Comparative Analysis of Multi Layered Composite Plates using Higher Order Theories

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Abstract: ***The present work deals with the modeling and analysis of laminated composites plate. In the recent years, significant research has been carried out in the area of composite structure owing to its widespread application in aerospace and smart as well as intelligent structures. In the current analysis, classical and higher order plate theory is considered for studying the deformation of composite plates with symmetric and anti-symmetric orientation. The equations of equilibrium are obtained by using the principle of virtual displacement. Solutions are obtained in closed form using Navier technique. The analysis is carried out for different side to thickness and aspect ratio. The non-dimensional deflections at mid plane and at various nodes have been calculated for different loading conditions (viz uniformly distributed and static sinusoidal distributed load). Finally the model is implemented by Finite Element Method on a laminated composite plate of any lay-up. Finite element model is based on four node quadrilateral element in the present analysis, where the deflections are calculated at all the nodes. The computing is done by using MATLAB. The results of non-dimensional deflection obtained by classical and higher order theories are compared for different aspect ratio with the variation of side to thickness ratio.***

Keywords: CLPT, FSDT, ESL, BVP, MATLAB.

1. Introduction

The use of composite material in structural components is increasing due to their attractive properties such as high strength-to-weight ratio, ability to tailor the structural properties, etc. Plate structures find numerous applications in the aerospace, military and automotive industries. The effects of transverse shear deformation are considerable for composite structures, because of their high ratio of extensional modulus to transverse shear modulus.

Most of the structural theories are used till now to characterize the behavior of composite laminate fall into category of equivalent single layer (ESL) theories. In these theories, the material properties of the constituent layers are combined to form a hypothetical single layer whose properties are equivalent to-through-thickness integrated sum of its constituents. This category of theory has been found to be adequate predicting global response characteristic of laminates, like maximum defections, maximum stresses, fundamental frequencies, or critical buckling load.Some study has been done over the different type of theories of plate regarding Non-dimensional deflection. In this context, some studies over the different fields are given here.

They had proposed model assumed piecewise linear variation of in-plane displacement components and constant transverse displacement through the thickness of the plate. They had used the assumed displacement field, strain- displacement relations and 3D constitutive equations of lamina. Equation of motion was derived by using Hamilton's principle. A original MATLAB computer program coded for analytical and finite element solutions of the theory. They had verified that the proposed model was capable to accurately predict both global and local response of laminated composite and sandwich plates, when compared to 3D elasticity solution and LW model, while ESL theories, such as classical (CLPT) and first-order shear deformation theory (FSDT) were inapplicable for the analysis of highly anisotropic laminates.

The presented novel semi-analytical model for composite and sandwich laminates. Formulation was based on solution of a two-point boundary value problem (BVP) governed by a set of linear first-order ordinary differential equations (ODEs) through the thickness of a laminate. These first-order ODEs numerically were integrated by using fourth-order routine. Results obtained through this technique compared with the available three dimensional (3D) elasticity and other two dimensional (2D) analytical and 2D/3D finite element (FE) solutions. They showed that through numerical investigation the result obtained by this approach was highly accurate. Another important feature of this approach was that both the displacements and the stresses computed simultaneously were the same degree of accuracy.

2. Objective of the Present Work

In the light of the literature survey presented above, the following objectives are identified for the present work;

1) Compare the result of non-dimensional deflection for a simply supported orthotropic plate obtained by using the Classical laminated plate theory, First-order shear deformation theory.
2) The results are compared in terms of side to thickness ratio, aspect ratio, loading condition, different stacking sequence and different numbers of layers.
3) The computing is done by using MATLAB.

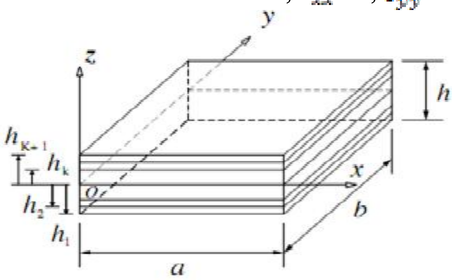
3. Classical Laminated Plate Theory

The classical laminated plate theory is an extension of the classical plate theory to composite laminates. In the classical laminated plate theory (CLPT) it is assumed (means an assumption is necessary for the development of the mathematical model, whereas a restriction is not a necessary condition for the development of the theory) that the Kirchhoff hypothesis holds:

(a). Straight lines perpendicular to the mid surfaces (i.e., transverse normal) before deformation remain straight after deformation.
(b). The transverse normal do not experienced elongation (i.e., they are inextensible).
(c). The transverse normal rotates such that they remain perpendicular to the mid surfaces after deformation.

The first two assumptions imply that the transverse displacement is independent the transverse (or thickness) coordinate and the transverse normal strain E_n is zero. The third assumption

Results in zero transverse shear strain, ε_{xz} =0, ε_{yy}=0.



