Implementation of Non Shannon Entropy Measures for Color Image Segmentation and Comparison with Shannon Entropy Measures

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Abstract: *Segmentation of color images is important for various image analysis problems and is somewhat more involved than the segmentation of gray scale images. Preserving the original colors in different segments of the original image is one of the main problems in color image segmentation. Here in this paper, presents an approach for color image segmentation, which almost preserves the colors in different segments of input color image. In this projected approach, threshold selection in each of the three component (RGB) images is done on the basis of different entropy measures such as the Shannon, Renyi, Havrda-Charvat, Kapur and Vajda entropy measures. Simulation results for all above mentioned entropy measures are also presented and it is observed that Havrda Charvat and Shannon entropy measures are better than other measures for color image segmentation problems.*

Keywords: Image segmentation, Threshold Techniques, Shannon Entropy, Renyi Entropy, Havrda-Charvat Entropy, Vajda Entropy, Kapur Entropy.

1. Introduction

Color image segmentation problem is a little more convoluted than the segmentation of gray scale images. The applications of image segmentation comprise identifying of objects, feature extraction etc. [1]-[12]. One of the problems associated with color image segmentation technique is related to preserving the colors in the different segments of the original image.

Numerous methods for segmentation of color images have been presented in the literature [1]-[4]. In [1], the maximization of the between-class variance (MVI) and the entropy have been used as a benchmark to determine an optimal threshold to segment the images. In [2], the watershed algorithm is used to segment the regions of the images while preserving the desirable discontinuity characteristics of image. K-means clustering technique recovered the problem of undesirable over-segmentation caused by watershed algorithm. A segmentation model for color images which is based on the geodesic active contour (GAC) modeling is proposed in [3]. Definitions of various entropy measures (Shannon and non-Shannon entropy measures) have also been conveyed in the [11]. The main advantage of non-Shannon measures over Shannon entropy measure is that non-Shannon measures have some parameters (α in case of Havrda-Charvat and Renyi and α, β in case of kapur) that can be used as free parameters. These parameters can play an important role in the image segmentation problems [4].

In [12], a comparison of Shannon and non-Shannon entropy measures for threshold selection in gray level images is done and and it is comprehended that the sum of Havrda and Charvat entropies as a criterion for threshold selection can result in an improved image segmentation. None of the existing techniques based on entropy measures have, however, been applied for color image segmentation problem.

Purpose of this paper is to study and present a relative study of different entropy measures for threshold selection purpose in color image segmentation problems using co-occurrence matrix. Here we calculate the threshold for each color plane using the minima of each entropy measures (Shannon, Renyi, Havrda-Charvat, Kapur and Vajda), which in turn is computed via the co-occurrence matrix and use it for segmentation of image. The simulation results using different entropy measures are also presented here. It is observed that the threshold values obtained from these plots are dependent on the particular definition of the entropy chosen, which in turn affects the segmentation results.

The rest of the paper is structured as follows. Section 2 describes the classification of several entropy measures which are evaluated for threshold selection in image segmentation problems. Simulation results are presented in section 3. Section 4 contains conclusions.

2. Proposed Methodology Founded on Entropy Measures

The proposed work of image segmentation methodology using the gray level co-occurrence matrix ($C_{m1,m2}$) and Shannon entropy measure is discussed in [5]. Now In this paper we extend this methodology using the co-occurrence matrix with non-Shannon entropy measures (such as Renyi, Havrda-Charvat, Kapur and Vajda entropy) on color images. The basic steps of the algorithm are reproduced here for the sake of convenience [5]:

(i) First of all, the co-occurrence matrix $C_{m1,m2}$ [5] of the image to be segmented is computed for each color channel.

(ii) The probability distribution $P_{m1,m2} = C_{m1,m2}/MN$ is then calculated from its co-occurrence matrix $C_{m1,m2}$

(iii) The entropy function for each entropy definitions, as defined below, are then calculated for each $t \in [0,1,2,....,L-2]$ for a given image to be segmented using the probability distribution $P_{m1,m2}$.

(iv) The numbers of minima points are determined from the entropy function versus gray level (t) plot. The color component level corresponding to the smallest minima may be taken as a threshold for image segmentation problems.

Next, we discuss different entropy measures [8]-[11], which are used in this paper for a reasonable study in image segmentation problems.

