Study of Various modes in a Solid State Laser

Kireet Semwal¹, S. C. Bhatt²

¹Department of Physics, G.B. Pant Engineering College, Pauri (Garhwal), Uttarakhand, India
²Department of Physics, HNB Garhwal Central University, Srinagar (Garhwal), Uttarakhand, India

Abstract: The light emitted by most lasers contains several discrete optical frequencies, separated from each other by frequency differences, which can be associated with different modes of the optical resonator. It is common practice to distinguish two types of resonator modes: “Longitudinal” modes differ from one another only in their oscillation frequency; and “Transverse” modes differ from one another not only in their oscillation frequency, but also in their field distribution in a plane perpendicular to the direction of propagation. Corresponding to a given transverse mode are a number of longitudinal modes which have the same field distribution as the given transverse mode but which differ in frequency.

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1. Introduction

To describe the electromagnetic field variations inside optical resonators, the symbol TEMmnq or TEMpq are used. The capital letters stands for “Transverse Electromagnetic Waves” and the first two indices identify a particular transverse mode, whereas q describes a longitudinal mode. Because resonators that are used for typical lasers are long compared to the laser wavelength, they will, in general, have a large number of longitudinal modes. Therefore, the index q, which specifies the number of modes along the axis of the cavity, will be very high. The indices for the transverse modes, which specify the field variations in the plane normal to the axis, are very much lower and sometimes may be only the first few integers.

The spectral characteristics of a laser, such as linewidth and coherence length, are primarily determined by the longitudinal modes, whereas beam divergence, beam diameter, and energy distribution are governed by the transverse modes. In general, lasers are multimode oscillators unless specific efforts are made to limit the number of oscillating modes. The reason for this lies in the fact that a very large number of longitudinal resonator modes fall within the bandwidth exhibited by the laser transition and a large number of transverse resonator modes can occupy the cross section of the active material [1].

1.1 Transverse Modes

The output spot of the laser beam is termed the transverse electromagnetic mode (TEM). Transverse modes are defined by the designation TEMmn for Cartesian coordinates. The integers m and n represent the number of nodes or zeros of intensity transverse to the beam axis in the vertical and horizontal directions. In cylindrical coordinates the modes are labelled TEMpl and are characterized by the number of radial nodes p and angular nodes l. The higher the values of m, n, p, and l, the higher the mode orders. The lowest-order mode is the TEM00 mode, which is a round mode with a Gaussian-like intensity profile in cross-section, with its maximum on the beam axis. However it is possible to operate on a wide variety of other transverse mode configurations. In these configurations, the output spot will have a much more peculiar shape [2]. For mode with subscripts of 1 or more, intensity maxima occur that are off-axis in a symmetrical pattern. To determine the location and amplitudes of the peaks and nodes of the oscillation modes, it is necessary to employ higher-order equations, which either involve Hermit or Laguerre polynomials. The Hermit polynomials are used when working with rectangular coordinates, while Laguerre polynomials are more convenient when working with cylindrical coordinates. The transverse mode structure will be calculated using the paraxial approximation to the electromagnetic wave equations [2] [1].

2. The Paraxial Approximation

In general electromagnetic wave equation for a laser material is given as

\[
v^2 \partial^2 \tilde{\mathbf{E}} = \left( \mu_0 \varepsilon_0 \mathbf{h} \frac{\partial^2 \tilde{\mathbf{P}}}{\partial t^2} + \mu_0 \varepsilon_0 \frac{\partial^2 \tilde{\mathbf{P}}}{\partial t^2} + \mu_0 \sigma \frac{\partial \tilde{\mathbf{E}}}{\partial t} \right) (1)
\]

where \( \tilde{\mathbf{E}} \) is the electric intensity vector and \( \tilde{\mathbf{P}} \) is the electric polarization vector given by [3]

\[
\tilde{\mathbf{P}} = \varepsilon_0 \chi \tilde{\mathbf{E}} \tag{2}
\]

where \( \chi \) is the complex susceptibility, \( \sigma \) is the conductivity, an \( \varepsilon_0 \) is the electric permittivity of the host crystal alone [4].

