

Nature of Electromagnetic Wave in Uniform Dusty Plasma

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Abstract: *In magnetized dusty plasma, Kinetic Alfvén wave (KAW) is one of the most important wave in electromagnetic regime and is ubiquitous in space environments for transporting electromagnetic energy. Dust charge fluctuation (DCF) effect on KAW have been studied under space and laboratory conditions using fluid description of the dusty plasma. Using Two Potential Theory, we have obtained the dispersion relation of KAW in a homogeneous dusty plasma under external uniform magnetic field by considering strongly magnetized electrons, finite Larmor radius (FLR) effects of ion and cold, unmagnetized but mobile dust grains. The dispersion relation shows a cutoff frequency for the KAW. DCF effect has been considered to analyze the damping of the electromagnetic KAW which is associated with the electrostatic parallel component of the waves.*

Keywords: Dust charge fluctuation effect, Kinetic Alfvén wave, Finite Larmor radius effect, Two potential theory, Cutoff frequency.

1. Introduction

Dusty plasma is generally a combination of normal electron-ion plasma with an additional charged component of micron or submicron-sized particulates. It is likely that, 99% of the matter in our universe is in the form of plasma in which dust is one of the omnipresent ingredients. They are quite common in astrophysical environments and laboratory situations [1]-[4]. The addition of highly charged dust grains into the usual electron-ion plasma in both space and laboratory environments play a very important role in changing the physical processes of the whole system to a large scale. New types of waves and instabilities cover a wide range of modern findings and fascinations in the field of dusty plasma. Wave propagation in dusty plasmas has been the subject of much interest in recent years because of its relevance to astrophysical plasmas, interstellar-gas, supernova-remnants, proto-stars, planetary magnetospheres as well as to laboratory plasmas. Charged dust grains are found to modify or even dominate wave propagation, wave scattering, wave instability, etc.

Alfvén waves are transverse magnetic tension waves which travel along magnetic field lines and can be excited in any electrically conducting fluid permeated by a magnetic field. They are known to be an important mechanism for transporting energy and momentum in many geophysical and astrophysical hydromagnetic systems [5]. They occur in interstellar clouds, cometary tails, planetary atmospheres and rings, solar corona and winds, Earth's ionosphere and magnetosphere, etc. [6]. They are also important in fusion devices for resonant heating of tokamak plasmas, current drives etc. [7]. The ubiquitous nature of Alfvén waves and their role in communicating the effects of changes in electric currents and magnetic fields have ensured that they remain the focus of increasingly detailed laboratory investigations.

There are basically two kinds of Alfvén waves below the ion-cyclotron frequency - (i) Compressional Alfvén wave

and (ii) Shear Alfvén wave. The compressional Alfvén wave propagates along the external magnetic field and the shear Alfvén wave propagates making some angle with the external magnetic field direction. Shear Alfvén waves with FLR effects are known as the kinetic Alfvén waves (KAW). The kinetic Alfvén wave develops a longitudinal parallel electrostatic field due to the finite-Larmor radius (FLR) effects and can transfer the wave energy to electrons via Landau damping resulting in the heating of plasmas or accelerate electrons along the magnetic field direction.

There is a characteristic damping mechanism for electrostatic waves in dusty plasmas [8]-[10]. This damping arises due to the DCF effect involving the dust dynamics. Although no charge density perturbation is associated with a pure transverse electromagnetic wave, because of the longitudinal electrostatic component, the kinetic Alfvén wave can be damped by the dust charge fluctuation effects in a dusty plasma. Earlier, KAW have been studied using fluid description of the dusty plasma [11]. Charge fluctuation effect on KAW have also been studied by involving FLR effects of the dust component neglecting ion FLR effects in the plasma [12]. Massive dust grains were taken magnetized and hot, and the frequency regime of the KAW was considered below the dust cyclotron frequency in the space and laboratory conditions.

