Estimation of a Co-integration Model Using Ordinary Least Squares (A Simulation Study)

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Abstract: ***Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models are usually used to analyse time series data with high volatility clustering. In this paper, it is proposed that if two time series follow GARCH(1,1), the two series are cointegrated and accordingly, we simulate data using the GARCH model which are used to prove this proposition. The choice of the simulation models is based on its ability to capture volatility and heteroskedasticity. Co-integration and Ordinary Least Squares methods are used; and the model parameters investigated for adequacy. Results from Augmented Dickey Fuller (ADF), Phillips Perron (PP) and Kwiatkowski Philips Schmidt Shin (KPSS) tests indicates stationarity in the data as expected. The Engle-Granger two-step method for testing co-integration is used. A linear co-integration model is estimated with coefficient of co-integration, λ, being 1.60477. Residuals from the fitted linear model are also stationary. A high R^2 value of 0.8633 is obtained which indicates adequacy of the model. This completes the proof and we conclude that the proposition holds; and also that co-integration models can be used to analyse time series data with high volatility and heteroskedasticity. Such data include share prices and exchange rates. It is recommended that a similar study be undertaken but with a combination of Auto Regressive Moving Average Process (ARMA) and GARCH models.***

JEL Classification: 60K35

Keywords: Co-integration, Ordinary Least Squares, Simulation, GARCH

1 Introduction

Over the recent decades, there have been considerable research on time series analysis, with several scholars building different models for inter-data movements; most of which are volatility models which attempts to describe the movements with time. These models have given an insight into the stochastic nature of the underlying data. Unfortunately, very few of these models have exhaustively discussed the source and existence of these volatilities. Nevertheless, [10], [17], [12], amongst others, concur that volatility posses a great challenge in forecasting of time series data, and that there exists a great need to analyze these shocks in two phases, the short and long-term, in order to capture the movements exhaustively; where imputation of the two relationships can be analyzed. [5] first proposed a procedure, called co-integration, which addresses this issue. The procedure involves estimation of a long-term and short-term relationship in existence. For these purposes and gaps in knowledge, a cointegration analysis procedure has been proposed.

Co-integration is a procedure which seeks to investigate the relationship between one time series and another. An important feature investigated is usually where they share a common drift. The technique measures the equilibrium of the time series in the long run. It utilizes the concept of relationship between non stationary time series, [3].

Earlier, [5] introduced the concept of co-integration. In his study, he showed that if two time series are unit-root non-stationary and their first difference is stationary such that a linear combination of the original non-stationary series is stationary, then the two series are said to be co-integrated. In his work, he was analyzing a balance in an Error Correction Model (ECM) where he realized that there was an imbalance in Integration of Order 0 or stationarity ($I(0)$) and Integration of Order 1 ($I(1)$) series, [6]. On analysis of the series, he realized that a linear combination of non-stationary series formed a stationary one. He then termed this result co-integration.

Later,[7] used co-integration techniques to analyze the risk factors that emanates from the Australian stock market. He identifies five key risk factors; interest rates, dividend yield, corporate profitability, industrial production and global market influences. [7] suggests the inclusion of these factors into the risk management portfolio for all investors. It was noted that regressing non-stationary time series often led to spurious regression, with an exception of co-integration indicating a long-run relationship.

Economics, particularly agriculture have gained from the concept of co-integration.[8] applied these procedures in the analysis of beans markets in Tanzania and Kenya, the main aim being to establish if there was any integration relationship within the markets, and if so, the impact of this integration. Pearson's correlation coefficients were used. The occurrence of spurious regression was as well appreciated and co-integration was opted as a correction mechanism.

These concepts have also been applied in the study of real exchange rate equilibrium and misalignment by [12]. In their study, the Johansen's co-integration test was applied and the error correction model computed. Ordinary Least Squares (OLS) technique was used to estimate co-integration regression parameters, then the model residuals tested for unit root. Based on the tests, [12] concluded that there existed enough evidence which showed that the real exchange rate maintained a level which was above its equilibrium. Nevertheless, within the study period, the country experienced sky-rocketing in the exchange rates market.