The field \( \tilde{\mathbf{E}} \) is basically a plane wave of the form

\[
\tilde{\mathbf{E}}_0 e^{i (\mathbf{k} \cdot \mathbf{r} - \omega t)}
\]

and Equation (1) is often written in the phasor form

\[
\nabla^2 \tilde{\mathbf{E}} + \omega^2 \mu_0 \varepsilon_0 \left( \mathbf{k} \cdot \mathbf{E} \right) \left( 1 + \frac{\sigma}{\omega \varepsilon_0 \mathbf{k} \cdot \mathbf{E}} \right) \tilde{\mathbf{E}} = 0 \tag{3}
\]

\[

abla^2 \tilde{\mathbf{E}} + k_c^2 \tilde{\mathbf{E}} = 0 \tag{4}
\]
where \( k_c = \alpha \sqrt{k_0 \sigma_m} \), \( \epsilon m = \epsilon 0 \sigma x \), \( \sigma x = 1 + \chi \) and \( \vec{E} \) is the phasor representing the electric field.

For deriving the transverse mode behaviour, consider the transverse variation to be given by \( U(x, y, z) \) so that the phasor expression for \( \vec{U}(x, y, z) \) has the form
\[
E = \vec{U}(x, y, z) e^{-jk_c z}
\]

where \( k_c = \alpha \sqrt{k_0 \sigma_m} \). Now expand the first term of the equation
\[
\nabla^2 \vec{U}(x, y, z) e^{-jk_c z} = \frac{\partial^2 \vec{U}(x, y, z)}{\partial^2} e^{-jk_c z} + \frac{\partial^2 \vec{U}(x, y, z)}{\partial^2} e^{-jk_c z}
\]

plus \( \frac{\partial^2 \vec{U}(x, y, z)}{\partial^2} e^{-jk_c z} \)

where \( \frac{\partial^2 \vec{U}(x, y, z)}{\partial^2} = \nabla^2 \vec{U}(x, y, z) \)

Rewriting this more conventionally,
\[
\frac{\partial^2 \vec{U}(x, y, z)}{\partial^2} e^{-jk_c z} = \left( \frac{\partial^2}{\sigma_x^2} + \frac{\partial^2}{\sigma_y^2} + (2j\kappa_z) \frac{\partial}{\partial z} \right) \vec{U}(x, y, z) e^{-jk_c z}
\]

In the paraxial approximation, it is assumed that the variation in the transverse structure with \( z \) is extremely small. Thus the term \( \frac{\partial^2}{\sigma_x^2} \vec{U}(x, y, z) e^{-jk_c z} \) is quite small in comparison with the remaining terms, and so the previous expression is written as
\[
\nabla^2 \vec{U}(x, y, z) e^{-jk_c z} = \left( \frac{\partial^2}{\sigma_x^2} + \frac{\partial^2}{\sigma_y^2} + (2j\kappa_z) \frac{\partial}{\partial z} \right) \vec{U}(x, y, z) e^{-jk_c z}
\]

Substituting this back into the original phasor form for the wave equation and making the appropriate cancellations gives the full paraxial approximation to the wave equation as
\[
\left( \frac{\partial^2}{\sigma_x^2} + \frac{\partial^2}{\sigma_y^2} + (2j\kappa_z) \frac{\partial}{\partial z} - k_c^2 \right) \vec{E} = 0
\]

and substituting in for \( k_c \) yields
\[
\left( \frac{\partial^2}{\sigma_x^2} + \frac{\partial^2}{\sigma_y^2} + (2j\kappa_z) \frac{\partial}{\partial z} + k_c^2 \right) \vec{E} = 0
\]

which is often termed the transverse wave equation.

