = $x_1 + x_2$
The solvency capital requirement for the aggregate risk = $P(x_1+x_2)$ The following inequality holds with probability 1

$$(x_1+x_2 - P(x_1) - P(x_2)) \leq \sum_{j=1}^{2}(x_j - P(x_j)) \quad (2)$$

(See [3])

This inequality states that, the shortfall of the merged portfolio is always smaller than the sum of the shortfall of the separate portfolios, when adding capitals. It expresses, that from point of view of the regulatory authorities that a merger adding the capital is to be preferred in the sense that the shortfall decreases. The underlying reason is that, within the merged portfolios, the shortfall of one of the entities can be compensated by the gain of the other one. This observation can be summarized as **"A merger decreases the shortfall risk".** It is important to note that the inequality (2) does not necessarily express that merger is advantageous for the owners of the business related to the portfolios. Let x_j be the loss related to that portfolio, j over the reference period and let k_j be its available capital. If the loss x_j is smaller than the capital k_j, the capital at the end of the reference period will be given by $k_j - x_j$ where as in case where the capital loss x_j exceeds the capital k_j, the business units related to this portfolio gets ruined and the end of the year capital will be zero. Hence, for portfolio j, the end of the year capital is given by $(k_j - x_j)$. it is straight forward to show that

$$(k_1 + k_2 - x_1 - x_2) \leq \sum_{j=1}^{2}(k_j - x_j) \ (3)$$

In terms of maximizing the end of the period's capital, it is necessary to keep the two portfolios separate. This situation may be preferred from the shareholders point of view, essentially because in this case, firewalls are built in ensuring that the ruin of one portfolio will not contaminate the other one. Notice that the optimal strategy from the shareholder's point of view is now just opposite of that of the regulators point of view. Hence (3) justifies the well known advice "***don't put all your eggs in one* basket** '. If the shareholders have a capital $k_1 + k_2$ at their disposal, and the riskiness of the business is given by (x_1, x_2), and given that their goals is to maximize the return of capital, then splitting the risk over two separate entities is always to be preferred.

However, when regulators talk about diversification, they mean the decrease in shortfall caused by merging. But when the shareholders are talking about diversification, they are talking about the increase in return caused by building in firewalls. In equation (2), we found that, from the point of view of maximizing the shortfall (this is the point of view of the regulator) it is better to merge and adding up the stand alone capitals. Moreover, taking into accounts the criterion of minimizing the shortfall, inequality (2) indicates that the capital of the merged portfolios can, to a certain extent, be smaller than the sum of the capital s of the two separate portfolios, as long as the merged shortfall does not become larger than the sum of the separate shortfalls. This observation has led to the belief (by researchers and practitioners) that a risk measure for setting capital requirements should be sub additive. It is Important to note that the requirement of sub additivity implies that

$$(x_1 + x_2 - P(x_1 + x_2)) \geq (x_1 + x_2 - P(x_1) - P(x_2)) \ (4)$$

Hence from (2) and (3) , we see that when adapting a subadditive risk measure in a merger, one could end up with a larger shortfall than the sum of the shortfall of the stand-alones.

2.2 Avoiding Merging Increases the Shortfall

Any theory that postulates that risk measures are sub additive should at least constraint this subadditivity ensuring that merging, which leads to a lower aggregate capital requirement, does not increase the shortfall risk. In order to ensure that the merger will indeed lead to a less risky situation, we need to investigate a number of requirements that could be imposed by the regulator in addition to the sub-additivity requirement

A first additional condition required by the regulator could be stated as follows;

For any couple (x_1, x_2), *the capital requirement*

P has to fulfill the condition

$$(x_1 + x_2 - P(x_1 + x_2)) \leq \sum_{j=1}^{2}(x_j - P(x_j)) \ (5)$$

This condition means that the regulator requires that the shortfall of any two merged portfolios with losses x_1 *and* x_2 respectively, is never allowed to be larger than the sum of the shortfall of the standalones.

