Estimation of a Co-integration Model Using Ordinary Least Squares (A Case Study of the Kenyan Market)

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Abstract: ***From a previous study on co-integration, it has been proposed that if two series follow a Generalized Auto-regressive Conditional Heteroskedasticity (GARCH(1,1)) model, then the two series are co-integrated. However, the proposition was carefully proved using a simulation study. In this paper, a proof of this proposition is presented by applying a case study of the Kenyan market. The dollar exchange and Interbank lending rates in Kenya are analyzed. The procedure described in the simulation study is carefully followed, and consequently all the tests and justifications given follow. Unit root tests (Augmented Dickey Fuller (ADF), Phillips Perron (PP) and Kwiatkowski Philips Schmidt Shin (KPSS)) on the data indicates non-stationarity. Differencing is applied to attain stationarity. Co-integrating factor is then estimated to be -0.490747, with its residuals being stationary. elatively same R^2 and adjusted R^2 values indicates adequacy of the model. This ascertains the proposition; and also that co-integration models can be used to analyse time series data with high volatility and heteroskedasticity. It is recommended that a similar study be undertaken with a combination of Auto Regressive Moving Average Process (ARMA) and GARCH models to capture both conditional variance and conditional expection properties.***

Keywords: Co-integration, Ordinary Least Squares, Unit Root, GARCH

1 Introduction

Researches have been done on time series analysis and models built in an attempt to examine the sources and impact of volatility in time series data. Forecasting has also been an area of interest with scholars trying to increase the precision of model predictions as much as possible. [11], [19], [14], amongst others, concur that volatility posses a great challenge in forecasting of time series data, and that there exists a great need to analyze these shocks in two phases, the short and long-term, in order to capture the movements exhaustively; where imputation of the two relationships can be analyzed.

Cointegration was first proposed by [5] to address the two-phased analysis procedure in an effort of capturing volatility, which involves estimation of a long-term and short-term relationship in existence. It is a procedure which seeks to investigate the relationship between one time series and another. An important feature investigated is usually where they share a common drift. The technique measures the equilibrium of the time series in the long run. It utilizes the concept of relationship between non stationary time series, [3].

Cointegration had been introduced earlier by [5] in his work where it was showed that if two time series are unit-root non-stationary and their first difference is stationary such that a linear combination of the original non-stationary series is stationary, then the two series are said to be co-integrated. [5]was analyzing a balance in an Error Correction Model (ECM) where it was established that there was an imbalance in a stationary ($I(0)$[1]) and an integrated of order one ($I(1)$) series, [6]. On analysis, it was appreciated that a linear combination of non-stationary series formed a stationary one. This result was then termed co-integration.

[12]applied co-integration techniques to estimate a co-integration model to track index and compare the co-integration approach to the time portfolio optimization approach. Some indices (FTSE100, DJ Industrial, DJ Composite Average) were used. A carefully done empirical analysis revealed that both the approaches yielded correct results. However, the co-integration approach was preferred due to its ability to capture data even with high volatility. Nevertheless, no particular model was preferred for index tracking analysis.

Earlier, [9] applied these procedures in the analysis of beans markets in Tanzania and Kenya, the main aim being to establish if there was any integration relationship within the markets, and if so, the impact of this integration. Pearson's correlation coefficients were used. The occurrence of spurious regression was as well appreciated and co-integration was opted as a correction mechanism.

[7]investigated co-integration relationship that existed between the swap spreads and various rates such as the London Interbank Offered Rate (LIBOR) rates, US corporate credit spreads and the treasury yield curve. Evidence of the existence of co-integration were found. In the study, it was showed that under the ECM[2] framework, the daily swap spreads reacted to the corrective long-run forces except from the short-term fluctuations in the variables. [7] concluded that the swap spread had a negative effect only on one measure, the treasury yield curve, but positive in all the other rates.

Later, [11] in his master's thesis in economics, analyzes the existence of co-integration in prices on different natural gas hubs in the European region. The study focused on four major hubs; NBP, Zeebrugge, TTF and Bunde. [11] estimates a co-integration model to investigate the pairwise relationships. Of these hubs, NBP and Zeebrugge were found to have the strongest relation due to their direct pipeline connecting the two hubs. This result was further ascertained by the ECM which indicated an exogenous connection between these two hubs.