In the present paper, we have investigated the nature of the low-frequency electromagnetic kinetic Alfvén waves and their damping due to the dust charge fluctuation effects by considering strongly magnetized electrons, FLR effect of ion and cold, unmagnetized but mobile dust grains.

2. Consideration of two-potential theory

We consider the propagation of an electromagnetic kinetic Alfvén waves (KAW) in homogeneous and collisionless dusty plasma in the presence of an external static magnetic field ($B_0 \parallel \hat{z}$). Let the electric field and propagation vector of the KAW lie in the same plane (XZ-plane) but not in the

same direction. For a low-β dusty plasma also, we can neglect the magnetic compression of the EM wave along the direction of the external magnetic field ($B_{1z} = 0$). In this consideration, we can assume the two different electrostatic potentials to represent the transverse and parallel components of the electric field of the KAW [6] [7] [13]

$$E_{\perp} = -\nabla_{\perp}\phi, \tag{1}$$

$$E_{\parallel} = -\frac{\partial\psi}{\partial z}, \tag{2}$$

where $\phi \neq \psi$ and the symbol \parallel (\perp) represents a quantity parallel (perpendicular) to the direction of the external magnetic field.

For strongly magnetized electrons, magnetized and hot ions and relatively highly charged and massive but cold and unmagnetized dust grains

$$\omega \ll \omega_{cj}, \quad j = e, i$$

$$k_{\parallel}v_{te} \gg \omega, \tag{3}$$

$$k_{\parallel}v_{ti} \ll \omega,$$

where (ω, k) are the angular frequency and the wave number vector of the KAW, $v_{ij} = (2T_j/m_j)^{1/2}$, is the thermal velocity and $\omega_{cj} = (q_j B_0/m_j c)$ is the cyclotron frequency of the j th species. Here, q_j, m_j, T_j and c are the charge, mass, temperature in energy units of the j th species, and the velocity of light in a vacuum, respectively.

The electric charge neutrality condition can be written as

$$n_{e0} = n_{i0} + Q_{d0}n_{d0}/e, \tag{4}$$

where $n_{\alpha 0}$ with $\alpha = e, i, d$ are the equilibrium number densities, $Q_{d0} = -Z_{dc}$ (with Z_d as the number of electronic charge on a grain) is the equilibrium charge on an average dust grain, and e is the electronic charge.

Denoting the perturbed quantities by a subscript 1, the linearized Poisson's equation is

$$\nabla_{\perp}^2\phi + \frac{\partial^2\psi}{\partial z^2} = 4\pi e \left[n_{e1} - n_{i1} - \frac{Q_{d0}}{e} n_{d1} - \frac{n_{d0}}{e} Q_{d1} \right]. \tag{5}$$

Combining the Ampere's and Faraday's laws, we can write

$$\frac{\partial}{\partial z} \nabla_{\perp}^2(\phi - \psi) = \frac{4\pi}{c^2} \frac{\partial}{\partial t} (J_{e1z} + J_{i1z} + J_{d1z}), \tag{6}$$

where J_{α} 's are the current densities.

3. Perturbed parts of the number densities and current densities for various species

Defining

$$n_{\alpha 1} = \int f_{\alpha 1} d\mathbf{v},$$

$$\mathbf{J}_{\alpha 1} = q_{\alpha} \int \mathbf{v} f_{\alpha 1} d\mathbf{v}, \tag{7}$$

where $\alpha = e, i, d$ and $f_{\alpha 1}$ is the perturbed distribution function of the species α and considering the variation of any perturbed quantity in the presence of the KAW as $\exp[-i(\omega t - \mathbf{k} \cdot \mathbf{x})]$, the coupled equations (5) and (6) reduce to

$$-(k_{\perp}^2\phi + k_{\parallel}^2\psi) = 4\pi e \left[n_{e1} - n_{i1} - \frac{Q_{d0}}{e} n_{d1} - \frac{n_{d0}}{e} Q_{d1} \right], \tag{8}$$

$$k_{\parallel}k_{\perp}^2(\phi - \psi) = \frac{4\pi\omega}{c^2} (J_{e1\parallel} + J_{i1\parallel} + J_{d1\parallel}). \tag{9}$$