Remark

The regulator wants the expected shortfall to be as small as possible, which means a preference for a high solvency capital requirement. On the other hand, he does not want to decrease the expected shortfall at any price, imposing an extremely large burden on the financial industry

2.3 Coherent Distortion Risk Measures

Let (Ω, A, P) be a probability space such that Ω is the space of outcomes or states of the world, *A* is the σ -algebra of events and *P* is the probability measure. For a measurable real-valued random variable *X* on this probability space, that is a map $X:\Omega \to R$, the probability distribution of *X* is defined and denoted by $F_X(x) = P(X \leq x)$.

In this paper, the random variable *X* represents net income or profit at time t. Given that $\omega \in \Omega$ the real number $X(w)$ is the realization of a loss and profit function, consequent upon the following conditions;

(a) if $X(w) \geq 0$ *for profit*
(b) if $X(w) \leq 0$ *for loss*

We denote by the functional $R[.]$ a risk measure (where X is given as the net income or profit) that assigns a real number to any random variable or its cumulative distribution function. A *risk measure* is a functional from the set of losses to the extended non-negative real numbers described by a map $R:\chi \to [0,\infty]$. A *coherent risk measure* is a risk measure, which satisfies the following axioms(see [1]) :

(M) (monotonicity) If $X, Y \in \chi$ are ordered in stochastic dominance of first order, that is $F_X(x) \geq F_Y(x)$ for all x, written $X \leq_{st} Y$, then $R[X] \leq R[Y]$

(P) (positive homogeneity) If $a > 0$ is a positive constant and $X \in \chi$ then $R[aX] = aR[X]$

(S) (subadditivity) If $X, Y, X+Y \in \chi$ then $R[X+Y] \le R[X] + R[Y]$

(T) (translation invariance) If c is a constant and $X \in \chi$ then $R[X+c] = R[X] + c$

In general setting, axiom (M) can be criticized. If that the Esscher premium is monotonic, that is, it does not hold that if X is first order stochastically dominated by Y, denoted by $X \le_{st} Y$, then $\phi_x(h) \le \phi_x(h)$ for all $h \in R$ (or even all $h \le 0$). Hence, axiom (M) does not guarantee monotonicity of the function $R[.]$ [7] replaced axiom (M) by the more restrictive axiom of respect for Laplace transform order, which does guarantee monotonicity of the functional $R[.]$. we say that X is smaller than Y in Laplace transform order if $E[e^{hX}] \ge E[e^{hY}]$ for all $h \le 0$. We write $X \le_{Lt} Y$. Indeed $X \le_{st} Y$ implies $X \le_{Lt} Y$. In the expected utility model, the Laplace transform order represents preference of decision makers with a negative exponential utility function given by

$u(x) = 1 - e^{hx}$ $h < 0$

and $R_g[X] = \int_0^\infty [1 - F_X^g(x)]$

Taking the mean value with respect to the distorted distribution of a loss $X \in x$ with probability distribution $F_X(x)$, one obtains the *distortion (risk) measure* $R_g[X] = \int_0^\infty [1 - F_X^g(x))]dx - \int_{-\infty}^0 F_X^g(x)dx$. (6)

This is equivalent to the exponential utility function given by $u(x) = 1 - e^{hx}$

Similarly, the dual distorted distribution defines the *dual distortion (risk) measure*

$$R_\gamma[X] = \int_0^\infty [1 - F_X^\gamma(x))]dx - \int_{-\infty}^0 F_X^\gamma(x)dx \quad (7)$$

One discovers that the dual transform $\gamma(x) = 1 - g(1-x)$ implies the following alternative dual representations of the distortion measures (6) and (7) in terms of the *distorted survival* function $\overline{F}_X^g(x) := g(\overline{F}_X(x)) = 1 - F_X^\gamma(x)$ and the *dual distorted survival* function $\overline{F}_X^\gamma(x) := \gamma(\overline{F}_X(x)) = 1 - F_X^g(x)$ associated to the survival function $\overline{F}_X(x) = 1 - F_X(x)$:

$$\overline{R}_g[X] = \int_0^\infty \overline{F}_X^g(x)dx - \int_{-\infty}^0 [1 - \overline{F}_X^g(x))]dx = R_\gamma[X] \quad (8)$$

$$\overline{R}_\gamma[X] = \int_0^\infty \overline{F}_X^\gamma(x)dx - \int_{-\infty}^0 [1 - \overline{F}_X^\gamma(x))]dx = R_g[X] \quad (9)$$

