

# Performance Comparison of L-MRC Receivers over Nakagami-M Fading and Hoyt Fading Channels

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**Abstract:** *We present exact closed-form expressions for the statistics of the sum of non-identical squared Nakagami-m random variables and it is shown that it can be written as a weighted sum of Erlang distributions. The analysis includes both independent and Nakagami-m cases with distinct average powers and integer-order fading parameters. The proposed formulation significantly improves previously published results which are in the form of infinite sums or higher order derivatives. The obtained formulae can be applied on the performance analysis of maximal-ratio combining diversity receivers operating over Nakagami-m fading channels.*

**Keywords:** Average bit error rate, equal correlation, Hoyt fading channel, MRC receiver, outage probability

## 1. Introduction

Performance analysis of digital wireless communications systems usually deals with complicated and cumbersome statistical tasks. One of them arises in the study of diversity combining receivers operating over Nakagami-m fading channels where the statistics of the sum of squared Nakagami-m random variables (RVs) (or equivalently the sum of Gamma RVs) are required. Well-known applications in the field of mobile radio systems where such sums could be useful are maximal-ratio combining (MRC) and post-detection equal-gain combining (EGC), or in the evaluation of the outage probability in cellular systems with co-channel interferences. The most general approach related to the distribution of the sum of Gamma RVs has been presented by Moschopoulos in where an infinite series representation for the probability density function (PDF) of the sum of independent Gamma RVs, with non-identical parameters, has been proposed. Alouini *et al.* in have extended the result of for the case of arbitrarily Nakagami-m Gamma RVs and studied the performance of MRC and post-detection EGC receivers, as well as the cochannel interference in cellular mobile radio systems. However, to the best of the authors' knowledge, there are not available in the open technical literature any simple closed-form expressions for both PDF and cumulative distribution function (CDF) of the sum of squared nonidentically distributed Nakagami-m RVs. Consequently, there have not been presented any closed-form expressions for the performance metrics of the above mentioned diversity receivers. In this paper, novel closed-form expressions for the PDF and the CDF of the sum of non-identical squared Nakagami-m RVs, with integer-order fading parameters, are derived. Our results include both the statistical independent and correlated cases. Furthermore, in order to reveal the importance of the proposed statistical formulation, we study the performance of L-branch MRC receivers, in the presence of Nakagami-m multipath fading. Exact formulae for the outage probability, the channel average spectral efficiency (SE) and the average symbol error probability (ASEP) for several coherent, non-coherent, binary and multilevel modulation signalings are obtained. After this short introduction, novel closed form

expressions for the PDF and the CDF of the sum of squared Nakagami-m RVs are obtained. The theoretical results of applied to derive useful expressions for performance metrics of MRC diversity receivers, operating over Nakagami-m fading channels. Finally, in Section V, useful concluding remarks are provided.

## 2. Existing System

The severity of fading on mobile communication channels calls for the combining of multiple diversity sources to achieve acceptable error rate performance. Traditional approaches perform the combining of the different diversity sources using either the conventional selective diversity combining (CSC), equal-gain combining (EGC), or maximal-ratio combining (MRC) schemes. CSC and MRC are the two extremes of compromise between performance quality and complexity. This paper presents a generalized diversity selection combining (GSC) scheme in which only those diversity branches whose energy levels are above a specified threshold are combined. Doing so, the proposed scheme will have a bit error (BER) performance that is upper- and lower-bounded by those of the CSC and MRC schemes respectively. Simulation results for the performances of this scheme over Nakagami-m (1960) fading channels are shown. In this paper, the performance of multiuser CDMA systems with different space time code schemes is investigated over Nakagami-m fading channel. Low-complexity multiuser receiver schemes are developed for space-time coded CDMA systems with perfect and imperfect channel state information (CSI). The schemes can make full use of the complex orthogonality of space-time coding to obtain the linear decoding complexity, and thus simplify the exponential decoding complexity of the existing scheme greatly. Moreover, it can achieve almost the same performance as the existing scheme. Based on the bit error rate (BER) analysis of the systems, the theoretical calculation expressions of average BER are derived in detail for both perfect CSI and imperfect CSI, respectively. As a result, tight closed-form BER expressions are obtained for space-time coded CDMA with orthogonal spreading code, and approximate closed-form BER expressions are attained for space-time coded CDMA with quasi-orthogonal spreading