For calculating the number densities and the current densities in Eqs.(8) and (9), we have to find the appropriate distribution functions for various species. Since electrons are assumed strongly magnetized, we can neglect the finite Larmor radius (FLR) effects of electrons for long perpendicular wavelength as $k_{\perp}v_{te} \ll \omega_{ce}$ is satisfied and consider only the parallel dynamics of the hot electrons. Thus, the perturbed distribution function for the hot and strongly magnetized electrons is given by

$$f_{e1} = \frac{ek_{\parallel}\psi}{m_e(\omega - k_{\parallel}v_{\parallel})} \frac{\partial f_{e0}}{\partial v_{\parallel}}, \tag{10}$$

where f_{e0} can be taken as Maxwellian.

Since the ions are hot and magnetized and Larmor radius is larger ($\rho_i \gg \rho_e$), where ρ_j is the Larmor radius of the j th species, the ion FLR effect must be taken into account. We solve the Vlasov equation for the hot and magnetized ions in terms of the guiding center coordinates [14] [15] and obtain the perturbed distribution function for any electromagnetic wave as

$$f_{i1} = e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} \left(\frac{n_{i0}e}{T_i} \right) \sum_l \sum_n \frac{k_{\parallel}v_{\parallel}\psi + n\omega_{ci}\phi}{\omega - n\omega_{ci} - k_{\parallel}v_{\parallel}}$$

$$e^{i(n-l)\theta} J_n \left(\frac{k_{\perp}v_{\perp}}{\omega_{ci}} \right) J_l \left(\frac{k_{\perp}v_{\perp}}{\omega_{ci}} \right) f_{i0}, \tag{11}$$

where J_n is the Bessel function of first kind of order n and f_{i0} is the Maxwellian distribution function for the ions.

Since the dust component of the plasma is considered cold and unmagnetized, we can employ the hydrodynamical fluid equations for finding the dust number density and current density perturbations.

Using Eq. (10) in Eq. (7), we obtain

$$n_{e1} = \frac{en_{e0}\psi}{T_e} [1 + \xi_{e0}Z(\xi_{e0})], \tag{12}$$

$$J_{e1\parallel} = \frac{e^2n_{e0}\psi}{m_e v_{te}} \xi_{e0}Z'(\xi_{e0}), \tag{13}$$

$$J_{e1x} = J_{e1y} = 0, \tag{14}$$

where $\xi_{e0} = \omega/k_{\perp}v_{te}$, Z' is the derivative of Z with respect to its argument, and Z is known as the plasma dispersion function [16].

Using Eq. (11) in Eqs.(7), we obtain

$$n_{i1} = -\frac{n_{i0}e}{T_i} \frac{1}{k_{\parallel}v_{ti}} \sum_n [k_{\parallel}v_{ti}(1 + \xi_{in}Z(\xi_{in}))\psi$$

$$+ n\omega_{ci}Z(\xi_{in})\phi] I_n e^{-b_i}, \tag{15}$$

where

$$\xi_{in} = (\omega - n\omega_{ci})/k_{\parallel}v_{ti},$$

$$b_i = k_{\perp}^2 v_{ti}^2 / 2\omega_{ci}^2,$$

and I_n is the modified Bessel function of first kind of order n and argument b_i . The oscillatory factor is implied in n_{i1} , ψ and ϕ . The components of the current density for ions turn out to be

$$J_{i1x} = -\frac{n_{i0}e^2}{T_i k_{\parallel} v_{ti} k_{\perp}} \sum_n n\omega_{ci} [k_{\parallel} v_{ti} (1 + \xi_{in} Z(\xi_{in})) \psi + n\omega_{ci} Z(\xi_{in}) \phi] I_n e^{-b_i}, \quad (16)$$