For propagation in free space, this is often written in a form that emphasizes its similarity to Schrodinger’s equation [5][6]. In this formalism, \( \Psi = \vec{E} \) and \( \chi = \sigma = 0 \) as
\[
\left( \frac{\partial^2}{\sigma_x^2} + \frac{\partial^2}{\sigma_y^2} + (2j\kappa_z) \frac{\partial}{\partial z} \right) \chi = 0
\]

3. Mathematical Treatment of the Transverse Modes

There are two distinct sets of solutions to the higher-order modes. These solutions depend on whether the laser has a square transverse geometry (excimers, waveguide CO2 lasers, etc.) or circular transverse geometry (the majority of other lasers). For a circular transverse geometry, the general transverse wave equation is expressed in cylindrical coordinates as [5]
\[
1 \frac{\partial}{r} \left( \frac{\partial \psi}{\partial r} \right) - 2 j k_c \frac{\partial \psi}{\partial z} = 0
\]

and has the general solution (where \( p \) is the radial integer, \( l \) is the angular integer, and \( r \) and \( \phi \) are polar coordinates) [6]
\[
\varphi_{p, l} = \frac{\Gamma}{\sqrt{\tau_{l} a}} L_p \left( \frac{\tau_{l} a}{\tau} \right) \exp \left( - \frac{\tau_{l} a}{\tau} \right) \left( \begin{array}{c}
\frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) \exp \left( - \frac{\tau_{l} a}{\tau} \right)
\end{array} \right)
\]

where \( r = x^2 + y^2 \), \( w(z) \) is the 1/e beam radius (waist) as a function of \( z \), where \( z \) is the propagation direction of the beam, \( w_0 \) is the minimum beam waist, and \( R(z) \) is the beam radius of curvature. In cylindrical coordinates, the radial intensity distribution of allowable circularly symmetric TEMpl modes is given by the expression [6][1]
\[
I_{pl}(r, \phi, z) = I_0 (\rho^2)(2 \cos^2 \phi) \exp(-\rho)
\]

where \( \rho = 2r_2 (z)/w_2(z) \), and \( r, \phi \) are the polar coordinates in a plane transverse to the beam direction. The radial intensity distributions are normalized to the spot size of a Gaussian profile; that is, \( w(z) \) is the spot size of the Gaussian beam, defined as the radius at which the intensity of the TEM00 is 1/ e2 of its peak value on the axis. \( \rho^2 \) is the general Laguerre polynomial of order \( p \) and index \( l \).

The intensity distribution given in (14) is the product of radial part and an angular part. For modes with \( l = 0 \) (i.e. TEM00), the angular dependence drops out and the mode pattern contains \( p \) dark concentration rings, each ring corresponding to a zero of \( \rho_p^2(\rho) \). The radial intensity distribution decays due to the factor \( \exp(-\rho) \). The center of a \( pl \) mode will be bright if \( l = 0 \), but dark otherwise because of the factor \( \rho \). These modes, besides having \( p \) zeros in the radial direction, also have \( 2l \) nodes in azimuth. The only change in a \( pl \) mode distribution comes through the dependence of the spot size \( w(z) \) on the axial position \( z \). However, the modes preserve the general shape of their electric field distributions for all values of \( z \). As \( w \) increases with \( z \), the transverse dimensions increase so that the sizes of the mode patterns stay in constant ratio to each other.
From (14) it is possible to determine any beam mode profile. Figure-1 shows the cylindrical and rectangular transverse mode patterns. For cylindrical modes, the first subscript indicates the number of dark rings, whereas the second subscript indicates the number of dark bars across the pattern. For rectangular patterns, the two subscripts give the number of dark bars in the x and y directions. Figure-1a depicts various cylindrical transverse intensity patterns as they would appear in the output beam of a laser, and the area occupied by a mode increases with the mode number. A mode designation accompanied by an asterisk indicates a mode which is a linear superposition of two like modes, one rotated 90° about the axis relative to the other. For example, the TEM mode designated 01* is made up of two TEM0,1 modes [7][1][2]. The intensity distribution of the modes shown in Figure-1a can be calculated if we introduce the appropriate Laguerre polynomials into (14), i.e., from Rodrigues formula