These concepts have also been applied in the study of real exchange rate equilibrium and misalignment by [14]. In their study, the Johansen's co-integration test was applied and the error correction model computed. Ordinary Least Squares (OLS) technique was used to estimate co-integration regression parameters, then the model residuals tested for unit root. Based on the tests, [14] concluded that there existed enough evidence which showed that the real exchange rate maintained a level which was above its equilibrium. Nevertheless, within the study period, the country experienced sky-rocketing in the exchange rates market.

In financial development and economic growth, [13] puts to light the importance of unit-root tests for co-integration in analyzing the in-

[1]integrated of order zero
[2]Error Correction Model

consistencies arising from recent empirical studies on the field. In his view, though multivariate methods and not bi-variate have been applied, there still existed a lot of inconsistencies and bias in their estimation. According to [13], unit-root tests were fundamental irrespective of method, whether co-integration or causality analysis. Further, there exists a need for unit root and co-integration tests before causality analysis can be done. Meaning, the results of the causality analyses were pegged on the unit root and co-integration tests. Therefore, according to [13], unit-root test is a powerful fundamental test.

Hedging has also been an area of application for co-integration. [4] used co-integration in their study of hedge funds. They criticized the conventional approaches of model constructions for asset-class indices to be applied in hedging. Seven factors were identified from which a model was built. On analysis of parameter stability, [4] applies the cumulative recursive residual method and plots on a time scale to investigate the reversion of the model parameter in the risk factor model . The factors were co-integrated and hence, they were able to propose a seven factor model to be applied for hedging.

The study by [4] was extrapolated by [19] who analyzed the same hedge funds in view of further examining the validity of the method used in deriving the seven factors which had been suggested by [4] for inclusion in an hedging portfolio. In his research, he reports that [4] did not provide enough evidence to proof that the procedure used in choosing the factors is quite different from the Sharpe and Fama-French which only relies on one characteristics of the entire market. Contrary to [4], [19] bases his parameter stability on the adjusted R^2 statistic. [19] does not mention the reason for his selection of R^2 statistic instead of the cumulative recursive residual. He identifies nine hedge indices which can be included in the hedging strategy. A full rank co-integration in the industry was as well established, and an eight factor model to be used for hedging strategies as the most powerful model, is proposed.

2 Organization of the Paper

In this paper, the proposition in section (3) is proved by a case study on the Kenyan market, presented in section (4). Tests and definitions of variables and parameters are therefore given in section (3.1) which are key for the proof. Important representations of the GARCH model is given in section (3.1.1) while the key tests to be used in the analysis is reviewed in section 3.1.2. The paper concludes by a discussion of the results and recommended areas for further research.

3 The Proposition

If two time series follow a GARCH(1,1) model, then the two series are co-integrated, and can simply be given as

$$X_t = \alpha + \lambda Y_t \tag{1}$$

where the estimate of λ, $\widehat{\lambda}$, is the Ordinary Least Squares (OLS)[3] *estimate of Equation (1), and (X_t, Y_t) are the two time series under consideration. Further, the two series will not drift too much from each other.*

[3] *Ordinary Least Squares*
[4] Generalized Auto Regressive Conditional Heteroskedasticity
[5] *Ordinary Least Squares*

3.1 Definitions

3.1.1 *Review of GARCH Representations*

According to [1], a GARCH model can generally be given by

$$\omega_t = v_t \sqrt{h_t} \tag{2}$$

in which

$$h_t = \alpha_0 + \alpha_1 \omega_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

where v_t is a white noise with $\sigma_v^2 = var(v_t) = 1$, $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

From Equation (3), the term $\alpha_1 \omega_{t-1}^2$ is the Auto Regressive Conditional Heteroskedasticity (ARCH) representation while $\beta_1 h_{t-1}$ is the GARCH[4] representation. The heteroskedasticity comes from the h_t term which is the one period ahead forecast of the variances whereas v_t represents the shocks.