$$J_{i1y} = -\frac{in_{i0}e^2}{T_i k_{\parallel}} \left(\frac{k_{\perp} v_{ti}}{2\omega_{ci}} \right) \sum_n [k_{\parallel} v_{ti} (1 + \xi_{in} Z(\xi_{in})) \psi + n\omega_{ci} Z(\xi_{in}) \phi] (I_n - I'_n) e^{-b_i}, \quad (17)$$

$$J_{i1\parallel} = -\frac{n_{i0}e^2}{T_i k_{\parallel}} \sum_n [(1 + \xi_{in} Z(\xi_{in})) (k_{\parallel} v_{ti} \xi_{in} \psi + n\omega_{ci} \phi)] I_n e^{-b_i}. \quad (18)$$

For the cold and unmagnetized dust component using the momentum balance equation and the continuity equation, we obtain

$$n_{d1} = \frac{n_{d0} Q_{d0}}{m_d \omega^2} (k_{\perp}^2 \phi + k_{\parallel}^2 \psi),$$

$$J_{d1x} = \frac{n_{d0} Q_{d0} k_{\perp} \phi}{m_d \omega},$$

$$J_{d1y} = 0,$$

$$J_{d1\parallel} = \frac{n_{d0} Q_{d0} k_{\parallel} \psi}{m_d \omega}. \quad (19)$$

4. Dust density perturbation due to dust charge fluctuation effects

The EM kinetic Alfvén wave is a mixed mode consisting of a pure EM wave and an electrostatic component which propagates along the magnetic field direction and arises due to the FLR effects of ions. The density perturbation arises because of this electrostatic part, which is described by the Poisson's equation. In this section, we have calculated the dust density perturbation due to the dust charge fluctuation effects. The charging equation for dust grains in a dusty plasma is given by

$$\frac{d}{dt} Q_{d1}(\omega, \mathbf{k}) = I_{e1} + I_{i1}, \quad (20)$$

where $Q_d = Q_{d0} + Q_{d1}$ and I_{e1} and I_{i1} are the perturbed charging currents for electrons and ions.

Let us assume $\rho_{e,i} \leq a_0$ where $\rho_{e,i}$ are the Larmor radii of electrons/ions and a_0 is the radius of the grains. We also

assume $\lambda_D \sim a_0$, so that we can use the orbit-limited motion avoiding sheath limited effect.

In the presence of the wave perturbation, the charging currents are given by [17]

$$I_{j1}(\mathbf{x}, t) = \int \int \mathbf{J} \cdot d\mathbf{S} = 2\pi a_0^2 q_j \int (v_{\perp} \cos\theta + v_{\perp} \sin\theta + v_{\parallel}) f_{j1} dv, \quad (21)$$

where $j = e, i$ and f_{e1} and f_{i1} are given by Eqs.(10) and (11). Thus,

$$Q_{d1} = -\frac{i}{\omega} \beta, \quad (22)$$

where

$$\beta = \beta_1 \psi + \beta_2 \phi, \quad (23)$$

with

$$\beta_1 = -2\pi a_0^2 \left[e^{e\Phi_G/T_e} \frac{n_{e0} e^2}{m_e v_{te}} \xi_{e0} Z'(\xi_{e0}) - e^{(-e\Phi_G/T_i)} \frac{n_{i0} e^2}{T_i k_{\parallel}} \sum_n \left\{ \left(\frac{n\omega_{ci}}{k_{\perp} v_{ti}} I_n e^{-b_i} - \frac{k_{\perp} v_{ti}}{i2\omega_{ci}} (I_n - I'_n) e^{-b_i} \right) \cdot k_{\parallel} v_{ti} (1 + \xi_{in} Z(\xi_{in})) - \frac{k_{\parallel} v_{ti}}{2} \xi_{in} Z'(\xi_{in}) I_n e^{-b_i} \right\} \right] \quad (24)$$

$$\beta_2 = 2\pi a_0^2 e^{-e\Phi_G/T_i} \frac{n_{i0} e^2}{T_i k_{\parallel}} \sum_n \left[n\omega_{ci} Z(\xi_{in}) \left(\frac{n\omega_{ci}}{k_{\perp} v_{ti}} I_n e^{-b_i} - \frac{k_{\perp} v_{ti}}{i2\omega_{ci}} (I_n - I'_n) e^{-b_i} \right) + n\omega_{ci} (1 + \xi_{in} Z(\xi_{in})) I_n e^{-b_i} \right]. \quad (25)$$

In deriving Eq.(22), we have assumed that the dust grain surface potential is constant, $\Phi_G = Q_{d0}/a_0$ where Q_{d0} is the equilibrium charge on a spherical grain of radius a_0 .

