Generalized Spatial Kernel based Fuzzy C-Means Clustering Algorithm for Image Segmentation

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Abstract: ***Image segmentation plays an important role in image analysis. It is one of the first and most important tasks in image analysis and computer vision. This proposed system presents a variation of fuzzy c- means algorithm that provides image clustering. Based on the Mercer kernel, the kernel fuzzy c-means clustering algorithm (KFCM) is derived from the fuzzy c-means clustering algorithm (FCM).The KFCM algorithm that provides image clustering and improves accuracy significantly compared with classical fuzzy C-Means algorithms. This proposed system makes the use of the advantages of KFCM and also incorporates the local spatial information and gray level information in a novel fuzzy way. The new algorithm is called Generalized Spatial Kernel based Fuzzy C-Means (GSKFCM) algorithm. The major characteristic of GSKFCM is the use of a fuzzy local (both spatial and gray level) similarity measure, aiming to guarantee noise insensitiveness and image detail preservation as well as it is parameter independent. The purpose of designing this system is to produce better segmentation results for images corrupted by noise, so that it can be useful in various fields like medical image analysis, such as tumor detection, study of anatomical structure, and treatment planning.***

Keywords*:* Image analysis, clustering, FCM, KFCM, GSKFCM.

1. Introduction

Image segmentation plays crucial role in many applications, such as image analysis and comprehension, computer vision, image coding, pattern recognition and medical images analysis. Many algorithms and methods have been proposed for object segmentation and feature extraction [1]. In this system, a clustering method for medical and other image segmentation will be considered. Clustering is a process of partitioning or grouping a given set of unlabelled objects into a number of clusters such that similar objects are allocated to one cluster. There are two main approaches to clustering [2]. One method is crisp clustering (or hard clustering), and the other one is fuzzy clustering. A characteristic of the crisp clustering method is that the boundary between clusters is fully defined. However, in many real cases, the boundaries between clusters cannot be clearly defined. Some patterns may belong to more than one cluster. In such cases, the fuzzy clustering method provides a better and more useful method to classify thesepatterns.The FCM employs fuzzy partitioning such that a data pixel can belong to all groups with different membership grades between0 and1.FCM is an iterative algorithm. The aim of FCM is to find cluster centers (centroids) that minimize objective function. The KFCM is derived from the original FCM based on the 'kernel method' [3]. A common philosophy behind these algorithms is based on the following kernel (substitution) trick. KFCM algorithm is extended which incorporates the neighbor term into its objective function [4]. In the literature, various numbers of techniques are described for clustering and image segmentation. Ever since Zadeh presented the fuzzy set theory in his seminal paper in 1965[5], fuzzy set theory also found applications [6] in clustering. Fuzzy clustering is a widely applied method for acquiring fuzzy patterns from data and become the main method of unsupervised pattern recognition. It has been applied successfully in many fields, such as pattern analysis, extraction of fuzzy rules, image segmentation and so on.Fuzzy clustering method (FCM) was originally proposed by Dunn and later extended by Bezdek [7]. Recently, many researchers have modified the original FCM algorithm. Drawback for FCM algorithm is the sensitivity to noise or outlier. Drawbacks of FCM were solved by introducing KFCM. A number of powerful kernel-based methods were proposed and have found successful applications in pattern recognition and function approximation.In Wu and Gao'spaper[8], the Mercer kernel based method was investigated. They proposed the KFCM algorithm which is extended from FCM algorithm.It is shown to be more robust than FCM. N. A. Mohamed, M.N. Ahmed et al.[9] described the application of fuzzy set theory in medical imaging. A modified fuzzy c-means classification algorithm is used to provide a fuzzy partition. Ruoyu Du and Hyo Jong Lee[10], [11] have discussed that the medical image segmentation seems to be tedious and also a more challenging task due to the intrinsic nature of the images. In this paper, we propose a Generalized Spatial Kernel based Fuzzy C-Mean clustering (GSKFCM) which is extended from KFCM which incorporates the neighbor term into its objective function. This algorithm deals with real and synthetic images and resolves the issues of noise and intensity inhomogeneity and also it is independent of parameter selection. However the methods discussed above are based on initial parameter selection hence A non-parametric segmentation method is needed which can determine the parameters like no of clusters automatically to make it more efficient. Deng-Yaun Haung, Ta- Wei Lin, Wu-Chih-Hu [12] proposed an effective method of histogram-based valley estimation is presented for determining the no of clusters for an image.Thus, GSKFCM has the following attractive characteristics: it is relatively independent of the types of noise, and as a consequence, it is a better choice for clustering in the absence of prior knowledge of the noise. The fuzzy local constraints incorporate simultaneously both the kernel function, spatial and the local gray level relationship in a fuzzy way. This system can provide significant robustness to the noisy images and partially improve the performance of segmentation.