[15], in his term project, examines the relationship between the two elements in Turkey by investigating Gross Domestic Product (GDP) as an economic indicator, on a quarterly basis. The theoretical Johansen's test procedure for analyzing time series is discussed in details, but the results presented were analyzed using Schwarz Criteria. Residuals did not show any auto-correlation. There existed no long run association between the two variable. [15] explains this as a failure of existence of a strong financial systems with developed financial markets. It was suggested that, if the same procedure is done in a well developed financial markets, then a perfect positive relation will be observed.

In financial development and economic growth,[11] puts to light the importance of unit-root tests for co-integration in analyzing the inconsistencies arising from recent empirical studies on the field. In his view, though multi-variate methods and not bi-variate have been applied, there still existed a lot of inconsistencies and bias in their estimation. According to [11], unit-root tests were fundamental irrespective of method, whether co-integration or causality analysis. Further, there exists a need for unit root and co-integration tests before causality analysis can be done. Meaning, the results of the causality analyses were pegged on the unit root and co-integration tests. Therefore, according to [11], unit-root test is a powerful fundamental test.

Hedging has also been an area of application for co-integration. [4] used co-integration in their study of hedge funds. They criticized the conventional approaches of model constructions for asset-class indices to be applied in hedging. Seven factors were identified from which a model was built. On analysis of parameter stability, [4] applies the cumulative recursive residual method and plots on a time scale to investigate the reversion of the model parameter in the risk factor model . The factors were co-integrated and hence, they were able to propose a seven factor model to be applied for hedging.

The study by [4] was extrapolated by [17] who analyzed the same hedge funds in view of further examining the validity of the method used in deriving the seven factors which had been suggested by[4] for inclusion in an hedging portfolio. In his research, he reports that [4] did not provide enough evidence to proof that the procedure used in choosing the factors is quite different from the Sharpe and Fama-French which only relies on one characteristics of the entire market. Contrary to [4], [17] bases his parameter stability on the adjusted R^2 statistic. [17] does not mention the reason for his selection of R^2 statistic instead of the cumulative recursive residual. He identifies nine hedge indices which can be included in the hedging strategy. A full rank co-integration in the industry was as well established, and an eight factor model to be used for hedging strategies as the most powerful model, is proposed.

[8]proposes the use of Granger Causality model in the analysis of the beans markets in Tanzania and Kenya. The existence of a co-integration relationship implies that there exists atleast one causal relationship ([8]). Causality test is an indication of the direction in which the variables affect each other. It regresses an explanatory variable (A) against its differenced series and another variable (B). If B is significant, it explains the variation in A and we say that B dynamically causes A.

2 The Main Proposition

In this paper, we make the following proposition;

Proposition

If two time series follow a GARCH(1,1) model, then the two series are cointegrated, and can simply be given as

$$X_t = \alpha + \lambda Y_t \tag{1}$$

where the estimate of λ, $\widehat{\lambda}$, is the OLS estimate of equation 1, and (X_t, Y_t) are the two time series under consideration. Further, the two series will not drift too much from each other.

This proposition is proven by an empirical study in section 3. Tests and definitions of variables and parameters are therefore given in section 2.1 which are key for the proof.

2.1 Definitions

Important representations of the GARCH model is given in section 2.1.1 while the key tests to be used in the analysis is reviewed in section 2.1.2, as follows:

2.1.1 Review of GARCH Representations

According to [1], a GARCH model can generally be given by

$$\omega_t = v_t \sqrt{h_t} \tag{2}$$

in which

$$h_t = \alpha_0 + \alpha_1 \omega_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

where v_t is a white noise with $\sigma_v^2 = var(v_t) = 1$, $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

From equation 3, the term $\alpha_1 \omega_{t-1}^2$ is the Auto Regressive Conditional Heteroskedasticity (ARCH) representation while $\beta_1 h_{t-1}$ is the GARCH representation. The heteroskedasticity comes from the h_t term which is the one period ahead forecast of the variances whereas v_t represents the shocks.