Suppose we introduce a transformation such that $\alpha_1 \omega_{t-1}^2 = \varepsilon_t$, then the series becomes a general case of the AR (1) process. Define Z_t as the AR(1) process given by

$$Z_t = \alpha Z_{t-1} + \varepsilon_t \tag{4}$$

It can be shown that under the null hypothesis of unit root, it can be reduced to a random walk process given by

$$(1-B)Z_t = a_t \tag{5}$$

where a_t is a Gaussian white noise process. It then turns out that Z_t becomes the sum of independent and identically distributed random variables $\{a_i\}_{i=1}^n$. Now let these series form nonwhite noise stationary process X_t given by the transformation

$$X_t = \sum_{t=0}^{\infty} \psi_j a_{t-j} = \psi(B) a_t \tag{6}$$

where

$$\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j, \quad \psi_0 = 0, \ and \ \sum_{j=0}^{\infty} j \left|\psi_j\right| < \infty. \tag{7}$$

Then, according to [18], to test for a unit root in this general case, we can fit the following OLS[5] regression

$$Z_t = \phi Z_{t-1} + X_t \tag{8}$$

and consider the estimator

$$\hat{\phi} = \frac{\sum_{t=1}^n Z_{t-1} Z_t}{\sum_{t=1}^n Z_{t-1}^2} \tag{9}$$

under the null hypothesis, $H_0 : \phi = 1$, we have

$$\hat{\phi} = \frac{\sum_{t=1}^n Z_{t-1} Z_t}{\sum_{t=1}^n Z_{t-1}^2} = 1 + \frac{\sum_{t=1}^n Z_{t-1} X_t}{\sum_{t=1}^n Z_{t-1}^2} \tag{10}$$

and

$$n\left(\hat{\phi} - 1\right) = \frac{n^{-1} \sum_{t=1}^n Z_{t-1} X_t}{n^{-2} \sum_{t=1}^n Z_{t-1}^2} \tag{11}$$

The estimates of the parameters can thus be obtained by recursive substitution.

3.1.2 *Review of Unit Root tests*

The first step in co-integration analysis is to test for stationarity of the two series. It is a condition that for the two series to be co-integrated, they must be non stationary. Three tests (ADF, KPSS and PP tests) are used.

Review of the Augmented Dickey Fuller test

This is a generalized form of the Dickey Fuller test ([2]). It relies on the assumption that the residuals are independent and identically distributed. For a series y_t, ADF uses the model

$$\Delta y_t = \alpha + \lambda t + \eta y_{t-1} + \delta_1 \triangle y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \quad (12)$$

which reduces to a random walk when $\alpha = 0$ and $\lambda = 0$; and a random walk with a drift when $\lambda = 0$. The ADF[6] test thus detrends the series before testing for unit root. It uses lagged difference terms to address serial correlation. The ADF test clearly depends on differenced series. This thus possess a need for another validating test.

An inspection of the p-value also determines whether the null hypothesis of non-stationarity will be accepted. A small p-value[7] leads to the rejection of the null hypothesis. An inspection of the Dickey-Fuller value is as well important as this indicates the mean-reverting property. It is normally a negative value. The larger its absolute value, the lower the chance of occurrence of mean-reverting property.

Review of the Kwiatkowski Philips Schmidt Shin test

Contrary to ADF test, KPSS ([10]) tests for the null hypothesis of level or trend stationarity. It gives a way to specify whether to test with a trend or without, in its test statistic. A regression model with linear combination of a deterministic trend[8], a random walk and a stationary residual series

$$Y_t = \alpha + \beta t + \lambda \sum_{i=1}^{t} \varepsilon_i + \delta_t \quad (13)$$

is used where δ_t is stationary, βt is the trend component while $\sum_{i=1}^{t} \varepsilon_i$ is the random walk. $\beta t = 0$ if we assume a without-trend regression. The series in Equation (13) will be stationary if $\lambda = 0$. Regression is used to obtain the estimate of δ_t, that is $\widehat{\delta_t}$, from which we compute

$$\Omega_{resid} = \sum_{i=1}^{t} \widehat{\delta_i} \quad (14)$$

The test statistic for KPSS test is then calculated as

$$R = \frac{\sum_{i=1}^{n} \Omega_i^2}{n^2 \widehat{\theta}_T^2} \quad (15)$$

where the spectral density function estimator

$$\widehat{\theta}_T^2 = \widehat{\sigma}_\delta^2 + 2 \sum_{k=1}^{T} \left(1 - \frac{k}{T-1}\right) \widehat{\omega}_t \quad (16)$$

is a linear combination of the variance estimator $\widehat{\sigma}_\delta^2$ and covariance estimator

$$\widehat{\omega}_t = \frac{\sum_{t=k+1}^{n} \delta_t \delta_{t-k}}{n} \quad (17)$$

The test turns to a prudential choice of T in Equation (16) above.