2. Preliminary Theory

2.1 Kernel Method

A common philosophy behind the algorithms using kernel method is based on the following kernel (substitution) trick, that is, firstly with a (implicit) nonlinear map, from the data space to the mapped feature space, $\Phi : X \rightarrow F\ (x \rightarrow \Phi(x))$, a dataset $\{ x_1 \ldots, x_n \} \subseteq X$ (an input data space with low dimension) is mapped into a potentially much higher dimensional feature space or inner product F, which aims at turning the original nonlinear problem in the input space into potentially a linear one in rather high dimensional feature space so as to facilitate problem solving. Due to many successes in applying kernel methods such as support vector machines (SVM), Mercer kernels have recently become more popular to real world problems.

Mercer Theorem: Any continuous, symmetric, positive semi definite kernel function $k(x, y)$ can be expressed as a dot product in a high-dimensional space.

The sample $S = x_1, x_2, ..., x_m$ includes m examples. The Kernel (Gram) matrix K is an m X m Matrix including inner products between all pairs of examples i.e., $K_{ij} = k(x^i, x^j)$.
K is symmetric since $k(x, y) = k(y, x) = \Phi(x) . \Phi(y)$ [10].

A symmetric function k (. , .) is a kernel iff for any finite sample S the kernel matrix for S is positive semi-definite. There the Mercer kernels are used to make it practical, in the following, the image of a input data X_i,i=1,2,……N in the high dimensional feature space is denoted by $\Phi(X_j)$,j=1,2,……M, where Φ (**.**)is nonlinear mapping function. Three commonly-used kernel functions in literature are:

- Polynomial kernel: $K(x, y) = (1 + (x, y))^d$
- Gaussian Radial basis function (GRBF) kernel: $K(x, y) = \exp \dfrac{\left(-\| x - y \|^2\right)}{\sigma^2}$
- Sigmoid kernel : $K(x, y) = \tanh(\alpha(x, y) + \beta)$

Where d, α, β are the adjustable parameters of the above kernel functions.

2.2 The Fuzzy C Means Clustering Algorithm (FCM)

The fuzzy c-means (FCM) algorithm is one of the most traditional and classical image segmentation algorithms**.** The FCM algorithm can be minimized by the following objective function. Consider a set of unlabeled patterns X, let $X=\{x_1,,x_2,. ..,x_N\}$, $x \in Rf$, where N is the number of patterns and f is the dimension of pattern vectors (features). The FCM algorithm focuses on minimizing the value of an objective function. The objective function measures the quality of the partitioning that divides a dataset into c clusters. The algorithm is an iterative clustering method that produces an optimal c partition by minimizing the weighted within group sum of squared error objective function J_m.

$$J_m = (U, W) = \sum_{J=1}^{C} \sum_{i=1}^{N} U_{ij}{}^m d_{ij}^2 \quad (1)$$

Where:
N: No of pattern in X
C: No of clusters
U_{ij} : Degree of membership X_i of in the j[th] cluster
W_i: Prototype of the center of the cluster j
$d_{ij}^2 = (\|X_i - C_j\|^2)$: a distance measure between object X_i and cluster center W_j;
m: the weighting exponent on each fuzzy membership.

The FCM algorithm focuses on minimizing Jm, subject to the following constraints on U:

$$U_{ij} \in [0,1] \quad i=1,2,......N, \text{ and } j = 1, 2,......C \ (2)$$

$$\sum_{j=1}^{C} U_{ij} = 1, \quad i=1,2,.....N \ (3)$$

$$0 < \sum_{i=1}^{N} U_{ij} < 1 \quad i=1,2,.....C \ (4)$$

Function (1) describes a constrained optimization problem, which can be converted to an unconstrained optimization problem by using the Lagrange multiplier technique.

$$U_{ij} = \frac{1}{\sum_{i=1}^{C} \left\langle d_{ij} \middle| d_{il} \right\rangle^{2/m-1}} \quad , i=1,.....,N \ j=1,....,C \ (5)$$

if $d_{ij} = 0$ then $U_{ij} = 1$ and $U_{ij} = 0$ for $1 \neq j$ (6)

$$W_j = \frac{\sum_{i=1}^{N} U_{ij}{}^m X_i}{\sum_{i=1}^{N} U_{ij}^m} \quad , j = 1,........,C \ (7)$$

Jm can be obtained through an iterative process, which is carried as follows.
Step 1**:** Set values for C, m and ε.
Step 2: Initialize the fuzzy partition matrix $U^{(0)}$.
Step 3: Set the loop counter b = 0.
Step 4: Calculate the C cluster centers $W_j^{(b)}$ with $U^{(b)}$ by using function (7)
Step 5: Calculate the membership matrix $U^{(b+1)}$ by using function (5)
Step 6: If $\{U^{(b)} - U^{(b+1)}\} < \varepsilon$ then stop, otherwise set b=b+1 and go to step 4.