Suppose we introduce a transformation such that $\alpha_1 \omega_{t-1}^2 = \varepsilon_t$, then the series becomes a general case of the AR (1) process. Define Z_t as the AR(1) process given by

$$Z_t = \alpha Z_{t-1} + \varepsilon_t \tag{4}$$

It can be shown that under the null hypothesis of unit root, it can be reduced to a random walk process given by

$$(1-B)Z_t = a_t \tag{5}$$

where a_t is a Gaussian white noise process. It then turns out that Z_t becomes the sum of independent and identically distributed random variables $\{a_i\}_{i=1}^n$. Now let these series form nonwhite noise stationary process X_t given by the transformation

$$X_t = \sum_{t=0}^{\infty} \psi_j a_{t-j} = \psi(B)\, a_t \tag{6}$$

where

$$\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j, \quad \psi_0 = 0, \ and \ \sum_{j=0}^{\infty} j \left| \psi_j \right| < \infty. \tag{7}$$

Then, according to [16], to test for a unit root in this general case, we can fit the following OLS regression

$$Z_t = \phi Z_{t-1} + X_t \tag{8}$$

and consider the estimator

$$\hat{\phi} = \frac{\sum_{t=1}^n Z_{t-1} Z_t}{\sum_{t=1}^n Z_{t-1}^2} \tag{9}$$

under the null hypothesis, $H_0 : \phi = 1$, we have

$$\hat{\phi} = \frac{\sum_{t=1}^n Z_{t-1} Z_t}{\sum_{t=1}^n Z_{t-1}^2} = 1 + \frac{\sum_{t=1}^n Z_{t-1} X_t}{\sum_{t=1}^n Z_{t-1}^2} \tag{10}$$

and

$$n\left(\hat{\phi} - 1\right) = \frac{n^{-1} \sum_{t=1}^n Z_{t-1} X_t}{n^{-2} \sum_{t=1}^n Z_{t-1}^2} \tag{11}$$

The estimates of the parameters can thus be obtained by recursive substitution.

2.1.2 Review of Unit Root tests

The first step in co-integration analysis is to test for stationarity of the two series. It is a condition that for the two series to be co-integrated, they must be non stationary. Three tests (ADF, KPSS and PP tests) are used.

Review of the Augmented Dickey Fuller test

This is a generalized form of the Dickey Fuller test ([2]). It relies on the assumption that the residuals are independent and identically distributed. For a series y_t, ADF uses the model

$$\Delta y_t = \alpha + \lambda t + \eta y_{t-1} + \delta_1 \triangle y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \tag{12}$$

which reduces to a random walk when $\alpha = 0$ and $\lambda = 0$; and a random walk with a drift when $\lambda = 0$. The ADF test thus detrends the series before testing for unit root. It uses lagged difference terms to address serial correlation. The ADF test clearly depends on differenced series. This thus possess a need for another validating test.

An inspection of the p-value also determines whether the null hypothesis of non-stationarity will be accepted. A small p-value[1] leads to the rejection of the null hypothesis. An inspection of the Dickey-Fuller value is as well important as this indicates the mean-reverting property. It is normally a negative value. The larger its absolute value, the lower the chance of occurrence of mean-reverting property.

[1] less than 0.05 or 0.01 depending on the statistician

Review of the Kwiatkowski Philips Schmidt Shin test

Contrary to ADF test, KPSS ([9]) tests for the null hypothesis of level or trend stationarity. It gives a way to specify whether to test with a trend or without, in its test statistic. A regression model with linear combination of a deterministic trend[2], a random walk and a stationary residual series

$$Y_t = \alpha + \beta t + \lambda \sum_{i=1}^{t} \varepsilon_i + \delta_t \tag{13}$$

is used where δ_t is stationary, βt is the trend component while $\sum_{i=1}^{t} \varepsilon_i$ is the random walk. $\beta t = 0$ if we assume a without-trend regression. The series in equation 13 will be stationary if $\lambda = 0$. Regression is used to obtain the estimate of δ_t, that is $\widehat{\delta}_t$, from which we compute

$$\Omega_{resid} = \sum_{i=1}^{t} \widehat{\delta}_i \tag{14}$$

The test statistic for KPSS test is then calculated as

$$R = \frac{\sum_{i=1}^{n} \Omega_i^2}{n^2 \widehat{\theta}_T^2} \tag{15}$$

where the spectral density function estimator

$$\widehat{\theta}_T^2 = \widehat{\sigma}_\delta^2 + 2 \sum_{k=1}^{T} \left(1 - \frac{k}{T-1}\right) \widehat{\omega}_t \tag{16}$$

is a linear combination of the variance estimator $\widehat{\sigma}_\delta^2$ and covariance estimator

$$\widehat{\omega}_t = \frac{\sum_{t=k+1}^{n} \delta_t \delta_{t-k}}{n} \tag{17}$$