Using Eqs.(12-19) in Eqs. (8) and (9) and after simplification, we obtain the coupled equations as

$$A\phi + B\psi = 0, \quad (26)$$

$$C\phi + D\psi = 0, \quad (27)$$

where

$$A = k_{\perp}^2 + \frac{1}{\lambda_{Di}^2} \frac{1}{k_{\parallel} v_{ti}} \sum_n n\omega_{ci} Z(\xi_{in}) I_n e^{-b_i} + i \frac{2\pi a_0^2 e^{-e\Phi_G/T_i}}{\omega} \frac{n_{d0}}{k_{\parallel} \lambda_{Di}^2} \sum_n \left\{ n\omega_{ci} Z(\xi_{in}) \left(\frac{n\omega_{ci} I_n}{k_{\perp} v_{ti}} \right) \cdot \frac{k_{\perp} v_{ti}}{2\omega_{ci}} (I_n - I'_n) e^{-b_i} + n\omega_{ci} I_n e^{-b_i} (1 + \xi_{in} Z(\xi_{in})) \right\} - \frac{k_{\perp}^2 \omega_{pd}^2}{\omega(\omega + i\nu_d)}, \quad (28)$$

$$B = k_{\parallel}^2 + \frac{1}{\lambda_{De}^2} (1 + \xi_{e0} Z(\xi_{e0})) + \frac{1}{\lambda_{Di}^2} \sum_n (1 + \xi_{in} Z(\xi_{in})) I_n e^{-b_i}$$

$$\begin{aligned}
 & -\frac{i2\pi a_0^2 n_{d0}}{\omega} \left[\frac{e^{e\Phi_G/T_e} \omega_{pe}^2}{v_{te}} \xi_{e0} Z'(\xi_{e0}) - \frac{e^{-e\Phi_G/T_i}}{k_{\parallel} \lambda_{Di}^2} \right. \\
 & \left. + k_{\parallel}^3 v_A^2 \omega_{pd}^2 \left(1 + \frac{3}{4} b_i \right) \right] \quad (40) \\
 & \sum_n \left\{ k_{\parallel} v_{ti} (1 + \xi_{in} Z(\xi_{in})) \left(\frac{n\omega_{ci} I_n}{k_{\perp} v_{ti}} + i \frac{k_{\perp} v_{ti}}{2\omega_{ci}} (I_n - I_n') \right) e^{-b_i} \right. \\
 & \left. - \frac{k_{\parallel} v_{ti}}{2} I_n e^{-b_i} \xi_{in} Z'(\xi_{in}) \right\} - \frac{k_{\parallel}^2 \omega_{pd}^2}{\omega(\omega + i\nu_d)}, \quad (29)
 \end{aligned}$$

$$C = k_{\perp}^2 c^2 k_{\parallel} + \frac{\omega}{k_{\parallel} \lambda_{Di}^2} \sum_n n \omega_{ci} (1 + \xi_{in} Z(\xi_{in})) I_n e^{-b_i}, \quad (30)$$

$$\begin{aligned}
 D = & -k_{\perp}^2 c^2 k_{\parallel} - \omega \left[\frac{\omega_{pe}^2}{v_{te}} \xi_{e0} Z'(\xi_{e0}) + \frac{1}{\lambda_{Di}^2} \right. \\
 & \left. \sum_n \frac{v_{ti}}{2} \xi_{in} Z'(\xi_{in}) I_n e^{-b_i} + \frac{\omega_{pd}^2 k_{\parallel}}{\omega + i\nu_d} \right], \quad (31)
 \end{aligned}$$