2.3 The fuzzy kernel C-means (KFCM) algorithm

The KFCM algorithm adds kernel information to the traditional fuzzy c-means algorithm and it overcomes the disadvantage that FCM algorithm can't handle the small differences between clusters. The main idea of fuzzy kernel c-means algorithm (KFCM) is described as follows. The kernel method maps nonlinearly the input data space into a high dimensional feature space.

From the Mercer theorem, it is known that a Mercer kernel induces an implicit function space Rq. The Euclidian

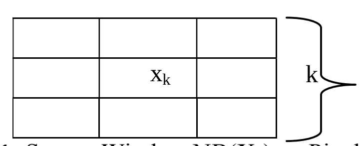


Figure 1: Square Window $NB(X_k)$ on Pixel X_k

distance between examples X_i and X_j in the feature space of the kernel K can be definition as follow

$$d_{ij} = dist\left(\phi(X_i), \phi(X_j)\right) = \sqrt{\| \phi(X_i) - \phi(X_j) \|^2} \quad (8)$$

distances can be computed directly from kernel function

$$d_{ij} = \sqrt{\Phi(X_i)\Phi(X_j) - 2\Phi(X_i)\Phi(X_j) + \Phi(X_j)\Phi(X_j)}$$

$$= \sqrt{K(X_i)K(X_i) - 2K(X_i, X_j) + K(X_j)K(X_j)}$$

$$= \sqrt{2 - 2K(X_i - X_j)} \quad (9)$$

Based on the original FCM algorithm, the object function J_{km} is going to be like this:

$$J_{km} = 2 \sum_{j=1}^{C} \sum_{i=1}^{N} U_{ij}^{m} \left(1 - K\left(X_i, W_j\right)\right) \quad (10)$$

According to equation (2), (3) and (4), by minimizing (10) using Lagrangian optimization as before, the following new iterative center Wj and membership U_{ij} update equations

$$U_{ij} = \frac{\left(1 - K\left(X_i, W_j\right)\right)^{1/(m-1)}}{\sum_{k=1}^{C} \left(1 - K\left(X_i, W_K\right)\right)^{1/(m-1)}} \quad (11)$$

$$W_j = \frac{\sum_{i=1}^{n} U_{ij}^{m} K\left(X_i, W_j\right) X_i}{\sum_{i=1}^{n} U_{ij}^{m} K\left(X_i, W_j\right)} \quad (12)$$

J_{km} can be obtained through an iterative process, which is carried as follows.

Step 1) Set values for C, m, and ε.

Step 2) Initialize the fuzzy partition matrix $U^{(0)}$.

Step 3) Set the loop counter b=0.

Step 4) Calculate the C cluster centers $W_j^{(b)}$ with $U^{(b)}$ by using function (12)

Step 5) Calculate the membership matrix $U^{(b+1)}$ by using function (11)

Step 6) If $\{U^{(b)} - U^{(b+1)}\} < \varepsilon$ then stop, otherwise, set b=b+1 and go to step 4.

The KFCM algorithm maps nonlinearly the input data space into a high dimensional feature space. It is confirmed that performance of KFCM algorithm is better than FCM algorithm in image segmentation.

2.4 Spatial Relationship

One of the important characteristics of an image is that neighbouring pixels are highly correlated. In other words, these neighbouring pixels possess similar feature values, and the probability that they belong to the same cluster is great. This spatial relationship is important in clustering, but it is not utilized in a standard FCM and KFCM algorithm. With the proposed method, the spatial constraint is adaptively taken into consideration by incorporating it in the membership function. Where NB (x_k) represents the square window. Here 3X3 window is used. Centre on the pixel x_k in the spatial domain. The spatial function is nothing but the summation of the membership function in the neighbourhood of each pixel that is taken under consideration.