The test turns to a prudential choice of T in equation 16 above.

Review of the Phillip Perron test

The Phillips Perron approach ([13]) applies a nonparametric correction to the standard ADF test statistic, allowing for more general dependence in the errors, including conditional heteroskedasticity. If there were strong concerns over heteroskedasticity in the ADF residuals, this might influence an analyst to go for PP. If the addition of lagged differences in ADF did not remove serial correlation, then this again might suggest PP as an alternative.

2.1.3 The Engle-Granger two-step Method for Testing Co-integration

Two series A_t and B_t are co-integrated if it can be written in the form

$$A_t + \lambda B_t = \Theta_t \tag{18}$$

where Θ_t is stationary. [3] proposes a two-step procedure for this estimation. In the first step, we ensure the individual series are $I(1)$. If not, apply differencing to attain $I(1)$. Estimate a linear relationship using OLS between the two $I(1)$ series. That is, we estimate λ in equation 18 above. Secondly, we extract the residuals from the estimated *OLS* equation and test for stationarity. Co-integration exists if the residuals obtained from the *OLS* estimation are stationary.

3 Empirical Analyses and Results

The proof of the main proposition in section 2 is done as an empirical study in this section, and proceeds as follows:

3.1 Proof

A GARCH(1,1) is used to simulate data to be used in the analysis. The choice of the simulation procedure is aimed at capturing the heteroskedasticity which has been noted to exist in a series of returns[3], where volatility clustering exists. The classical inferential property of a high-low series is violated and a series of high levels tend to cluster together. In which case, variances tend to be related across different periods and hence leading to the result

$$var(X_t) = E\left(X_t^2\right) \tag{19}$$

that is the variance of the series at a given time, say t, is the same as the the expectation of the square of the series. This is the basis for heteroskedasticity and hence an indication that auto-correlation still exists in the squares of the returns.

Figure 1 below shows the plot of the simulated series. The values of α_0, α_1 and β_1 are chosen arbitrarily to satisfy the conditions[4] for a GARCH(1,1) model.

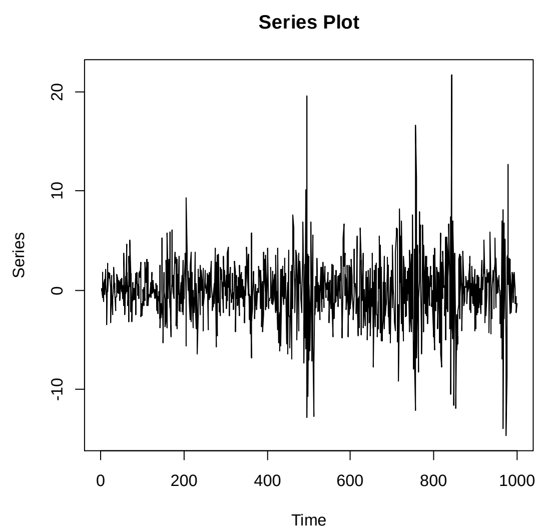

Figure 1: A Plot of the Simulated Series Using GARCH(1,1).

Another GARCH(1,1) series is simulated and its plot superimposed[5] on the first series. An inspection of the superimposed plot suggest an existence of co-integration, as in figure 2 below.