with

$$\lambda_{Dj}^2 = \frac{v_{tj}^2}{2\omega_{pj}^2}, \quad j = e, i.$$

Thus, the dispersion relation for the kinetic Alfvén waves is obtained as

$$AD - BC = 0 \quad (32)$$

Now, applying the realistic approximations for the dusty magnetoplasmas of astrophysical and laboratory conditions in Eq.(3) and neglecting the complex parts, we obtain

$$A_r = \frac{k_{\perp}^2 f_i}{\omega^2} \left(1 - \frac{3}{4} b_i \right) (\omega^2 - \omega_{dth}^2), \quad (33)$$

$$B_r = \frac{1}{\omega^2 \lambda_{De}^2} (\omega^2 - k_{\parallel}^2 \lambda_{De}^2 \omega_{pd}^2), \quad (34)$$

$$C_r = c^2 k_{\perp}^2 k_{\parallel}, \quad (35)$$

$$D_r = \frac{2\omega^2 \omega_{pe}^2}{k_{\parallel} v_{te}^2} - k_{\parallel} (\omega_{pd}^2 + k_{\perp}^2 c^2), \quad (36)$$

where $f_i = \frac{\omega_{pi}^2}{\omega_{ci}^2}$, $\omega_{dth}^2 = \frac{\omega_{pd}^2 \omega_{ci}^2}{\omega_{pi}^2}$, and the subscript r denotes a real quantity. In deriving Eqs.(33-36), we assumed $k_{\parallel} \lambda_{De} < 1$.

Thus, from the dispersion relation Eq.(32), we obtain a bi-quadratic equation for ω in terms of the plasma parameters,

$$P\omega^4 + Q\omega^2 + R = 0, \quad (37)$$

where

$$P = \frac{2\omega_{pe}^2}{k_{\parallel} v_{te}^2}, \quad (38)$$

$$\begin{aligned}
 Q = & -\frac{2\omega_{pd}^2 \omega_{dth}^2}{k_{\parallel} v_{te}^2} - k_{\parallel} (\omega_{pd}^2 + k_{\perp}^2 c^2) \\
 & - \frac{k_{\parallel} v_A^2}{\lambda_{De}^2} \left(1 + \frac{3}{4} b_i \right), \quad (39)
 \end{aligned}$$

$$R = \omega_{dth}^2 k_{\parallel} (\omega_{pd}^2 + k_{\perp}^2 c^2)$$

From Eqs. (38-40), one can easily show that $Q^2 > 4PR$. Thus, the dominant root of Eq.(37) reduces to

$$\omega^2 = \omega_{dth}^2 + k_{\parallel}^2 v_A^2 \left[1 + \left\{ \frac{3}{4} + \frac{T_e}{T_i} \delta \left(1 + \frac{\omega_{pd}^2}{k_{\perp}^2 c^2} \right) \right\} b_i \right], \quad (41)$$

where $v_A = \frac{c\omega_{ci}}{\omega_{pi}}$ is the Alfvén speed and the non-neutrality parameter, $\delta = \frac{n_{i0}}{n_{e0}}$.

Eq. (41) is the dispersion relation of the EM kinetic Alfvén waves in the presence of unmagnetized and cold but mobile dust grains. In absence of the dust component of the plasma, this dispersion relation reduces to that for the usual kinetic Alfvén wave in an electron-ion plasma where the FLR effects arise through the ion-dynamics. Thus, the dust dynamics introduces a new cutoff frequency to the KAW in the dusty plasma and a modification of the FLR effect through the magnetized and hot ions.