3. Generalized Spatial Kernel Based Fuzzy C-Means Clustering Algorithm(GSKFCM)

Though the conventional FCM algorithm works well on most noise-free images, and KFCM algorithms have excellent performance in the applications by given appropriate kernel function and reasonable parameters. But, the KFCM algorithm described in previous section still has one drawback: it is very sensitive to noise and other imaging artifacts, since it does not consider any information about neighbourhood term. Using the KFCM algorithm on image segmentation, the calculation of J_{km} only consider the pixels of X_i, in fact, the neighbour around of the X_i have the implied relationship to the X_i. As a consequence the KFCM algorithm is unsuitable for images corrupted by impulse noise. In order to overcome this problem, we propose a generalized spatial kernel based fuzzy c means (GSKFCM) algorithm which incorporates local information into its objective function, defined in terms of J_{GSKFCM} (U,W) as follows:

$$J_{GSKFCM}(U,W) = \sum_{j=1}^{C} \sum_{i=1}^{N} U_{ij}^{m} \left(1 - K\left(X_i, W_j\right)\right) \left(\frac{N_R - \alpha \sum_{k \in N_i} U_{ik}}{N_R} \right) \quad (13)$$

Where N_R is its cardinality and N_i stands for the set of neighbours falling into a window around pixel X_i. where the i^{th} pixel is the center of the local window（for example, 3×3 or 5×5）.The parameter α is used to control the effect of the neighbours term which is getting higher with the increase of image noise(0<α<1). According to equation (2), (3) and (4), by minimizing (13) using Lagrangian optimization, the following new iterative center W_j and membership U_{ji} update equations [13]:

$$U_{ij} = \frac{\left(\left(1 - K\left(X_i, W_j\right)\right) \left(\frac{N_R - \alpha \sum_{l \in N_i} U_{jl}}{N_R} \right) \right)^{\frac{1}{(m-1)}}}{\sum_{k=1}^{C} \left(\left(1 - \left(X_i, W_K\right)\right) \left(\frac{N_R - \alpha \sum_{l \in N_i} Ukl}{N_R} \right) \right)^{\frac{1}{(m-1)}}} \quad (14)$$

$$W_j = \frac{\sum_{i=1} U_{ij}^{m} K\left(X_i, W_j\right) X_i}{\sum_{i=1}^{N} U_{ji}^{m} K\left(X_i, W_j\right)} \quad (15)$$

3.1 Peak point Analysis Algorithm

The segmentation method should be nonparametric, and should take the local and global feature distribution into consideration. This is a nonparametric algorithm that detects the peaks of Clusters in the color histogram of an image. The histogram bins rather than the pixels themselves to find the peaks of clusters; thus, the algorithm can find the peaks efficiently. Then, the algorithm associates the pixels of a detected cluster based on the local structure of the cluster. The approximate no of clusters can be determined based on following algorithm:

step 1: Input color image is browsed and selected that needs segmentation.

step2: Selected input image is divided in to three channels which is RGB i.e. Red, Green and Blue.

step3: Then windowing technique is applied to each channel separately. Each channel is divided into no of blocks, i.e. for each channel window is applied from start of image to the end of image with window move on. We set No of bins= No. of window.

step4: The intensity value of each pixel in to the window is compare with the specified range and if the intensity values are nearer to the range in the window then we club it into one window i.e. Neighboring pixels that lead to the same peak are grouped together.
step5: Otherwise we keep the pixel in the other window. Output of this algorithm is image with separated color values.
step6: Histogram of this image is computed..
step7: Then the peakfinder () method is used to determine the no of distinct peaks in the histogram which determines approximate no of clusters

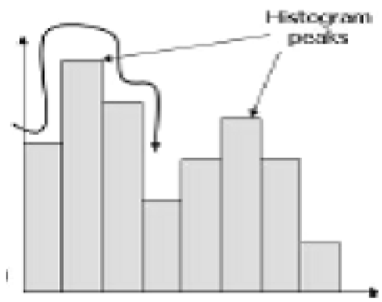


Figure 2: Finding Peaks in Histogram

J_{GSKFCM} can be obtained through an iterative process, which is carried as before.
Step 1: Set values m and ε .
Step 2: Find the No. of clusters using the peak point analysis method.
Step 3: Initialize the fuzzy partition matrix $U^{(0)}$.
Step 4: Set the loop counter b=0.
Step 5: Calculate the C cluster centers $W_j^{(b)}$ with $U^{(b)}$ by using function (15)
Step 6: Calculate the membership matrix $U^{(b+1)}$ by using function (14)
Step 7: If $\{U^{(b)} - U^{(b+1)}\} < \varepsilon$ then stop, otherwise set b=b+1 and go to step 4.