(*see* [24])

Which implies that the risk measures (7) and (8) are coherent risk measures provided that $g(x)$ is a concave ($\gamma(x)$ is a convex) function ([24]). This implies that (6) and (7) are coherent provided that $g(x)$ is a convex ($\gamma(x)$ is a concave) function. For completeness, let us also mention a further duality between losses and gains, the latter being defined as negative losses. With this result, it suffices to study risk measures of either losses or gains.

Lemma 1 Let $X \in \chi$ be a loss random variable, $g(x)$ a distortion function, and $\gamma(x) = 1 - g(1-x)$ the dual distortion function, hence the relationships.

$$R_g[-X] = -R_\gamma[X], \quad \overline{R}_g[-X] = -\overline{R}_\gamma[X]. \quad (10)$$

Proof. Using that $F_{(-X)}(x) = 1 - F_X(-x)$ and making the substitution $x = -t$ one obtains

$$R_g[-X] = \int_0^\infty [1 - g(1 - F_X(-x))]dx - \int_{-\infty}^0 g(1 - F_X(-x))dx$$

$$= \int_{-\infty}^0 \gamma(F_X(t))dt - \int_0^\infty [1 - \gamma(F_X(t))]dt = -R_\gamma[X].$$

3. Weibull Distortion Measure

For a loss random variable x, with distribution function $f(x)$, we define a new risk measure for capital requirements, for a preselected security $\alpha = 1 - \lambda$ where λ = 1- α to be $W_B(\alpha) = (\lambda r)^{-1} X^\alpha$, (11)

Where α is the parameter of scale, λ is the shape parameter and r is the measure of risk aversion [17]

3.1 Tail-Free Distortion Risk Measures

Besides monotonicity, that is preservation of stochastic dominance of first order, it is known that a distortion measure R_i with concave distortion function preserves the stop-loss order or increasing convex order (e.g.).This is a desirable property because increased risk should be penalized with an increased measure. With equal means and variances, a stop-loss order relation between different random variables cannot exist. In this situation, increased risk can be modeled by the degree three stop-loss order or equivalently, by equal mean and variance, the degree three convex order. Thus, one is interested in distortion measures, which preserve this higher degree orders. As suggested by, such measures should be called free of tail risk or simply tail-free distortion measures. Some more formal definitions and properties are required.

For any real random variable X with distribution $F_X(x)$, consider the higher order partial moments $\pi_X^n(x) = E[(X-x)_+^n]$, $n = 0,1,2,...,$ called *degree n stop-loss transforms*. For n=0 the convention is made that $(x-d)_+^0$ coincides with the indicator function $1_{\{x>d\}}$, hence $\pi_X^0(x) = \overline{F}_X(x) = 1 - F_X(x)$ is simply the survival function of *X*. For n=1 this is the usual stop-loss transform $\pi_X(x)$, written without upper index. It is not difficult to establish the recursion (see [11])

$$\pi_X^n(x) = n \cdot \int_x^\infty \pi_X^{n-1}(t)dt, \; n = 1,2,.... \quad (12)$$

It will be useful to consider the following variants of the higher degree stop-loss orders (see [16,12]).

Definitions 2. For n=0,1,2,..., a random variable X precedes Y in *degree n stop-loss transform order*, written $X \le_{slt}^{(n)} Y$, if for all x one has $\pi_X^n(x) \le \pi_Y^n(x)$. A random variable X precedes Y in *degree n stop-loss order*, written $X \le_{sl}^{(n)} Y$, if $X \le_{slt}^{(n)} Y$ and the moment inequalities $E[X^k] \le E[Y^k]$, $k = 1,\ldots,n-1$, are satisfied. With equal moments $E[X^k] = E[Y^k]$, $k = 0,\ldots,j$, for some $j \in \{0,\ldots,n\}$, the relation is written $X \le_{sl,j}^{(n)} Y$. In particular, the one extreme case $\le_{sl,0}^{(n)} \equiv \le_{sl}^{(n)}$ defines a general degree n stop-loss order and the other one $\le_{sl,n}^{(n)} \equiv \le_{(n+1)-cx}$ defines the so-called *(n+1)-convex order* recently studied by [4]. Note that the special case $\le_{sl}^{(0)}$ is identical with the usual stochastic order or stochastic dominance of first order, also denoted $\le_{st}$. For n=1 the stochastic order $\le_{sl}^{(1)}$ coincides with the usual stop-loss order $\le_{sl}$ or equivalently increasing convex order $\le_{icx}$.