(A) Shannon Entropy

Shannon's entropy measure H_s ($P_{mi, m2}$) provides an absolute limit on the best possible lossless compression of a signal under certain constraints [11]. It is defined as:

$$H_s\left(p_{m_1,m_2}\right) = -\sum_{m_1}\sum_{m_2} p_{m_1,m_2} \log p_{m_1,m_2} \tag{1}$$

Where $P_{m1,m2}$ is the probability distribution associated with the 2-D random variable. In this paper we have calculated the values of $P_{m1,m2}$ from the entries of the gray level co-occurrence matrix ($C_{m1,m2}$) [5], [6] of the given image as given by the following relation $P_{m1,m1} = C_{m1,m2} / MN$ where M, N represents image dimensions along X, Y directions respectively. The entropy function for the purpose of the calculation of threshold for image segmentation is then computed from the expression given as:

Where L represents the maximum number of gray level presents in a particular image and m_1, $m_2 \in [0, 1, 2...L-1]$ and $t \in [0, 1...L-2]$.

$$Entropy(t) = -\sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1} p_{m_1,m_2} \log p_{m_1,m_2} - \sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t} p_{m_1,m_2} \log p_{m_1,m_2} \tag{2}$$

(B) Kanpur Entropy

Kapur's entropy H_k ($P_{m1,m2}$) of order α and type β defined as [3],[8]

$$H_k\left(p_{m_1,m_2}\right) = \left(\frac{\sum_{m_1}\sum_{m_2} p_{m_1,m_2}^{\alpha+\beta-1}}{\sum_{m_1}\sum_{m_2} p_{m_1,m_2}^{\beta}} - 1\right)(2^{1-\alpha}-1)^{-1}$$

$$Entropy(t) = \sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1}\left(\frac{p_{m_1,m_2}^{\alpha+\beta-1}}{p_{m_1,m_2}^{\beta}} - 1\right)(2^{1-\alpha}-1)^{-1}$$

$$+ \sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t}\left(\frac{p_{m_1,m_2}^{\alpha+\beta-1}}{p_{m_1,m_2}^{\beta}} - 1\right)(2^{1-\alpha}-1)^{-1}$$

(C) Vajda Entropy (4)

Vajda entropy measure H_v ($P_{m1,m2}$) is a special case of Kapur's entropy where $\beta = 1$ is taken. It provides the advantage of faster calculations over Kapur's entropy measure and is defined as:

$$H_v\left(p_{m_1,m_2}\right) = \left(\frac{\sum_{m_1}\sum_{m_2} p_{m_1,m_2}^{\alpha}}{\sum_{m_1}\sum_{m_2} p_{m_1,m_2}} - 1\right)(2^{1-\alpha}-1)^{-1}, \tag{5}$$

and the corresponding entropy function is given by

$$Entropy(t) = \left(\frac{\sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1} p_{m_1,m_2}^{\alpha}}{\sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1} p_{m_1,m_2}} - 1\right)(2^{1-\alpha}-1)^{-1}$$

$$+ \left(\frac{\sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t} p_{m_1,m_2}^{\alpha}}{\sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t} p_{m_1,m_2}} - 1\right)(2^{1-\alpha}-1)^{-1}$$

(6)

(D) Renyi Entropy

The Rényi entropy H_r ($P_{m1,m2}$) which is a generalization of Shannon entropy is one of a family of functional for quantifying the diversity, uncertainty or randomness of a system. It is defined as [3];

$$H_r\left(p_{m_1,m_2}\right) = \frac{\log\left(\sum\sum\left(p_{m_1,m_2}\right)^{\alpha}\right)}{1-\alpha}, \alpha \neq 1, \alpha > 0 \tag{7}$$

and the corresponding entropy function is given by

$$Entropy(t) = -\sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1}\frac{\log\left(\sum\sum(p_{m_1,m_2})^{\alpha}\right)}{1-\alpha} - \sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t}\frac{\log\left(\sum\sum(p_{m_1,m_2})^{\alpha}\right)}{1-\alpha} \tag{8}$$

(E) Havrda- Charvat Entropy

The Havrda–Charvát entropy H_{hc} ($P_{m1,m2}$) of degree α introduced by Havrda and Charvát and later on modified by Daróczy is often used in statistical physics and is defined as follows

$$H_{hc}(p_{m_1,m_2}) = \frac{\sum\sum p_{m_1,m_2}^{\alpha} - 1}{2^{1-\alpha}-1} \tag{9}$$

and the corresponding entropy function is given by

$$Entropy(t) = \frac{1}{2^{1-\alpha}-1}\left(\sum_{m_1=0}^{t}\sum_{m_2=t+1}^{L-1} p_{m_1,m_2}^{\alpha} - 1\right) + \frac{1}{2^{1-\alpha}-1}\left(\sum_{m_1=t+1}^{L-1}\sum_{m_2=0}^{t} p_{m_1,m_2}^{\alpha} - 1\right)$$