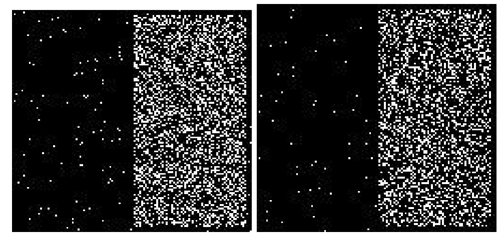
4. Experiments and Discussion

4.1 Experiment I:

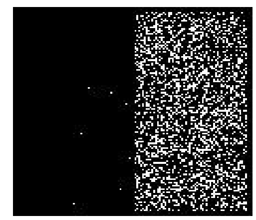
We apply all the algorithms to a synthetic test image (Fig..3 (a): 128 ×128 pixels, two classes with two gray level values 0 and 225) , We firstly added 15% salt and pepper noise to the original image). The algorithm for finding no of clusters is applied and the output is achieved by applying the algorithms mentioned in the table1**.**

Figure 3 illustrates the clustering results of a corrupted image. Original image [Fig.3 (a)], corrupted image [Fig3 (b)], FCM result [Fig. 3(c)], KFCM result [Fig3 (d)], GSKKFCM result [Fig3 (e)].

(a) **(b)**



(c) **(d)**

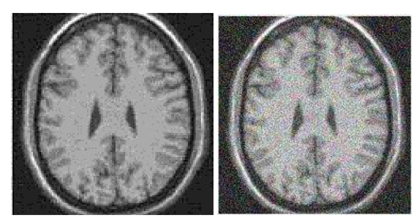


(e)

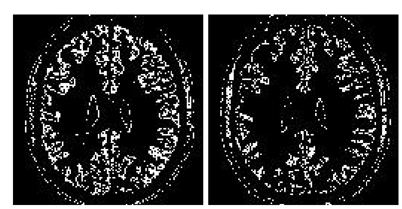
Figure.3: Clustering of a synthetic image. (a) Original image, (b) the same Image with Salt & pepper (15%), (c) FCM result, (d) KFCM result, (e) GSKFCM result

4.2 Experiment II

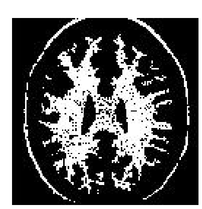
We apply all the algorithms to brain image (Fig.4.(a): brain image downloaded from net). We firstly added 3% Gaussian noise to the original image). The algorithm for finding no of clusters is applied and the output is achieved by applying the algorithms mentioned in the table 1.



(a) (b)



(c) (d)



(e)

Figure 4: Clustering of a brain image. (a) Original image, (b) the same image with Gaussian noise (3%), (c) FCM result, (d) KFCM result, (e) GSKFCM result

4.3 Performance Evaluation:

To evaluate the performance of the clustering algorithms, Clustering Accuracy Rate ((CAR) is used which is defined as:

$$\text{CAR}=\sum_{j=1}^{C}\frac{Ai\cap Cj}{\sum_{i=1}^{C}Ci}\ \text{X } 100\%$$

Where C is the number of clusters, A_i represents the set of pixels belonging to the j^{th} class found by the algorithm, while C_i represents the set of pixels belonging to the j^{th} class in the reference segmented image. CAR∈ [0, 1], thus the clustering performance is getting better when the value of CAR is higher.

Clustering Accuracy Rate can also be calculated as formula below:

CAR = Number of correctly classified pixels / Total number of pixels

Table 1: Comparison of Clustering Accuracy Rate (CAR%) of Four algorithms on Synthetic Images

Digital Image	*FCM*	*KFCM*	*GSKFCM*
Salt & Pepper [(15%)	91.79	94.10	95.27
Gaussian 3% (brain)	84.04	94.92	95.38

5. Conclusion and Future Scope

Clustering is one of the efficient techniques in medical and other image segmentation. The primary advantage of the project work is that it includes the kernel method, the effect of neighbour pixel information and gray level information to improve the clustering accuracy of an image, and to overcome the disadvantages of the known FCM algorithm which is sensitive to the type of noises. The proposed algorithm Generalized Spatial Kernel based Fuzzy C-Means (GSKFCM) is independent of parameter selection. Due to the factor that the gray value of the neighbour pixels generally affect the result of segmentation; the new algorithm puts kernel method and the effect of neighbours together. It provides noise-immunity and preserves image details. It can be useful in various fields like medical image analysis, such as tumor detection, study of anatomical structure, and treatment planning. For this algorithm, Gaussian kernel is selected as a kernel function in used in clustering algorithms. However, other heuristics or approaches for estimating the kernel parameter may be considered in future for improving the result

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