\[ L_p^0 (\rho) = \frac{1}{p!} \rho^p \frac{1}{d\rho} \frac{d}{d\rho} \left( -\rho \right)^p \]

\[ L_0^0 (\rho) = 1, \quad L_0^0 (\rho) = 1 - \rho, \quad L_0^0 (\rho) = 1 - 2\rho + \frac{1}{2}\rho^2. \]

A plot of the intensity distribution of the lowest-order TEM0,0 mode and the next two higher-order transverse modes, i.e., TEM01*, and TEM10 is shown in Figure-2.

The radii are normalized to the beam radius w00 of the fundamental mode [6][2][1].

For a rectangular transverse geometry, the general transverse wave equation (11) has the general solution (where m and n are the mode numbers for the x and y directions, respectively [6][2][1]

\[ \psi_{m,n} = H_m \left( \frac{\sqrt{2}x}{w(z)} \right) H_n \left( \frac{\sqrt{2}y}{w(z)} \right) \exp \left( - \frac{r^2}{2w^2(z)} \right) \exp \left( \frac{j k r^2}{2w^2(z)} \right) \]

where \( r^2 = x^2 + y^2 \), \( w(z) \) is the 1/e beam waist (radius) as a function of z, w0 is the minimum beam waist at the location z0, and R(z) is the beam radius of curvature.\n
\[ H_m \left( \frac{\sqrt{2}x}{w(z)} \right) \quad \text{and} \quad H_n \left( \frac{\sqrt{2}y}{w(z)} \right) \]

are Hermit polynomial functions of \( \left( \sqrt{2}x/w(z) \right) \) and \( \left( \sqrt{2}y/w(z) \right) \) given by Rodrigues formula

\[ H_m (\rho) = (-1)^m e^{-\rho^2} \frac{d^m}{d\rho^m} e^{\rho^2} \]

The values of low-order Hermit Polynomials are H0(s) = 1; H1(s) = 2v; H2(s) = 4s2 – 2; H3(s) = 8s3 – 12. v. (19)

In rectangular coordinates the intensity distributions of a (m, n) mode is given by

\[ I_{mn}(x,y,z) = \left[ H_m \left( \frac{\sqrt{2}x}{w(z)} \right) \right] \left[ H_n \left( \frac{\sqrt{2}y}{w(z)} \right) \right] \exp \left( - \frac{r^2}{2w^2(z)} \right) \exp \left( \frac{j k r^2}{2w^2(z)} \right) \]

As before, \( w(z) \) is the spot size at which the transverse intensity decreases to 1/e2 of the peak intensity of the lowest-order mode. The function \( H_m(s) \) is the mth-order Hermit polynomial. At a given axial position z, the intensity distribution consists of the product of a function of x alone and a function of y alone. The intensity patterns of rectangular transverse modes are shown in Figure-1b. The m, n values of a single spatial mode can be determined by counting the number of dark bars crossing the pattern in the x and y directions. The fundamental mode \( (m = n = 0) \) in this geometry is identical with the fundamental mode in cylindrical geometry [2][1].

![Figure 3: Linearly polarized resonator mode configurations for square and circular mirrors.](image-url)
According to Koechner, [1] the transverse modes shown in Figure-1 can exist as linearly polarized beams, as shown in Figure-3. By combining two orthogonally polarized modes of the same order, it is possible to synthesize other polarization configurations; this is shown in Figure-4 for the TEM0,1 mode.

