

# Measurement of Heart Rate Variability by Methods based on Chaos Theory

Hoang ChuDuc<sup>1</sup>, Thuan NguyenDuc<sup>2</sup>

<sup>1</sup>Washington University in St. Louis, School of Medicine,  
13th floor Northwest Tower, 660 S. Euclid Ave. Saint Louis, Missouri 63110, USA  
[hoang.chuduc@wustl.edu](mailto:hoang.chuduc@wustl.edu)

<sup>2</sup>Hanoi University of Science and Technology, School of Electronics and Telecommunications,  
No1 Dai Co Viet, Hanoi, Vietnam  
[thuan.nguyenduc@hust.edu.vn](mailto:thuan.nguyenduc@hust.edu.vn)

**Abstract:** Heart rate variability (HRV) is the physiological phenomenon of variation in the time interval between heartbeats. It is measured by the variation in the beat-to-beat interval. HRV parameters providing information about the scaling behavior or the complexity of the cardiac system were included. In addition, Chaos theory is a field of study in mathematics, with applications in several disciplines including physics, engineering, economics, biology, and philosophy. Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, an effect which is popularly referred to as the butterfly effect. The goal was to investigate the influence of gender, age and day-night variation on these nonlinear HRV parameters. Numerical titration yielded similar information as other nonlinear HRV parameters do. However, it does not require long and cleaned data and therefore applicable on short (15 minutes) noisy time series. A higher nonlinear behavior was observed during the night while nonlinear heart rate fluctuations decline with increasing age. Our results support the involvement of the autonomic nervous system in the generation of nonlinear and complex heart rate dynamics..

**Keywords:** heart rate variability, circadian variations, chaos theory.

## 1. Introduction

Cardiovascular structures and functions change with age, increasing the risk of developing cardiovascular disease [1]. Even during day and night periods, the autonomic cardiovascular modulation is different. Although the exact mechanisms underlying this circadian profile of adverse vascular events are still unknown. How the autonomic nervous system (ANS) exactly modulates the heart rate remains an open question. Heart rate variability (HRV) can be used to quantify several aspects of the autonomic heart rate modulation [2]. Standard time and frequency domain methods of HRV are well described by the Task Force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology [3], but they fail to show the dynamic properties of the fluctuations. Therefore, nonlinear methods are typically designed to assess the quality, scaling and correlation properties, rather than to assess the magnitude of variability like standard HRV methods do. However, in agreement with Lefebvre et al [4] and Yamamoto and Hughson [5], it seems likely that the cardiovascular system follows some nonlinear dynamics which need to be explored further. Indeed, an important feature of a healthy cardiovascular system is adaptation, which can be defined as the capacity to respond to unpredictable stimuli. Consequently, a nonlinear behavior would offer greater flexibility than a linear behavior. The use of nonlinear techniques will probably give additional information related to the dynamical changes in cardiovascular control.

While the day-night difference as well as the influence of gender and age on the autonomic modulation of heart rate have often been studied, most studies were limited to the standard HRV parameters. Ramaekers et al [6] already

reported a significant difference between day and night standard HRV, reflecting a higher vagal modulation during the night while Beckers et al [7] described a tendency for higher nonlinearity during night time. Schwartz et al [8] found a decreasing autonomic modulation with advancing age, which already starts in childhood.

In this paper, not only linear but also a large set of nonlinear techniques are applied to quantify scaling behavior and complexity. In particular, we focus on the recently developed numerical noise titration technique [9], which provides a highly sensitive test for deterministic chaos and a relative measure for tracking chaos of a noise-contaminated signal in short data segments, for example 5 minutes of data. The main goal of this study is to examine the nonlinear dynamics in autonomic heart rate control according to gender, age and day-night periods. Therefore, the study includes a large number of healthy subjects between adolescence and old age.

## 2. Methods

### 2.1 Data acquisition

In this paper, a data mining technique is used on Cardiod based person identification mechanism using electrocardiogram (ECG). Recent studies in Cardiod based ECG biometric excites a new dimension of efficient patient authentication, which places new hope in faster patient care. However, existing research suffers from lower accuracy due to random biometric template selection from fixed points in Cartesian coordinate. In this paper, we have extracted the ECG features using set of Euclidean distances with the help of data mining techniques. Euclidean distances, being independent of fixed points (as opposed to existing research) maintains higher accuracy in biometric identification when Bayes Network was implemented for classification purposes. A total of 26 ECG recordings from MIT/BIH Normal Sinus

Rhythm database (NSRDB) and MIT/BIH Arrhythmia database (MITDB) are used for development and evaluation. Our experimentation on these two sets of public ECG databases shows the proposed data mining based approach on Euclidean distances obtained from Cardioid graph results to 98.60% and 98.30% classification accuracy respectively. This way, type A errors (QRS detected prematurely when in fact a sinus conducted wave has not occurred) and B type errors (failing to detect an R wave that is present) could be largely avoided, as well as irregular sinus rhythms [10]. A 20%-filter was used, meaning that every RR interval that differ more than 20% from the previous one, is replaced by an interpolated value, defined via spline interpolation over the 5 previous and 5 next intervals. Finally, a file containing the consecutive RR intervals, called tachogram, was exported for later processing.