Review of the Phillip Perron test

The Phillips Perron approach ([15]) applies a nonparametric correction to the standard ADF test statistic, allowing for more general dependence in the errors, including conditional heteroskedasticity. If there were strong concerns over heteroskedasticity in the ADF residuals, this might influence an analyst to go for PP. If the addition of lagged differences in ADF did not remove serial correlation, then this again might suggest PP as an alternative.

3.1.3 *The Engle-Granger two-step Method for Testing Co-integration*

Two series A_t and B_t are co-integrated if it can be written in the form

$$A_t + \lambda B_t = \Theta_t \quad (18)$$

where Θ_t is stationary. [3] proposes a two-step procedure for this estimation. In the first step, we ensure the individual series are $I(1)$[9]. If not, apply differencing to attain $I(1)$. Estimate a linear relationship using OLS between the two $I(1)$ series. That is, we estimate λ in Equation (18) above. Secondly, we extract the residuals from the estimated *OLS* equation and test for stationarity. Co-integration exists if the residuals obtained from the *OLS* estimation are stationary.

4 Empirical Analyses and Results

The proof of the main proposition in section (3) is done as an empirical study in this section, and proceeds as follows:

4.1 Proof

Data on exchange and interbank lending rates are used in the analysis, which are investigated for GARCH(1,1) properties. The study is aimed at capturing heteroskedasticity property, a characteristic which commonly exist in time series data[10], where volatility clustering occur. In such a case, the statistical property of data gradually decreasing from high to low densities and vice versa does not apply. Rather, high densities tend to cluster together, in which case, variances tend to be related across different periods and hence leading to the result:

$$var(X_t) = E\left(X_t^2\right) \quad (19)$$

That is the variance of the series at a given time, say t, is the same as the the expectation of the square of the series. This is the basis for heteroskedasticity and hence an indication that auto-correlation still exists in the squares of the returns.

[6] Augmented Dickey Fuller test
[7] less than 0.05 or 0.01 depending on the statistician
[8] if test statistic is with a trend
[9] Integrated of Order 1
[10] Particularly in price returns

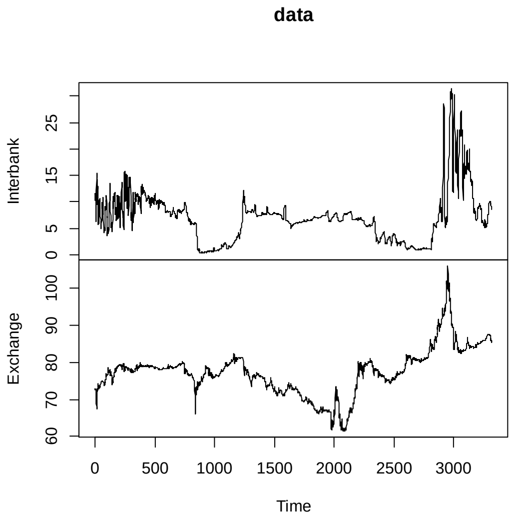

Figure 1: A Plot of the Exchange and Lending Rates. The Series is Plotted Against Time. It is Clearly a Non-stationary Series by Visual Inspection.

Figure (1) above shows the plot of the exchange and interbank lending rates. The movements of the two series seem to be similar, though it has been plotted on different scales.

The two series are plotted on the same scale for clearity in inspection and visual comparison. This is done by superimposing the lending rates series on the exchange rates series. A visual inspection of the superimposed plot suggest an existence of co-integration, as in Figure (2). Nevertheless, a visual inspection does not give any concrete judgment.

