Mathematical Modeling of Three-Link Bipedal Robot

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Abstract: ***Modeling of biped robot is one of the important issues to be discussed before deciding the control technique for the stable walking of the biped. This paper includes the insight of mathematical modeling of three link biped robot along with different control techniques employed to the same. That can help to design suitable control law for stable walking of the same.***

Key words: Modeling, hypotheses, point foot robot, Three-link biped

1. Introduction

Modeling of three link biped with point feet is discussed. There are certain assumptions to be made for modeling a three link biped. Those assumptions are also discussed here. The robots are assumed to consist of rigid links with mass, connected via rigid, frictionless, revolute joints to form a single open kinematic chain lying in a plane. Each leg end is terminated in a point so that, in particular, either the robot does not have feet, or it is walking tiptoe. All motions will be assumed to take place in the sagittal plane and consist of successive phases of single support and double support in the case of walking consisting of continuous dynamics and a re-initialization rule at the impact event. With general robotic terminology defined [1], complete lists of hypotheses are now assembled for the robot model, the desired walking gaits, and the impact model.

2. Dynamics

The multiple support phases present in a bipedal walking cycle naturally lead to a mathematical model that consists of at least two parts: a set of differential equations describing the dynamics during the single support phase, and a discrete model of the contact event when double support is initiated.
Assume furthermore that the stance leg end acts as an ideal pivot. Under these assumptions, the standard robot equations apply, resulting in,

$$D(q)\ddot{q}+C(q,\ \dot{q})\dot{q}+G(q) \;=\; Bu \qquad (1)$$

Where q is a set of generalized coordinates and u denotes the vector of actuator torques [1]-[3].

An impact occurs when the swing leg touches the walking surface. The resulting forces that are generated between the robot and the walking surface depend on whether the surface is springy, like a trampoline, viscous, like a muddy edge of a pond, or essentially rigid, like a solid floor. The ground reaction forces are replaced with impulses, resulting in a discontinuity in the velocity components of the robot's state. The ultimate result of the impact model is a new initial condition from which the single support model evolves until the next impact, written as

$$x^{+} = (\Delta x^{-}) \qquad (2)$$

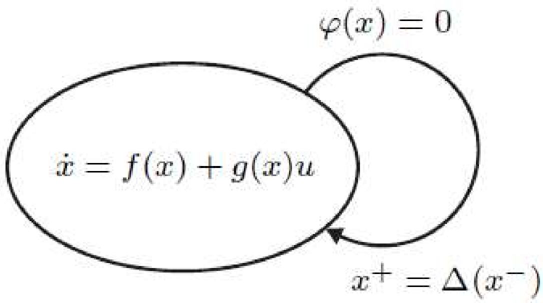

Figure 1: Single-mode hybrid model of walking that corresponds either to walking with point feet or to flat-footed walking

Key elements are the continuous dynamics of the single support phase, written in state space form as - $\dot{x} = f(x) + g(x)\ u$, the switching or impact condition, $\phi = 0$, which detects when the height of the swing leg above the walking surface is zero, and the re-initialization rule coming from the impact map, Δ [2].

3. Hypotheses

3.1 Robot with Point Feet Hypotheses:

The robot is assumed to be;

- planar, with motion constrained to the sagittal plane
- bipedal, with two symmetric legs connected at a common point called the hip, and both leg ends are terminated in points
- comprised of N rigid links connected by (N−1) ideal revolute joints (i.e., rigid and frictionless) to form a single open kinematic chain; furthermore, each link has nonzero mass and its mass is distributed (i.e., each link is not modeled as a point mass)
- independently actuated at each of the (N −1) ideal revolute joints
- unactuated at the point of contact between the stance leg and ground

3.2 Gait Hypotheses for Walking:

- there are alternating phases of single support and double support
- during the single support phase, the stance leg end acts as an ideal pivot
- the double support phase is instantaneous and the associated impact can be modeled as a rigid contact
- at impact, the swing leg neither slips nor rebounds, while the former stance leg releases without interaction with the ground
- in steady state, the motion is symmetric with respect to the two legs
- in each step, the swing leg starts from strictly behind the stance leg and is placed strictly in front of the stance leg at impact
- walking is from left to right and takes place on a level surface

3.3 Rigid Impact Model Hypotheses

- an impact results from the contact of the swing leg end with the ground
- the impact is instantaneous
- the impact results in no rebound and no slipping of the swing leg
- in the case of walking, at the moment of impact, the stance leg lifts from the ground without interaction, while in the case of running, at the moment of impact, the former stance leg is not in contact with the ground
- the externally applied forces during the impact can be represented by impulses
- the actuators cannot generate impulses and hence can be ignored during impact, and
- the impulsive forces may result in an instantaneous change in the robot's velocities but there is no instantaneous change in the configuration.

4. Modeling of three-link walker

A three-link walker is the robot which has no knees. It is known to possess stable walking motions (i.e., asymptotically stable periodic orbits) when walking down a sufficiently gentle constant slope, this robot model does not possess any stable walking motions without control. The three-link walker provides the simplest example where torso stabilization is important. The model is given in two sets of coordinates.