**2.2 Nonlinear HRV parameters**

Nonlinear HRV parameters do not describe the amount of modulation as such, but are able to describe the scaling and complexity properties of the signal. Often used parameters which study the scaling of the system are 1/f slope, fractal dimension (FD) and detrended fluctuation analysis (DFA  $\alpha_1$  &  $\alpha_2$ ) while the complexity is addressed via the correlation dimension (CD) and approximate entropy (ApEn). Also a chaotic signature is calculated by means of the Lyapunov exponent (LE) and the numerical noise titration, a nonlinear data analysis that is recently developed by Poon and Barahona [8]. A short overview of these methods will be given as they have been used multiple times, except the noise titration technique which will be outlined in detail based on Barahona and Poon [11].

For any heartbeat RR time series  $y_n, n = 1, 2, \dots, N$ , a closed-loop version of the dynamics is proposed in which the output  $y_n$  feeds back as a delayed input. The univariate time series are analyzed by using a discrete Volterra autoregressive series of degree  $d$  and memory  $\kappa$  as a model to calculate the predicted time series :

$$y_n^{calc} = a_0 + a_1 y_{n-1} + a_2 y_{n-2} + \dots + a_\kappa y_{n-\kappa} + a_{\kappa+1} y_{n-1}^2 + a_{\kappa+2} y_{n-1} y_{n-2} + \dots + a_M y_{n-\kappa}^d \quad (1)$$

$$= \sum_{m=0}^{M-1} a_m z_m(n)$$

where  $M = (\kappa + d)! / (\kappa! d!)$  is the total dimension. Details of the Volterra–Wiener method are described in Barahona and Poon [11]. Briefly, the Volterra–Wiener algorithm produces a family of polynomial (linear and nonlinear) autoregressive models with varying memory and dynamical order, optimally fitted to predict the data. Thus, each model is parameterized by  $\kappa$  and  $d$  which correspond to the embedding dimension and the degree of nonlinearity of the model (i.e.  $d = 1$  for linear and  $d > 1$  for a nonlinear model). The coefficients  $a_m$  are recursively estimated from (1) by using the Korenberg algorithm [12].

Nonlinear detection (NLD). The goodness of fit of a model (linear vs. nonlinear) is measured by the normalized residual sum of squared errors:

$$\varepsilon(\kappa, d)^2 = \frac{\sum_{n=1}^N (y_n^{calc}(\kappa, d) - y_n)^2}{\sum_{n=1}^N (y_n - \mu_y)^2} \quad (2)$$

with  $\mu_y = \frac{1}{N} \sum_{n=1}^N y_n$  and  $\varepsilon(\kappa, d)^2$

representing a normalized variance of the error residuals. The best linear and nonlinear models are chosen according to the Akaike information–theoretic criterion [13] which minimizes:

$$C(r) = \log \varepsilon(r) + \frac{r}{N} \quad (3)$$

where  $r \in [1, M]$  is the number of polynomial terms of the truncated Volterra expansion from a certain pair  $(\kappa, d)$ . For each data series, the best linear model is obtained by searching for  $\kappa^{linear}$  which minimizes  $C(r)$  with  $d=1$ . Analogously, varying  $\kappa^{nonlinear}$  and  $d>1$  leads to the best nonlinear model. Likewise, the best linear and nonlinear models for surrogate randomized data sets with the same autocorrelation and power spectrum as the original series are obtained. This results in four competing models with error standard deviations  $\varepsilon_{original}^{linear}, \varepsilon_{original}^{nonlinear}, \varepsilon_{surrogate}^{linear}$  and  $\varepsilon_{surrogate}^{nonlinear}$ . The presence of nonlinear determinism is now indicated if  $d_{opt} > 1$ . Further corroboration is obtained with the following objective statistical criteria: for models with Gaussian residuals, a standard  $F$ -test will serve to reject, with a certain level of confidence, the hypothesis that nonlinear models are no better than linear models as one-step-ahead predictors. This Gaussian assumption was verified throughout the analysis by using a  $\chi^2$ -test with a 99% cut-off.