For fixed n, the above stop-loss order variants satisfy the following hierarchical relationship

$$\le_{(n+1)-cx} \equiv \le_{sl,n}^{(n)} \Rightarrow \le_{sl,n-1}^{(n)} \Rightarrow \ldots \Rightarrow \le_{sl,1}^{(n)} \Rightarrow \le_{sl,0}^{(n)} \equiv \le_{sl}^{(n)} \Rightarrow \le_{slt}^{(n)} .$$

(13)

Moreover, the higher degree stop-loss orders build a *hierarchical class* of partial orders
(see [, that is one has

$$\le_{sl,j}^{(n)} \Rightarrow \le_{sl,j}^{(n+1)}, \quad j \in \{0,\ldots,n\}. \quad (14)$$

Definition 3. A risk measure $R: \chi \to [0,\infty]$ is called a *degree n tail-free risk measure* if it is preserved under the *(n+1)*-convex order, that is if $X, Y \in \chi$ satisfy $X \le_{(n+1)-cx} Y$ then $R[X] \le R[Y]$.

Consequently, it is known that a distortion measure $\overline{R}_g[X]$ with concave distortion function preserves $\le_{(n+1)-cx}$ for $n = 0,1$, and is thus a tail-free risk measure of degree zero and one. In this paper, we are interested in specific concave distortion functions $g(x)$ such that $\overline{R}_g[X]$ is a degree two tail-free risk measure. For motivation, it is very important to emphasize the practical relevance of tail-free distortion measures, in which case, our field of application is risk management.

Example 1 : Comparison of Conditional value-at-risk versus Wang right-tail measure and Weibull distortion measure

Consider the coherent distortion measure (7) defined by the increasing concave distortion function $g_\varepsilon(x) = \min\{\frac{x}{\varepsilon}, 1\}$, where ε is a small probability of loss, say $\varepsilon = 0.05$. By definition, the measure associated to $X \in \chi$ is denoted $\overline{R}_{g_\varepsilon}[X]$. It is known that this risk measure coincides with several other known risk measures like the *conditional value-at-risk* measure and the *expected shortfall* measure (see [13]). In a standard notation, conditional value-at-risk at the confidence level $\alpha = 1 - \varepsilon$, written $CVaR_\alpha[X]$, coincides with $\overline{R}_{g_\varepsilon}[X]$. For comparison, consider the distortion function $g(x) = \sqrt{x}$. The coherent distortion measure (7), called *Wang right-tail measure* and denoted by $WRT[X] := \overline{R}_g[X]$, has been proposed by [20] as a measure of right-tail risk. For illustration, let Y be a loss consisting of two scenarios with loss amounts 20$, 2100$ such that $P(Y = 20) = 1 - P(Y = 2100) = \frac{25}{26}$. Through active risk management, assume that the lower amount can be eliminated and that the higher loss amount can be reduced to 1700$. By equal mean and variance, this results in a loss X such that $P(X = 0) = 1 - P(X = 1700) = \frac{16}{17}$. Suppose a risk manager is weighing the cost of risk management against the benefit of capital relief. Then CVaR does not promote risk management because

$$CVaR_\alpha[X] = 1700 > CVaR_\alpha[Y] = 20 + 2080 \cdot \left(\tfrac{20}{26}\right) = 1620,$$

which shows that there is a capital penalty instead of a capital relief for either removing or reducing the initial loss amounts. However, the Wang right-tail measure and Weibull distortion measure offers a capital relief because

$$WRT[X] = 1700 \cdot \sqrt{\tfrac{1}{17}} = 412.3 < WRT[Y] = 20 + 2080 \cdot \sqrt{\tfrac{1}{26}} = 427.9 .$$

WB(X) = $\frac{1}{\frac{16}{17} \times 1} (1700)^{1-\frac{16}{17}} =$ = 1.65< $WB(Y) = 20 + 2080. = \frac{1}{\frac{20}{26} \times 1} (2100)^{1-\frac{20}{26}}$ = 8.15.