[2] if test statistic is with a trend
[3] Particularly in price returns
[4] $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.
[5] figure 2

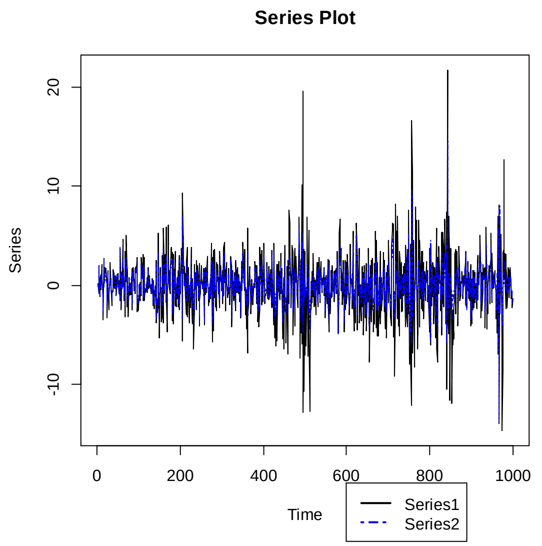

Figure 2: The Second Simulated GARCH(1,1) Series is Superimposed onto the Series Plot in Figure 1.

Before analysing the simulated series, an investigation of the GARCH properties should be done. All the properties must be met before the data can be used for any analysis. Being a GARCH model, it is expected to exhibit a high auto-correlation at lag one and insignificant correlation in higher orders. This translates to a spike at lag one in the ACF[6] plot which is evident in figure 3. An investigation on the heteroskedastic property of the series can as well be investigated from figure 3. An inspection of the ACF's of the squares of the two series indicate existence of serial correlation, an indication of heteroskedasticity property. The mean and variance properties of a GARCH model are all satisfied.

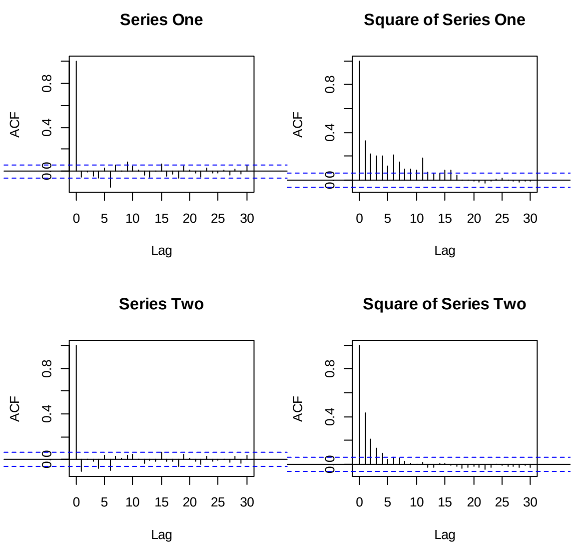

Figure 3: Auto Correlation Functions for the Series and Their Respective Squares, in that Order.

The model used for simulation should yield a stationary series, by definition. This is evident as the series plot resemble a mean zero white noise process. Nevertheless, unit root tests such as ADF, KPSS and PP tests discussed previously in section 2.1.2 are used to investigate stationarity in the series.

The ADF test tests the null hypothesis of non stationarity. The p-value indicates the amount of evidence against the null hypothesis. For the two simulated series, the ADF test output is as shown in table 1 below;

Table 1: Augmented Dickey Fuller test output for the two simulated series. Omega Represents the First Series Whereas Omega1 is the Second Series.

Data	Dickey-Fuller	Lag	P-value
Omega	-9.726	9	0.01
Omega1	-10.0075	9	0.01

From table 1, the p-value is 0.01. At 5% level of significance, the classical probability rule dictates rejection of the null hypothesis. The decision rule is that there exists sufficient evidence that the series might be stationary. Also the absolute values of the Dickey-Fuller values are relatively low thus we may conclude both series may be mean-reverting. In such a case, co-integration may exist. But ADF test has two downsides which has to be addressed

1. The model for an ADF test uses the differenced series.
2. It assumes that the residuals are independent and identically distributed.

KPSS test addresses the differenced model in ADF test. It tests the null of stationarity with respect to an existing trend. It is similar to the ADF test but does not detrend the series. That is, the long term general movement of the series is preserved. Table 2 presents the output of this test.