4. TEM0,0 Gaussian Beam Propagation

The lowest-order transverse mode is called the TEM0,0 mode, the 00-mode, the lowest-order mode, fundamental mode, or the Gaussian mode. This is the mode that is circular in transverse dimensions and has a Gaussian intensity profile. It is the mode that is most widely used in laser systems [8] [5] [6]. The decrease of the field amplitude with distance r from the axis in a Gaussian beam is described by Koechner [1] as:

\[ E(r) = \tilde{E}_0 \exp \left( -\frac{r^2}{w^2} \right) \]  

Thus, the distribution of power density is

\[ I(r) = I_0 \exp \left( -\frac{r^2}{w^2} \right) \]  

The quantity w is the radial distance at which the field amplitude drops to 1/e of its value on the axis and the power density is decreased to 1/e2 of its axial value. The parameter w is often called the beam radius or “spot size” and 2w, the beam diameter. According to Koechner [1] the fraction of the total power of a Gaussian beam which is contained in a radial aperture of r = w, r =1.5w, and r = 2w is 86.5 %, 98.9 % and 99.9 %. If a Gaussian beam is passed through a radial aperture of 3w, then only 10-6 % of the beam power is lost due to the obstruction. Therefore a radial aperture in excess of three spot sizes is means an “infinite aperture”.

Although the intensity distribution is Gaussian in every propagating beam cross section, the width of the intensity profile changes along the axis. The Gaussian beam contracts to a minimum diameter 2w0 at the beam waist where the phase front is planer. If one measures z from this waist, the expansion laws for the beam assume a simple form. The spot size a distance z from the beam waist expands as a hyperbola, which has the form

\[ w(z) = w_0 \left( 1 + \left( \frac{2z}{w_0^2} \right) \right)^{-1/2} \]  

Its asymptotes are inclined at an angle \( \Theta / 2 \) with the axis, as shown in Figure-5, and defines the far-field divergence angle of the emerging beam. The full divergence angle for the fundamental mode is given by

\[ \Theta = \lim_{z \to \infty} \frac{2w(z)}{z} = \frac{2\lambda}{\pi w_0} = 1.27 \frac{\lambda}{(2w_0)} \]  

From these considerations it follows that at large distances, the spot size increases linearly with z, and the beam diverges at a constant cone angle \( \Theta \). The smaller the spot size w0 at the beam waist, the greater the divergence.

At sufficiently large distances from the beam waist the wave has a spherical wave front appearing to emanate from a point on the beam axis at the waist. If R(z) is the radius of curvature of the wave front that intersects the axis at z, then

\[ R(z) = \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right) \right]^{-2} \]  

In a Gaussian beam the wavefront has the same phase across its entire surface. According to Koechner [1], sometimes the properties of a TEM00 mode beam are described by specifying a confocal parameter

\[ b = \frac{2\pi w_0^2}{\lambda} \]  

where b is the distance between the points at each side of the beam waist for which \( w(z) = \sqrt{2}w_0 \) (Figure-5).

A laser operating at the TEM00 mode will have a beam divergence according to Eq. (24). For a plane wavefront incident upon a circular aperture of diameter D, the full cone angle of the central (Airy) disc, defined at the first, minimum of the Fraunhofer diffraction pattern, is given by

\[ \Theta_p = \frac{2.44\lambda}{D} \]  

the energy contained within this angle is about 84 % of the total energy transmitted by the aperture. Equations (24) and (27) are often confused, because various conventions have been adopted by different authors, with the equation

...
\[ \Theta_R = \frac{1.22 \lambda}{D} \] (28)

which represents the half-cone angle of the Fraunhofer diffraction pattern, and also happens to be the “Rayleigh criterion” for the angular resolution of an optical instrument.

In laboratory work, a beam size is often obtained by measuring the diameter of the illuminated spot with a scale. This is not the spot size \(2w_0\) as defined by Eq. (23). There is no obvious visual cue to the magnitude of the spot size in the appearance of the illuminated spot. Thus, “spot size” and “size of the illuminated spot” are totally different concepts. The former is a property of the laser cavity; the latter is a subjective estimate. To measure the spot size, the illuminated spot is scanned with a photodetector behind a small pinhole. The resulting curve of intensity versus position of the pin-hole will yield a Gaussian curve from which the spot size of intensity versus position of the pin-hole will yield a photodetector behind a small pinhole. The resulting curve of intensity versus position of the pin-hole will yield a Gaussian curve from which the spot size \(w_0\) be extracted by mathematical methods [7] [6].