Table 1: The model parameters

Parameters	*Unit*
Torso length, l	m
Leg length, r	m
Torso mass, M_T	Kg
Hip mass, M_H	Kg
Leg mass, m	Kg
Acceleration due to gravity, g_0	m/s^2

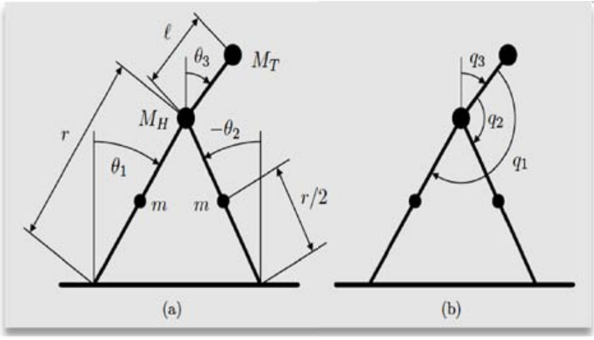

Figure 2: Three link bipedal walker

Applying the method of Lagrange yields the following data for the mode [5] l:

$$(D_s(q))_{1,1} = (\frac{5}{4}m + M_H + M_T)r^2$$

$$(D_s(q))_{1,2} = -\frac{1}{2}mr^2\cos(\theta_1 - \theta_2)$$

$$(D_s(q))_{1,3} = M_T rl\cos(\theta_1 - \theta_3)$$

$$(D_s(q))_{2,2} = \frac{1}{4}mr^2$$

$$(D_s(q))_{2,3} = 0$$

$$(D_s(q))_{3,3} = M_T l^2 \qquad (3)$$

The remaining entries of inertia matrix are completed by symmetry. The nonzero entries of Cs are:

$$(C_s(q,\dot{q}))_{1,2} = -\frac{1}{2}mr^2\sin(\theta_1 - \theta_2)\dot{q}_2$$

$$(C_s(q,\dot{q}))_{1,3} = M_T rl\sin(\theta_1 - \theta_3)\dot{q}_3$$

$$(C_s(q,\dot{q}))_{2,1} = \frac{1}{2}mr^2\sin(\theta_1 - \theta_2)\dot{q}_1$$

$$(C_s(q,\dot{q}))_{3,1} = -M_T rl\sin(\theta_1 - \theta_3)\dot{q}_1 \qquad (4)$$

The vector Gs and the input matrix Bs are given by:

$$G_s = \begin{bmatrix} -\frac{1}{2}g_0(2M_H + 3m + 2M_T)r\sin(\theta_1) \\ \frac{1}{2}g_0 mr\sin(\theta_2) \\ -g_0 M_T l\sin(\theta_3) \end{bmatrix} \qquad (5)$$

And

$$B_s = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} \qquad (6)$$

The impact model is given below:

$$\Delta q = R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$(\Delta\dot{q})_{1,1} = \frac{1}{den}[2M_T\cos(-\theta_1+2\theta_3-\theta_2) - (2m+4M_H+2M_T)\cos(\theta_1-\theta_2)]$$

$$(\Delta\dot{q})_{1,2} = \frac{m}{den}$$

$$(\Delta\dot{q})_{1,3} = 0$$

$$(\Delta\dot{q})_{2,1} = \frac{1}{den}[m-(4m+4M_H+2M_T)\cos(2\theta_1-2\theta_2) +2M_T\cos(2\theta_1-2\theta_3)$$

$$(\Delta\dot{q})_{2,2} = \frac{1}{den}2m\cos(\theta_1-\theta_2)$$

$$(\Delta\dot{q})_{2,3} = 0$$

$$(\Delta\dot{q})_{3,1} = \frac{r}{lden}[(2m+2M_H+2M_T)\cos(\theta_3+\theta_1-2\theta_2) -(2m+2M_H+2M_T)\cos(-\theta_1+\theta_3) +m\cos(-3\theta_1+2\theta_2+\theta_3)$$

$$(\Delta\dot{q})_{3,2} = -\frac{r}{lden}m\cos(-\theta_2+\theta_3)$$

$$(\Delta\dot{q})_{3,3} = 1$$

$$den = -3m-4M_H-2M_T+2m\cos(2\theta_1-2\theta_2) +2M_T\cos(-2\theta_2+2\theta_3)$$

(8)

Equation (1) can be solved according to the above equations of D_S, C_S, G_S and B_S.

Notations:

f(x): continues part of mathematical model of biped
g(x): discrete part of mathematical model f biped
φ(x): function of position of biped
x^+: position of biped just after impact
x^-: position of biped just before impact
Δ: impact function of biped model
$(q, \dot{q}) = x$: coordinates of position
D_S: inertia matrix
C_S:Coriolis matrix
G_S: gravitation force matrix
B_S: input matrix
u: control signal
θ_1: angle of swing leg
θ_2: angle of stance leg
θ_3: angle of torso
den: denominator

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