A Novel Study on Complex Fuzzy Logic

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Abstract: ***Complex fuzzy logic is a generalization of traditional fuzzy logic, based on complex fuzzy sets. In complex fuzzy logic, inference rules, the novelty of complex fuzzy logic is that the sets used in the reasoning process are complex fuzzy sets, characterized by complex-valued membership functions. Several mathematical properties of complex fuzzy sets, which serve as a basis for the derivation of complex fuzzy logic, are reviewed in this paper. These properties include basic set theoretic operations on complex fuzzy sets—namely complex fuzzy union and intersection, complex fuzzy relations and their composition, and a novel form of set aggregation—vector aggregation.***

Keywords: Complex fuzzy logic, complex fuzzy relations, complex fuzzy sets.

1. Introduction

Complex fuzzy logic is a unique framework, designed to maintain the advantages of traditional fuzzy logic, while benefiting from the properties of complex fuzzy sets. It is shown in the following sections that complex fuzzy logic is not merely a linear extension of conventional fuzzy logic. Rather, complex fuzzy sets allow a natural extension of fuzzy logic to problems that are either very difficult or impossible to address with one-dimensional grades of membership. The development of complex fuzzy relations and compositions serves as a basis for the derivation of complex fuzzy logic. Fuzzy logic: A form of knowledge representation suitable for notions that cannot be defined precisely, but which depends upon their contexts.Fuzzy set: Fuzzy set are whose elements have degree of membership.

2. Fuzzy Operations

- Fuzzy Union
- Fuzzy Intersection
- Fuzzy Complement

Fuzzy Union: In Fuzzy Logic, intersection, union and complement are defined in terms of their membership functions Fuzzy intersection and union correspond to ‘AND’ and ‘OR’, respectively, in classic/crisp/Boolean logic. These two operators will become important later as they are the building blocks for us to be able to compute with fuzzy if-then rules
The union (OR) is calculated using t-conorms

t-conorm operator is a function s(.,.)
Its features are

- s(1,1) = 1, s(a,0) = s(0,a) = a (boundary)
- s(a,b) ≤ s(c,d) if a ≤ c and b ≤ d (monotonicity)
- s(a,b) = s(b,a) (commutativity)
- s(a,s(b,c)) = s(s(a,b),c) (associativity)

The most commonly used method for fuzzy union is to take the maximum. That is, given two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$

$$\mu_{AUB}(x) = \max(\mu_A(x), \mu_B(x))$$

3. Fuzzy Intersection

The intersection (AND) is calculated using t-norms.
t-norm operator is a function t(.,.)

Its features

- t(0,0) = 0, t(a,1) = t(1,a) = a (boundary)
- t(a,b) ≤ t(c,d) if a ≤ c and b ≤ d (monotonicity)
- t(a,b) = t(b,a) (commutativity)
- t(a, t(b,c)) = t(t(a,b),c) (associativity)

The most commonly adopted t-norm is the minimum. That is, given two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

4. Fuzzy Complement

To be able to develop fuzzy systems we also have to deal with NOT or complement.
This is the same in fuzzy logic as for Boolean logic
For a fuzzy set *A*, *A* denotes the fuzzy complement of *A*
Membership function for fuzzy complement is

$$\mu_{A}(x) = 1 - \mu_A(x)$$

For Example

Suppose we have the following (discrete) fuzzy sets:

A = 0.4/1+0.6/2+0.7/3+0.8/4
B = 0.3/1+0.65/2+0.4/3+0.1/4

The union of the fuzzy sets *A* and *B*

= 0.4/1+0.65/2+0.7/3+0.8/4

The intersection of the fuzzy sets *A* and *B*

= 0.3/1+0.6/2+0.4/3+0.1/4

The complement of the fuzzy set A

= 0.6/1+0.4/2+0.3/3+0.2/4

FUZZY LOGIC REPRESENTATION using fuzzy sets for car:

Slowest (0.0-0.25)
slow (0.25-0.50)
fast (0.50-0.75)
fastest (0.75-0.100)

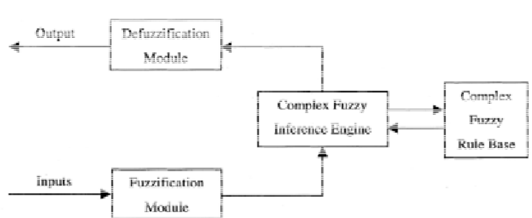
```
float speed;
get the speed
if ((speed >= 0.0)&&(speed < 0.25)) {
        // speed is slowest
}
else if ((speed >= 0.25)&&(speed < 0.5))
{
        // speed is slow
}
else if ((speed >= 0.5)&&(speed < 0.75))
{
        // speed is fast
}
else // speed >= 0.75 && speed < 1.0
{
//speed is fastest}
```

5. Properties of Fuzzy Logic

Involution	$\neg\neg A = A$
Commutativity	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotence	$A \cup A = A$ $A \cap A = A$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Absorption by X and $\emptyset$	$A \cup X = X$ $A \cap X = A$
Identity	$A \cup \emptyset = A$ $A \cap \emptyset = \emptyset$
DeMorgan's Laws	$\neg(A \cap B) = \neg A \cup \neg B$ $\neg(A \cup B) = \neg A \cap \neg B$

6. Complex Fuzzy Logic

Complex Fuzzy Implication and Inference:



General scheme of a CFLS (complex fuzzy logic system)

Complex Fuzzy Implication and Inference:

Complex fuzzy implication may be utilized for the construction of complex fuzzy inference rules, in the form of Generalized Modus Ponens.
Premise *1*: “X is A^*”; Premise 2: “X IF is A , THEN Y is B ”.
Consequence: “Y is B^*.”

of course, the sets A,B,A^*, and B^* are all complex fuzzy sets.

The output of a CFLS is determined in three stages, as illustrated in Fig. The first stage is Fuzzification, used to map crisp inputs into fuzzy input sets. These fuzzy sets may or may not be complex, depending on the application. The second stage, Fuzzy Inference, utilizes a complex fuzzy rule base to map the fuzzy input sets into fuzzy output sets. Defuzzification, as usual, is the final stage of the mapping performed by the CFLS. This stage involves defuzzification of the complex fuzzy output set to produce a crisp output.

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