The above mentioned entropy functions given in (2), (4), (6), (8) and (10) are calculated for each $t \in [0,1,2,....,L-2]$ for a given image to be segmented using the probability

distribution $P_{m1,m2}$ which in turn is calculated from its gray and color component level co- occurrence matrix $C_{m1,m2.}$

3. Simulation Results

Simulation results performed in MATLAB on the input images shown in Figure 1 is presented in this paragraph .The entropy function versus individual RGB component level plot is obtained for Shannon and non-Shannon entropy measures. A typical plot for entropy function using Havrda-Charvat entropy measure is shown in Table I. The value of the thresholds obtain from the Shannon and non- Shannon entropy functions are summarized in Table II . The segmentation results obtained from these thresholds for each color plane and final segmented image are given in Figure 2(A) , 2(B) , 2(C) , 2(D) respectively and for the sake of comparison the results obtained using Shannon and non-Shannon (Renyi entropy) entropy measures are given in Figure 3(A) and 3(B) .

Table 1: Entropy Versus Individual Rgb Component Level Plot For Havrda- Charvat Entropy

Color Planes	Typical Entropy versus Gray level plot
R Plane	
G Plane	
B Plane	

Table 2: Parameters of Entropy versus Individual RGB Component Level Plot For "Fig 1" Image

Entropy	Component		
	R	G	B
Havrda- Charvat Entropy	124	42	55
Shannon Entropy	17	39	106
Renyi Entropy	234	15	39

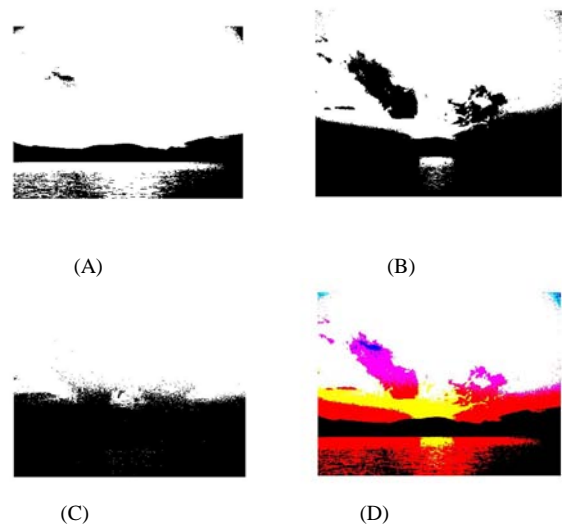**Figure 1:**   Original image for segmentation

Figure 2:(A) Segmented R Plane using Havrda Entropy Measure, (B) Segmented G Plane using Havrda Entropy Measure, (C) Segmented B Plane using Havrda Entropy Measure, (D) Final Segmented Image Using Havrda Entropy Measure.
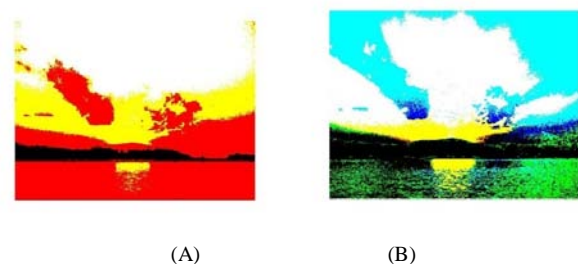

Figure 3: (A) Segmentation results of image in figure 1 using Shannon Entropy, (B)) Segmentation results of image in fig 1 using Renyi Entropy.

4. Conclusion

In this paper, several entropy measures for threshold selection purpose in color image segmentation problems are studied. The threshold values obtained are dependent on the particular definition of the entropy chosen, which in turn affects the segmentation results. It is further observed that the segmentation results obtained using Havrda-Charvat entropy

measures are better than other entropy measures in the sense of preservation of colors in different segments. Simulation results performed in MATLAB on the input images shown in Figure 1 is presented in this paragraph .The entropy function versus individual RGB component level plot is obtained for Shannon and non-Shannon entropy measures. A typical plot for entropy function using Havrda-Charvat entropy measure is shown in Table I. The value of the thresholds obtain from the entropy functions are summarized in Table II . The segmentation results obtained from these thresholds are given in Figure 2(A) to 2(D) and Figure 3(A) and 3(B).

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