Alternatively, the results are confirmed by using the nonparametric Mann – Whitney rank-sum statistic, which does not depend on the Gaussian assumption [14]. Taking into account this scheme, the relevance of nonlinear predictors is established when the best nonlinear model from the original data is significantly more predictive than the best linear model from the real data series as well as the best linear and nonlinear models obtained from the surrogate series.

To understand the nonlinear detection part completely, two important remarks are inevitable. Firstly, surrogate data are generated by preserving only the linear autocorrelation function of the original data series. The nonlinear autocorrelations are randomized and therefore adding nonlinear terms does not increase the prediction power. Consequently, surrogate data are always best approximated by a linear model:  $\varepsilon_{surrogate}^{linear} \approx \varepsilon_{surrogate}^{nonlinear}$ . Secondly, the time delay  $\tau$  for the embedding is another free parameter which has to be determined in case of continuous signals. The optimal time delay is chosen so as to maximize the difference between  $\varepsilon_{original}^{linear}$  and  $\varepsilon_{original}^{nonlinear}$ . Within the range

of acceptable time delays, generally  $\epsilon_{surrogate}^{linear} \approx \epsilon_{surrogate}^{nonlinear}$ , meaning that the prediction power of the linear model of a continuous signal derives mainly from its autocorrelation function. This holds for discrete maps as well. Consequently, and in contrast with other methods, surrogate data play only a confirmative role in this nonlinear detection procedure.

**3. Results**

All values, expressed as mean ± standard deviation, for mean RR and the nonlinear indices are listed in table 1, separately for day and night. For every HRV parameter, it is indicated whether the day-night difference is statistically significant or not. As expected during the night, heart rate was significantly lower (higher mean RR interval). A day-night variation was present in all nonlinear HRV parameters, although one has to remark that day and night period have a different recording length which can slightly affect some nonlinear HRV parameters.

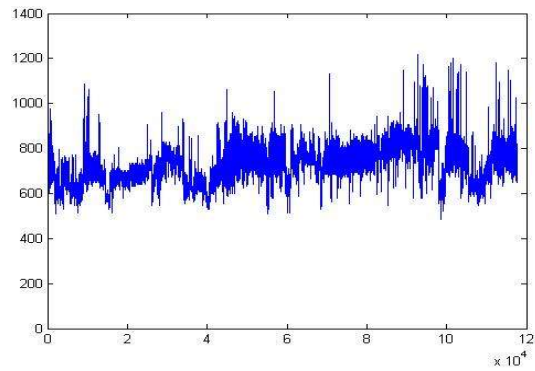
**Table 1:** Difference in chaotic behavior parameters (mean ± standard deviation) according to gender and day or night period.

	Time of a day		
	Day	Night	24h
NLmean [%]			
Men	19.20 ± 10	21.43 ± 8	19.96 ± 9
Women	17.88 ± 9	19.93 ± 8	18.60 ± 8
All	18.57 ± 10	20.72 ± 8	19.32 ± 8
P (gender)	Ns	0.0519	ns
NLdr [%]			
Men	53.07 ± 19	60.35 ± 14	55.85 ± 17
Women	48.21 ± 20	57.64 ± 20	51.94 ± 18
All	50.77 ± 20	59.06 ± 19	53.99 ± 18
P (gender)	0.0725	ns	ns

Abbreviations: P (gender) indicates the p-value for gender differences in case  $p < 0.1$ , otherwise ns = non-significant. More subtle changes during the transitions between day and night are reflected for all nonlinear HRV parameters by quantifying each of the 24 hours. Mean and standard error for each hour are plotted in figure 1 for HR and all nonlinear parameters, separately for men and women. In general, an evolution over 24 hours can be observed for all indices, except LE.

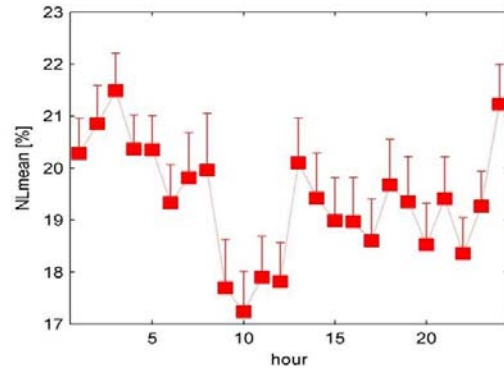
Heart rate starts already increasing extremely from 5 a.m., becoming stable from 8 a.m. on and again decreasing monotonously after 7 p.m. reaching the lowest peak at 4 a.m. The values of FD and DFA  $\alpha_1$  increase just before and during waking up, are almost constant afterwards and start to decrease slightly from late afternoon on.