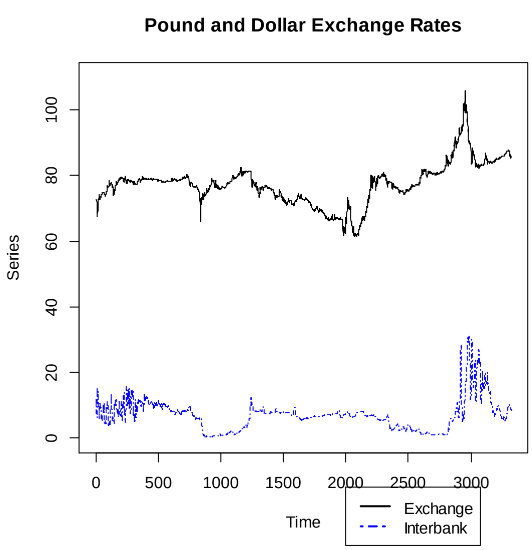

Figure 2: The Interbank Lending Rate is Superimposed onto the Exchange Rate Time Plot. A Visual Inspection Suggests a Co-integration Relationship

Since the series are non-stationary, differencing is applied to achieve stationarity. Also, following the proposition, an investigation of the GARCH properties should be done, to ensure all properties are satisfied before any other analysis can be done. Being a GARCH model, it is expected to exhibit a high auto-correlation at lag one and insignificant correlation in higher orders. This translates to a spike at lag one in the Auto-correlation Function (ACF) plot which is evident in Figure (3). An investigation on the heteroskedasticity property of the series can as well be investigated from Figure (3).

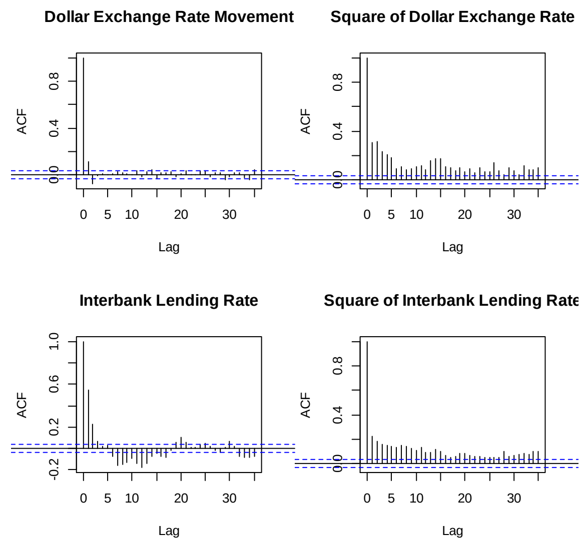

Figure 3: Auto Correlation Functions for the Series and Their Respective Squares.

An inspection of the ACF's of the squares of the two series indicate existence of serial correlation, an indication of heteroskedasticity property. The mean and variance properties of a GARCH model are satisfied. However, the ACF of the interbank lending rate does not seem to depict a classic GARCH(1,1) property.

The ACF for the interbank lending rates indicates higher order seasonal and serial correlation. It is important to remove high serial correlations in the data before further analysis. Logarithmic transformation removes this in the data. The ACF of the log-transformed series is given in Figure (4).

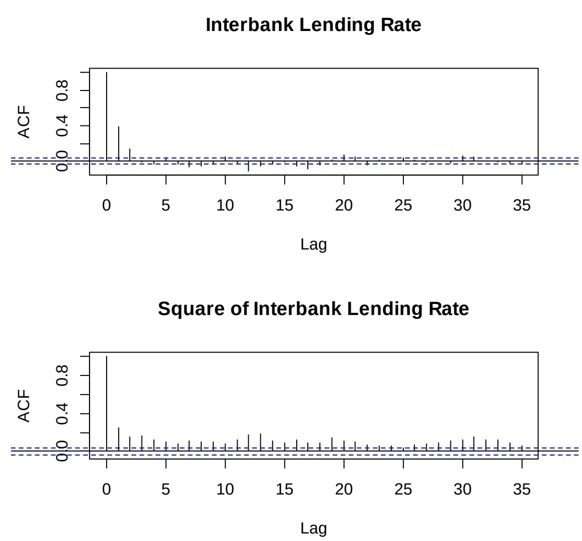

Figure 4: ACF of the Log-transformed Interbank Lending Rate

GARCH(1,1) conditions are satisfied. A visual inspection suggests that the two series are non-stationary. Nevertheless, unit root tests such as ADF, KPSS and PP tests discussed previously in section (3.1.2) are used to investigate stationarity in the series.

The ADF test tests the null hypothesis of non stationarity. The p-value indicates the amount of evidence against the null hypothesis. For the two simulated series, the ADF test output is as shown in Table (1).

Table 1: Augmented Dickey Fuller Test Output for the Two Series.