Now, including the dust charge fluctuations, Q_{di} given by Eq.(22) in the Poisson Eq. (8) and following the same procedure, we obtain $A_i \cong C_i = 0$ and $B_i \gg A_r, D_r/C_r$, where the subscript i denotes the complex part of a quantity. Thus, we obtain the damping rate of the KAW in the presence of dust charge fluctuation effects from $B_i \approx 0$ where

$$\begin{aligned}
 B_i = & \frac{\sqrt{\pi} \xi_{e0} e^{-\xi_{e0}^2}}{\lambda_{De}^2} + \frac{\sqrt{\pi} \xi_{i0} e^{-\xi_{i0}^2}}{\lambda_{Di}^2} + \frac{\sqrt{\pi}}{\lambda_{Di}^2} \\
 & \left(\xi_{i+} e^{-\xi_{i+}^2} + \xi_{i-} e^{-\xi_{i-}^2} \right) I_1(b_i) e^{-b_i} + \frac{4\pi a_0^2 n_{d0} \omega_{pe}^2}{\omega_r v_{te}} \\
 & \left[e^{e\Phi_G/T_e} \xi_{e0} - e^{-e\Phi_G/T_i} \frac{\delta}{2} \sqrt{\frac{T_e m_e}{T_i m_i}} \left(\frac{1}{\xi_{i0}} + \left(\frac{1}{\xi_{i+}} + \frac{1}{\xi_{i-}} \right) I_1 e^{-b_i} \right) \right] + \frac{2k_{\parallel}^2 \omega_{pd}^2 \gamma}{\omega_r^2 \omega_r}. \quad (42)
 \end{aligned}$$

Here, we assume $\omega = \omega_r + i\gamma$ with $\gamma \ll \omega_r$ and $\xi_{i\pm} = (\omega \pm \omega_{ci})/k_{\parallel} v_{ti}$.

Thus, we obtain the normalized damping rate of the KAW including collisionless Landau damping and the charge fluctuation effects, as

$$\begin{aligned}
 \frac{\gamma}{\omega_r} = & -\sqrt{\frac{\pi}{4}} \frac{\omega_r^2}{\omega_{pd}^2 k_{\parallel}^2 \lambda_{De}^2} \left[\xi_{e0} e^{-\xi_{e0}^2} + \delta \frac{T_e}{T_i} \left(\xi_{i0} e^{-\xi_{i0}^2} \right. \right. \\
 & \left. \left. + \left(\xi_{i+} e^{-\xi_{i+}^2} + \xi_{i-} e^{-\xi_{i-}^2} \right) I_1 e^{-b_i} \right) + \frac{2\sqrt{\pi} a_0^2 n_{d0} v_{te}}{\omega_r} \right. \\
 & \left. \left\{ e^{e\Phi_G/T_e} \xi_{e0} - \frac{\delta}{2} \left(\sqrt{\frac{T_e m_e}{T_i m_i}} \right) e^{-e\Phi_G/T_i} \left(\frac{1}{\xi_{i0}} + \left(\frac{1}{\xi_{i+}} + \frac{1}{\xi_{i-}} \right) I_1 e^{-b_i} \right) \right\} \right]. \quad (43)
 \end{aligned}$$

where ζ 's have been defined earlier. It is noticed from Eq.(43) that the collisionless Landau damping of the KAW is small and the charge fluctuation damping is mainly due to the parallel motion of unmagnetized electrons along the direction of the external magnetic field. By numerical calculation, Juli et al. [18] also showed that the Landau damping of the transverse ordinary Alfvén waves propagating parallel to the magnetic field is negligible.

5. Discussion

In the present paper, we have studied the electromagnetic kinetic Alfvén waves using two-potential theory in uniform dusty plasma in the presence of an external uniform magnetic field. We obtain the dispersion relation, Eq.(41), of the kinetic Alfvén waves in the dusty magnetoplasma. This relation shows a cutoff frequency for the kinetic Alfvén wave due to the hybrid dynamics of the unmagnetized and cold dust particles and the gyrating motion of the ions. Finally we obtain the damping of the kinetic Alfvén wave due to dust charge fluctuation effect. It is observed that the motion of the strongly magnetized electrons along the external magnetic field direction is the main reason for the damping of this electromagnetic wave.

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