While DFA  $\alpha_1$  increases in the morning hours till 10 a.m., DFA  $\alpha_2$  reflects exactly the opposite profile. 1/f and CD show an abrupt fall just before and during awakening, similar to heart rate, and a big jump in the late evening.

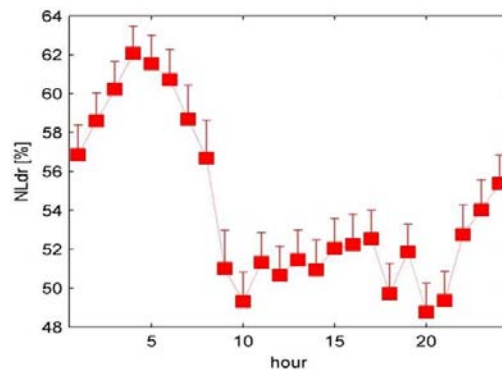


**Figure 1.** Long term HRV parameters.

LE did not show a clear 24h profile; however a transient dip was observed in the morning hours. When looking in particular to the noise titration parameters (Figure 2), deterministic chaos seems to increase monotonously in the evening, reaching a maximum early in the morning (3 – 5 am.), followed by a decrease and sharp fall around 7am (Figure 3).



**Figure 2.** Circadian evolution, expressed by mean and standard error, of the chaotic HRV parameters NLmean over the complete study population, ignoring gender and age variations.



**Figure 3.** Circadian evolution, expressed by mean and standard error, of the chaotic HRV parameters NLdr over the complete study population, ignoring gender and age variations.

**4. Discussion**

Heart rate variability is considered a parameter of autonomic cardiovascular control. In this study, the most commonly

used nonlinear HRV parameters were examined in a population of 256 healthy subjects between 20 and 65 years of age from MITDB, physonet. In addition, the recently developed method of numerical noise titration was applied leading to the new chaotic HRV parameters NLmean and NLdr. Nowadays, this technique has only been applied a few times and always to study relative differences between patient groups. Many previous studies have tried to assess day-night variations or gender- and age-related differences in HRV parameters, although most of these studies have major limitations: small groups, fixed age category, unequal amount of male and female subjects, short duration recordings and use of only a few parameters. All these shortcomings were taken into account in this study. In addition, via hour-by-hour analysis, we presented for each nonlinear HRV parameter the clear circadian profile as a function of age or gender.

It is generally acknowledged that there is a unique circadian distribution of cardiovascular events with a striking preponderance in the morning hours. A possible link to physiological rhythms with a similar diurnal variation is therefore inevitable. According to our knowledge, only Bonnemeier et al studied hour-by-hour HRV analysis of Holter recordings in healthy subjects as a function of age and gender, although restricted to only time domain HRV parameters. Our study is an extension and complementary to the previous study as we performed an analysis with nonlinear HRV parameters, giving the benefit of examining the daily profile more in detail instead of only day-night variations. Nevertheless many scaling behaviour or complexity parameters showed higher values for women than men, again confirming the added value of chaotic HRV parameters with respect to the other nonlinear HRV parameters.

## 5. Conclusion

Measurement of heart rate variability by methods based on chaos theory has been proved effective in evaluating autonomic modulation of the sinus node and in stratifying patients after MI. The marked reduction in HRV observed in high-risk patients seems to be associated with a significant attenuation of the linear and rhythmic components of HRV; thus, a nonlinear approach based on the evaluation of long- and short-term correlation of interbeat intervals seems to be more efficient in detecting the abnormal pattern of R-R fluctuations present in these patients and likely to reflect an abnormal autonomic modulation. This study showed the typical circadian profiles for several nonlinear HRV parameters as a function of age and gender. Not only parameters providing information about the scaling behavior or the complexity of the autonomic heart rate modulation are included, but also the chaotic behavior was quantified by means of the recently developed numerical noise titration technique. This method can be applied on short noisy time series, which can be a big advantage in clinical environment in the future.

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## Author Profile



**Hoang ChuDuc** is Researcher at Department of Electronics and Biomedical Engineering, School of Electronics and Telecommunication, Hanoi University of Science and Technology. He has 8 years experience as a researcher in biomedical equipment design, signal processing and teaching. From 2010, he is doing PhD research with project "Impact analysis between the ECG signal and the common defense of the Vietnamese".



**ThuanNguyenDuc** is Lecturer at Department of Electronics and Biomedical Engineering, School of Electronics and Telecommunication, Hanoi University of Science and Technology. He has over 30 years experience in biomedical equipment design, signal processing and teaching.