Data	Dickey-Fuller	Lag	P-value
Exchange Rate	-2.1937	14	0.4963
log of Interbank Rate	-2.5287	14	0.3545

From Table (1), the p-values are higher than 0.05. At 5% level of significance, the classical probability rule dictates failure to reject the null hypothesis. The decision rule is that there exists sufficient evidence that the series might be non-stationary. Differencing is the common procedure for stationarity attainment. The two series are differenced and ADF output for the differenced series presented in Table (2).

Table 2: ADF of the Differenced Series

Data	Dickey-Fuller	Lag	P-value
diff(Ex. Rate)	-14.2119	14	0.01
diff(log(Int. Rate))	-16.364	14	0.01

From the p-values, the null hypothesis of non stationarity is rejected. Also the absolute values of the Dickey-Fuller values are relatively low thus we may conclude both series may be mean-reverting. In such a case, co-integration may exist. But ADF test has two downsides which has to be addressed.

1. The model for an ADF test uses the differenced series.
2. It assumes that the residuals are independent and identically distributed.

KPSS test addresses the differenced model in ADF test. It tests the null of stationarity with respect to an existing trend. It is similar to the ADF test but does not detrend the series. That is, the long term general movement of the series is preserved. Table (3) presents the output of this test.

Table 3: KPSS Test Output for the Two Time Series

Data	KPSS Level	Lag	P-value
diff(Ex. Rate)	0.0755	13	0.1
diff(log(Int. Rate))	0.0477	13	0.1

The test yield a p-value of 0.1. Following the same decision rule, we fail to reject the null hypothesis at 5% significance level and conclude that the series might be stationary.

All the above tests are parametric. They all assume independence and identical distribution of residuals. A non-parametric test need to be done, which can be used in presence of heteroskedasticity. Philip Perron test is the best alternative. To ascertain these decisions, a Philip Perron test is done whose results are presented in Table (4).

Table 4: Phillip Perron Test Output for the Two Series

Data	Dickey-Fuller	Lag	P-value
diff(Ex. Rate)	-2765.71	9	0.01
diff(log(Int. Rate))	-1864.619	9	0.01

This test, just like ADF, test the null of non stationarity. Having the same p-value of 0.01 as that in the ADF test, the same decision rule is followed.

It can finally be concluded that the two differenced series are stationary and therefore satisfying all the underlying model assumptions. The requirement of the two series being integrated of order 1, $I(1)$, is fully satisfied since stationarity has been achieved by differencing the series once, indicated by the above tests. A relatively low Dickey-Fuller absolute values is an indication of a possibility of co-integration. It then remains to estimate the parameter λ from Equation (18) to estimate co-integration relationship.

Equation (18) can as well be rearranged and written as

$$A_t = \Theta_t - \lambda B_t \qquad (20)$$

which is a linear model in A_t and B_t. Classical Ordinary Least Squares (COLS) can therefore be used to estimate the parameter λ. A linear model is fitted which estimates λ to be -0.490747. This value remains an estimate till the residuals of the fitted model is tested for stationarity. Figure (5) below is a plot of the residuals. A visual inspection suggests stationarity as it has the form of a mean-zero white noise. The residual series is less random about the mean and hence its variance approaches unity.

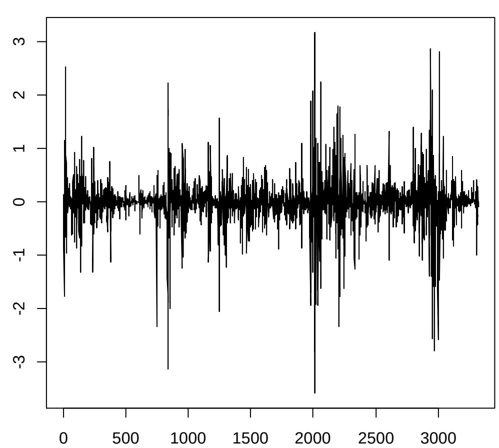

Figure 5: A Plot of the Residual Series. A Visual Inspection of the Plot Indicates a Stationary Process. It is a Replica of the Purely Random Process.

Similarly, stationarity tests are done on the residual series and Table (5) below contains the outputs from the three tests.