5. Mode Selection

Many applications of solid-state lasers, such as micromachining, nonlinear optical experiments, holography, and range finding, quite often require operation of the laser at the TEM00 mode since this mode produces the smallest beam divergence, the highest power density, and hence, the highest brightness. Furthermore, the radial intensity profile is uniform and uniphase. The latter property, i.e., the spatial coherence of the TEM00 mode, is particularly important for holographic applications. Focusing a fundamental-mode beam by an optical system will produce a diffraction-limited spot of maximum power per unit area. In many applications it is a high brightness (power/unit area/ solid angle) rather than large total emitted power that is desired from the laser.

Transverse mode selection generally restricts the area of the laser cross section over which oscillation occurs, thus decreasing the total output power. However, mode selection reduces the beam divergence so that the overall effect of mode selection is an increase in the brightness of the laser. For example, the beam diameter and beam divergence for a TEM\(mpl\) mode increases with the factor Cpl, which means that for the same output power the brightness decreases by a factor of (Cpl)-4 for the higher-order modes.

Most practical lasers tend to oscillate not only in higher-order transverse modes, but in many such modes at once. Because of the fact that higher-order transverse modes have a larger spatial extent than the fundamental mode, a given size aperture will preferentially discriminate against higher-order modes in a laser resonator. As a result, the question of whether or not a laser will operate only in the lowest-order mode depends on the size of this mode and the diameter of the smallest aperture in the resonator. If the aperture is much smaller than the TEM00 mode size, large diffraction losses will occur which will prevent the laser from oscillating. If the aperture is much larger than the TEM00 mode size, then higher-order modes will have sufficiently small diffraction losses to be able to oscillate.

The diffraction losses caused by a given aperture and the transverse mode selectivity achievable with an aperture of radius \(a\). The mode selectivity is strongly dependent on the resonator geometry, and is greatest for a confocal resonator and smallest for the plane-parallel resonator. The resonators of laser operating in the TEM00 mode will have Fresnel numbers on the order of approximately 0.5 to 2.0. For Fresnel numbers much smaller than these, the diffraction losses will become prohibitively high, and for much larger values of \(N\) mode discrimination will be insufficient.

6. Conclusion

These predictions are in agreement with the experimental observations. For example, typical ruby and Nd:YAG lasers have cavity lengths of 50 to 100 cm and TEM00 operation typically requires the insertion of an aperture in the cavity with a diameter between 1 and 2 mm. Without an aperture, a 50-cm-long resonator with a 0.62-cm-diameter Nd:YAG rod as the limiting aperture will have a Fresnel number of 19. In ruby lasers, where oscillator rods of 15-mm diameter are not uncommon, the Fresnel number would be 160 for the same resonator length.

Because the TEM00 mode has the smallest beam diameter of all the resonator modes, a number of techniques have been developed to increase the TEM00 mode volume in the active material, which is normally considerably larger in diameter than the mode size. A resonator designed for TEM00 mode operation will represent a compromise between the conflicting goal of large mode radius, insensitivity to perturbation, good mode discrimination, and compact resonator length.

References


**Author Profile**

Dr. Kireet Semwal is working as an Assistant Professor (Physics) in the Applied Science Department, G B Pant Engineering College, Pauri (Garhwal), Uttarakhand-246001, INDIA. His field of research is Laser and Nonlinear Optics. He did his M. Sc. from HNB Garhwal University, Srinagar (Garhwal), Uttarakhand, and MTech from G B Pant University of Agriculture and Technology, Pantnagar, Uttarakhand.