Since Y is evidently a higher loss than X, the CVaR measure fails to recognize this feature. Even more, in this simple example X precedes Y in the degree three convex order. This shows that through a meaningful counterexample that CVaR is not a degree two tail-free coherent risk measure. In view of the fact that CVaR ignores useful information in a large part of the loss distribution, [21] has proposed a new coherent distortion measure, which should adjust more properly extreme low frequency and high severity losses. However, as the following counterexample shows, Wang's most recent proposal does not generate a degree two tail-free coherent risk measure.

Example 2: Wang transform measure versus Wang right-tail measure versus Weibull distortion measure.

Consider the distortion function $g_{\varepsilon}(x) = \Phi\left(\Phi^{-1}(x) - \Phi^{-1}(\varepsilon)\right)$, where $\Phi(x)$ is the standard normal distribution and ε is a small probability of loss, say $\varepsilon = 0.05$. This interesting choice finds further motivation in [21,22] and defines the *Wang transform measure* $WT_{\alpha}[X] := \overline{R}_g[X]$, where $\alpha = 1 - \varepsilon$. Similar to Example 1, consider a biatomic loss Y such that $P(Y = 90) = 1 - P(Y = 100'100) = \frac{10'000}{10'001}$. Let X be a biatomic loss with the same mean and variance such that $P(X = 0) = 1 - P(X = 10'010) = \frac{100}{101}$. Obviously Y is a higher loss than X, but the Wang measure does not provide incentive for risk management because

$$WT_{\alpha}[X] = 10'010 \cdot g_{\varepsilon}\left(\tfrac{1}{101}\right) = 2468.5$$
$$> WT_{\alpha}[Y] = 90 + 100'100 \cdot g_{\varepsilon}\left(\tfrac{1}{10'001}\right) = 1993.3.$$

However, the Wang right-tail measure offers a capital relief because

$$WRT[X] = 10'010 \cdot \sqrt{\tfrac{1}{101}} = 996.0 < WRT[Y] = 90 + 100'100 \cdot \sqrt{\tfrac{1}{10'001}} = 1090.1 .$$

$$WB(X) = \frac{1}{\frac{100}{101}} \cdot (10010)^{1-\frac{100}{101}} \times 100 = 109.6 < WB(Y) = \frac{1}{\frac{10000}{10001}} \cdot (100100)^{1-\frac{10000}{10001}} \times 100 = 112.2$$

and Weibull distortion measure also offers a capital relief.

Therefore, Since X precedes Y, the Wang transform measure is not a tail-free coherent risk measure.

4. Conclusion

With the above two counter examples, we suggest that if a risk manager is weighing the cost of risk management against the benefit of capital return one can not only depend on the diversification principal (merging) but an alternative optimal Weibull distortion risk management which provide an incentive for risk management and offers a capital relief which is reminiscent of the handy and quite old slogan ‘Avoiding merging increases shortfall’ which justifies the well known advice ‘ don’t put all your eggs in one basket’.

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Author Profile

Godswill U. Achi is currently a principal lecturer and HOD Mathematics department, Abia State Polytechnic, Aba. He is a PhD holder in Mathematics of Finance and Ms.C Stochastic Optimization & Control. He has published several articles in a reputable journal.

Solomon Okechukwu holds M.Sc in Mathematics of Finance and B.Sc in Mathematics Education from Abia State University, Uturu, Nigeria. He has published three articles in a reputable journal Ogwo Obiageri has M. Sc in Mathematics and B.Sc in mathematics from Abia State University, Uturu, Nigeria.