Table 5: Residual Series Tests for Stationarity

Test	Test Statistic	Lag value	P-value
ADF	-14.0361	14	0.01
KPSS	0.0794	13	0.1
PP	-2775.608	9	0.01

Using the same decision rule, the residual series is stationary. Therefore a co-integration relation exists. It therefore remains to test the significance of the coefficient of co-integration. The hypotheses under consideration is

$$\begin{aligned} H_0 &: \lambda = 0 \\ &vs \\ H_1 &: \lambda \neq 0 \end{aligned} \tag{21}$$

From the summary of the fitted model presented in Table (6), the p-value is infinitesimally small.

Table 6: Summary of the Fitted Model

	Estimate	Std. Error	t value	$pr(>\|t\|)$
Intercept	0.003863	0.006684	0.578	0.563
diff(ILR)	-0.490747	0.110065	-4.459	8.52×10^{-6}
Residual Standard Error : 0.3852 on 3319 df				
Multiple R^2 : 0.005954				
Adjusted R^2 : 0.005655				
P-value = 8.519×10^{-6}				

.

NB : The fitted model is $A_t = \alpha + \beta B_t$ where $A_t = diff(Exchange)$ and $B_t = diff(log(Interbank))$

The null hypothesis is rejected and concluded that the co-integration factor is different from zero. Further, the intercept is not significant.

The Model

The fitted model can be given by

$$A_t = -0.490747 B_t \tag{22}$$

or

$$A_t + 0.490747 B_t = 0 \tag{23}$$

Conclusion

The residual standard error is considerably small. It can be concluded that the co-integration coefficient is significant in the model. However, despite the R^2 and the adjusted R^2 values being small, the two values are approximately the same, an indication that the sampled data characteristic does not differ much from the population data. The overall model is therefore significant.

4.2 Discussion

There exists a co-integration relation between the two series. A unit change in exchange rate results in a change in the interbank lending rate by 0.490747 units in the opposite direction. Though the series has a long-run equilibrium relationship, a short term relationship is more useful as it is evident from the R^2 and the adjusted R^2 values. An intercept of zero indicates that the two series, the exchange and interbank lending rates do not have any drift, and if it occurs, they cannot drift too far apart from the equilibrium because economic forces will act to restore the equilibrium relationship . This therefore completes the proof of the first part of the proposition that the two series will not drift too far from each other.

Next, it is noted that at equilibrium the value of A is 0.490747 times the value of B. Since A and B are interbank and exchange rates respectively then when the price of A exceeds 0.490747 times the value of B, we expect either;

1. The price of A to decrease so as to reach the point of equilibrium in the near future, or
2. The price of B to be pushed up for it to balance at equilibrium with that of A.

A small value of the residual standard error indicates that most of the variability in the data is captured by the co-integration model. This is an indication of the model's ability to capture intra-data clustering. Future shocks which might be experienced are therefore easily captured in the forecasts. It therefore indicates a high level of significance in the forecasts of this model.

Notably, the R^2 and the adjusted R^2 values are almost equal. It is a good indication of high precision forecasts. It is an indication that the characteristic exhibited by series is persistent. A sample of the set of data will always have the same characteristic as the population of the data. This is in line with heteroskedasticity as data tend to cluster in a similar manner throughout the data. It is therefore an indication of the reliability of the forecasts obtained if the model was to be used. This completes the final proof of existence of a co-integration relationship.

It can be concluded therefore that if two series follow a GARCH(1,1) model, they are co-integrated and they do not drift too far from each other. It can as well be concluded that co-integration is a powerful tool in the analysis of time series data and can be used to obtain optimal forecasts. A co-integration relationship can therefore be used to explain the source of variability in one series if the variability in the other series is known. Finally, heteroskedasticity does not influence the predictability of a co-integration model. Therefore, highly significant forecasts can still be obtained from a highly heteroskedastic series. This wraps up the proof to the main proposition in section (3). Recommendations for further research is given in section (4.3).

4.3 Recommendations

[1]As much as GARCH models captures heteroskedasticity, it is still a conditional variance model. GARCH models are therefore most appropriate for squared return series. On the other hand, ARMA[11] models are built on conditional expectation. They are therefore perfect for a normal return series. Therefore, A combination of the two series will be most appropriate when the series exhibit correlation in both first and second order. It is therefore recommended that a similar study be undertaken with an investigation of a combination of ARMA and GARCH models. It is as well recommended that further study be done on the Kenyan interbank lending rate to investigate the source of very high serial correlations.

[11] Auto Regressive Moving Average Process

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Appendices

Author Profile

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