Table 2: KPSS test output for the two simulated series

Data	KPSS Level	Lag	P-value
Omega	0.1328	7	0.1
Omega1	0.2012	7	0.1

The test yield a p-value of 0.1. Following the same decision rule, we fail to reject the null hypothesis at 5% significance level and conclude that the series might be stationary.

All the above tests are parametric. They all assume independence and identical ditribution of residuals. A non-parametric test need to be done, which can be used in presence of heteroskedasticity. Philip Perron test is the best alternative. To ascertain these decisions, a Philip Perron test is done whose results are presented in table 3. This test, just like ADF, test the null of non stationarity. Having the same p-value of 0.01 as that in the ADF test, the same decision rule is followed.

Table 3: Phillip Perron test output for the two simulated series

Data	Dickey-Fuller Z_α	Lag	P-value
Omega	-955.7931	7	0.01
Omega1	-1025.337	7	0.01

It can finally be concluded that the two series are stationary and therefore satisfying all the underlying model assumptions. The requirement of the two series being integrated of order 1, $I(1)$, is fully satisfied since the two series were simulated from a GARCH(1,1) series, and all the tests above indicate stationarity satisfaction. A relatively low Dickey-Fuller absolute values is an indication of a possibility of co-integration. It then remains to estimate the parameter λ from equation 18 to estimate co-integration relationship.

Equation 18 can as well be rearranged and written as

$$A_t = \Theta_t - \lambda B_t \tag{20}$$

[6] Auto Correlation Function

which is a linear model in A_t and B_t. Classical Ordinary Least Squares (COLS[7]) can therefore be used to estimate the parameter λ. A linear model is fitted which estimates λ to be 1.60477. This value remains an estimate till the residuals of the fitted model is tested for stationarity. Figure 4 below is a plot of the residuals. A visual inspection suggests stationarity as it has the form of a mean-zero white noise. Comparing it to figure 1, the residual series is less random about the mean and hence its variance approaches unity.

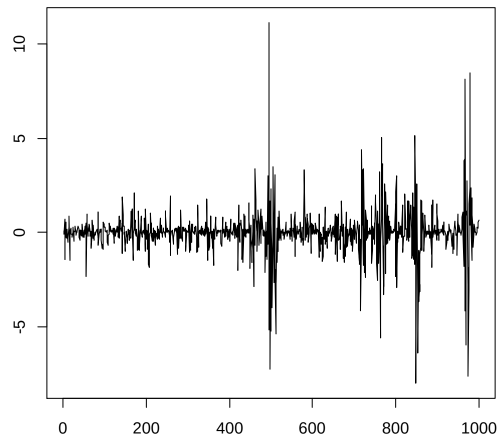

Figure 4: A Plot of the Residual Series. A Visual Inspection of the Plot Indicates a Stationary Process. It is a Replica of the Purely Random Process.

Similar, stationarity tests are done on the residual series and table 4 below contains the outputs from the three tests.

Table 4: Residual Series Tests for Stationarity

Test	Test Statistic	Lag	P-value
Augmented Dickey-Fuller	-12.338	9	0.01
KPSS	0.0728	7	0.1
Phillips-Perron Unit Root	-1011.148	7	0.01

Using the same decision rule, the residual series is stationary. Therefore a co-integration relation exists. It therefore remains to test the significance of the coefficient of co-integration. The hypotheses under consideration is

$$\begin{array}{c} H_0 : \lambda = 0 \\ vs \\ \lambda \neq 0 \end{array} \tag{21}$$

From the summary of the fitted model presented in table 5, the p-value is infinitesimally small. The null hypothesis is rejected and concluded that the co-integration factor is different from zero.

[7]Classical Ordinary Least Squares

Table 5: Summary of the Fitted Model

	Estimate	Std. Error	t value	$pr(>\|t\|)$
Intercept	-0.05039	0.03847	-1.31	0.191
Omega1	1.60477	0.02022	79.38	2.0×10^{-16}
Residual Standard Error : 1.217 on 998 df				
Multiple R^2 : 0.8633				
Adjusted R^2 : 0.8631				
P-value = 2.2×10^{-16}				

NB : The fitted model is $A_t = \alpha + \beta B_t$ where $A_t = Omega$ and $B_t = Omega1$

The Model

The fitted model can be given by

$$A_t = -0.05039 - 1.60477 B_t \tag{22}$$

or

$$-0.05039 = A_t + 1.60477 B_t \tag{23}$$

Conclusion

The residual standard error is considerably small. It can be concluded that the co-integration coefficient is significant in the model. The R^2 value is 0.8633 and the adjusted R^2 is 0.8631. The two values are approximately the same, an indication that the sampled data characteristic does not differ much from the population data. The overall model is therefore significant.