4. First Order Shear Deformation theory (FSDT)

The next theory in the hierarchy of ESL laminates theories is the first order shear deformation theory (or FSDT), which is also based on the displacement field:

$$u(x,y,z,t) = u_0(x,y,t) + z\varnothing_x(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,t) + z\varnothing_y(x,y,t)$$
$$w(x,y,z,t) = w_0(x,y,t)$$

Where $\varnothing_x$ and $\varnothing_y$ denote rotations about the y & x axes respectively. The FSDT extends the kinematics of the CLPT by including a gross transverse shear deformation in its kinematic assumptions, i.e., the transverse. Shear strain is assumed to be constant with respect to the thickness coordinate. Inclusion of the rudimentary form of shear deformation allows the normality restriction of the classical laminate theory to be relaxed. The first-order shear deformation theory requires shear correction factors, which are difficult to determine for arbitrarily laminated plate structures. The shear correction factors depend not only on the lamination and geometric parameters; but also on the loading and the boundary conditions.

In both CLPT and FSDT, the plane-stress state assumption is used and plane-stress reduced form of the constitutive law. In both theories the inextensibility and / or straightness of transverse normal can be removed. Such extensions lead to second and higher-order theories of plates.

Second and higher-order ESL laminated plate theories use higher-order polynomials in the expansions of the displacement components through the thickness of the laminate among many others. The higher-order theories introduce additional unknowns that are often difficult to interpret in physical terms. The second-order theory with transverse inextensibility is based on the displacement field.

$$u(x,y,z,t) = u_0(x,y,t) + z\varnothing_x(x,y,t) + z^2\Psi_x(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,t) + z\varnothing_y(x,y,t) + z^2\Psi_y(x,y,t)$$
$$w(x,y,z,t) = w_0(x,y,t)$$

5. Stacking Sequence

A laminate is a collection of lamina stacked to achieve the desired stiffness and thickness. For example, unidirectional fiber-reinforced lamina can be stacked so that the fiber in each lamina is oriented in the same or different direction. The sequence of various orientation of a fiber reinforced composite layer in a laminate is termed the lamination scheme or stacking sequence. The layers are usually bonded together with the same matrix material as that in a lamina. A unidirectional lamina (i.e., all lamina have the same orientation) will be very strong along the fiber direction and weak in transverse direction. In our problem we have assumed four and eight layers laminate with symmetric and anti-symmetric lamination scheme.

(a)	(b)
θ = 0°	θ = 0°
θ = 90°	θ = 90°
θ = 0°	θ = 0°
θ = 90°	θ = 90°

Figure 1: Four layers symmetric and Anti-symmetric cross-ply laminate

(a)	(b)
θ = 0°	θ = 0°
θ = 90°	θ = 90°
θ = 0°	θ = 90°
θ = 90°	θ = 0°
θ = 0°	θ = 0°
θ = 90°	θ = 90°
θ = 0°	θ = 90°
θ = 90°	θ = 0°

Figure 2: Eight layers symmetric and anti-symmetric cross-ply laminated plate.

6. Conclusion

This study considers the non-dimensional deflection response of laminated composite rectangular plates with simply supported boundary conditions. The laminated composite plates have varying side to thickness ratio, aspect ratio, different cross-ply orientation and different type of load conditions. From the present analytical study, the following conclusions can be made.

1. It was noted that different side to thickness ratio affected the non-dimensional deflection. The non-dimensional deflection decreases as side to thickness ratio increases.

The rate of decrease of the non-dimensional deflection is uniform with the rate of increase of side to thickness ratio.

2. As the aspect ratio increases, the non-dimensional deflection of the plate decreases. When the aspect ratio changed from less than unity to greater than unity, the variation in the non-dimensional deflection is very large.
3. It was seen that the different stacking sequence affected the non-dimensional deflection. When the stacking sequence changes from symmetric to anti-symmetric, the non-dimensional deflection decreases.
4. The non-dimensional deflection due to the uniformly distributed load is found to be greater than sinusoidal distributed load.
5. The results obtained by CLPT were not in good agreement because this theory neglects both transverse shear and transverse normal and the curve obtained by this theory as a constant straight line without any effect of the variation of side to thickness ratio.

In case of FSDT, the shear correction factor is necessary but a definite procedure to determine this factor has not yet been satisfactorily enunciated, thus introducing an element of arbitrariness in the analysis.

7. Future scope

In the present analysis the non-dimension deflection of the laminated plate was determined. The effect of side to thickness ratio, aspect ratio types of loading conditions and stacking sequence on non-dimension deflection was studied. The future scope of the present investigation can be expressed as follows,

(a). The non-dimension deflection of delaminated industry driven woven composite plates with and without cutouts.
(b). The non-dimension deflection of laminated composite plates can be determined by numerical approach for different boundary conditions.
(c). Dynamic stability of laminated composite plate can be determined with different types of loading and stacking sequence.

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