3.2 Discussion

There exists a co-integration relation between the two series. A change in the first series results in a change in the second series by 1.60477 units in the same direction. The series have a long-run equilibrium relationship. A negative small drift means that series A_t drifts in the opposite direction upon a movement in series B_t, and cannot drift too far apart from the equilibrium because economic forces will act to restore the equilibrium relationship . This therefore completes the proof of the first part of our proposition that the two series will not drift too far from each other.

Next, we note that at equilibrium the value of A is 1.60477 times the value of B. If A and B are prices, then when the price of A exceeds 1.60477 times the value of B, we expect either;

1. The price of A to decrease so as to reach the point of equilibrium in the near future, or

2. The price of B to be pushed up for it to balance at equilibrium with that of A.

A small value of the residual standard error indicates that most of the variability in the data is captured by the co-integration model. This is an indication of the model's ability to capture intra-data clustering. Future shocks which might be experienced are therefore easily captured in the forecasts. It therefore indicates a high level of significance in the forecasts of this model.

Notably, the R^2 and the adjusted R^2 values are almost equal. It is a good indication of high precision forecasts. It is an indication that the characteristic exhibited by series is persistent. A sample of the set of data will always have the same characteristic as the population of the data. This is in line with heteroskedasticity as data tend to cluster in a similar manner throughout the data. It is therefore an indication of the reliability of the forecasts obtained if the model was to be used. This completes the final proof of existence of a cointegration relationship.

It can be concluded therefore that if two series follow a GARCH(1,1) model, they are cointegrated and they do not drift too far from each other. It can as well be concluded that co-integration is a powerful tool in the analysis of time series data and can be used to obtain optimal forecasts. A co-integration relationship can therefore be used to explain the source of variability in one series if the variability in the other series is known. Finally, heteroskedasticity does not influence the predictability of a co-integration model. Therefore, highly significant forecasts can still be obtained from a highly heteroskedastic series. This wraps up the proof to the main proposition in section 2. Recommendations for further research is given in section 3.3.

3.3 Recommendations

It is recommended that the study be done on such time series as stock prices or exchange rates to evaluate the applicability of the main proposition in section 2. As much as GARCH models captures heteroskedasticity, it is still a conditional variance model. GARCH models are therefore most appropriate for squared return series. On the other hand, ARMA[8] models are built on conditional expectation. They are therefore perfect for a normal return series. Therefore, A combination of the two series will be most appropriate when the series exhibit correlation in both first and second order. It is therefore recommended that a similar study be undertaken with simulation models being a combination of ARMA and GARCH models.

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Appendices

Author Profile

Rotich Titus Kipkoech is an ambitious goal-oriented professional and academician with strong track-record of delivering top performance. He possesses large spectrum of experience in pure and applied mathematics, pure and applied statistics, corporate finance, actuarial science and data analysis. He has excellent project management skills. He is currently working on his Doctor of Philosophy **(PhD)**; has done Master of Science (Msc.) degree in Applied Statistics. Successfully cleared actuarial degree course with **first class honours.**

Nomenclature

ADF	Augmented Dickey Fuller
ARCH	Auto Regressive Conditional Heteroskedasticity
ARMA	Auto Regressive Moving Average Process
COLS	Classical Ordinary Least Squares
ECM	Error Correction Model
GARCH	Generalized AutoRegressive Conditional Heteroskedasticity
GDP	Gross Domestic Product
I(0)	Integration of Order 0 or stationarity
I(1)	Integration of Order 1
KPSS	Kwiatkowski Philips Schmidt Shin
OLS	Ordinary Least Squares
PP	Phillips Perron

[8] Auto Regressive Moving Average Process