code. Computer simulation for BER shows that the theoretical analysis and simulation are in good agreement. The results show that the space-time coded CDMA systems have BER performance degradation for imperfect CSI.

The channel has been assumed to be slow, frequency nonselective, with Hoyt fading statistics. The receiver has been provided with L antennas for spatial diversity reception of fading signals. The complex low pass equivalent of the signal received by the lth antenna,  $l=1, 2, \dots, L$ , over one bit duration  $T_b$  can be

### 3. Channel And Receiver

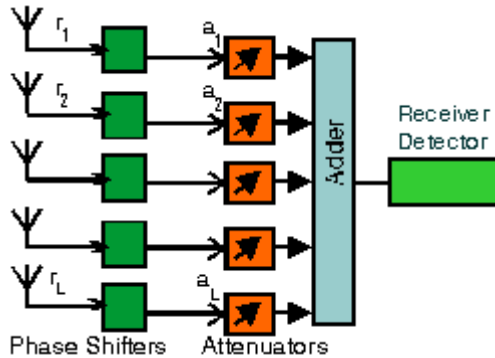


Figure 1

$$r_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t), \quad (1)$$

Where  $s(t)$  is the transmitted bit signal with energy  $E_b$  and  $n_l(t)$  is the complex Gaussian noise having zero mean and two-sided power spectral density  $2N_0$ . The RV  $\phi_l$  denotes the instantaneous phase and  $\alpha_l$  is the Hoyt distributed fading amplitude having PDF given by

$$p(\alpha_l) = \frac{(1+q^2)\alpha_l}{q\Omega_l} e^{-\frac{(1+q^2)\alpha_l^2}{4q^2\Omega_l}} I_0\left[\frac{(1-q^4)\alpha_l^2}{4q^2\Omega_l}\right], \alpha_l \geq 0, \quad (2)$$

Where  $\Omega_l = E[\alpha_l^2]$ ,  $q \in [0, 1]$  is the Hoyt fading parameter and  $I_0(\cdot)$  is the modified Bessel function of the first kind and zero order. In the MRC receiver, the received signals from all diversity antennas are co phased, proportionally weighted, and combined to maximize the output SNR. It is followed by a detector corresponding to the modulation scheme employed. The output SNR of the MRC combiner can be given by

$$\gamma_{mrc} = \sum_{l=1}^L \gamma_l = \frac{E_b}{N_0} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2), \quad (3)$$

$$\gamma_l = \frac{E_b}{N_0} \alpha_l^2$$

Where  $\gamma_l$  is the SNR of the  $l^{th}$  receiving branch and the average received SNR is

$$\bar{\gamma}_l = \frac{E_b}{N_0} E[\alpha_l^2] = \frac{E_b \Omega_l}{N_0}$$

We assume  $\bar{\gamma}_l = \bar{\gamma}, \forall l$

$$\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt. \quad (4)$$

#### 4.1. PDF of Output SNR

From (3), it can be observed that the PDF of  $\gamma_{mrc}$ . From [8], a Hoyt RV  $\alpha_l$  can be modeled as  $|\mathbf{Z}_l|$ , where  $\mathbf{Z}_l = X_l + jY_l$  with  $X_l$  and  $Y_l$  are independent Gaussian RVs with zero mean and unequal variances  $\sigma_x^2$  and  $\sigma_y^2$  respectively. In this representation the Hoyt RV  $\alpha_l$  has the PDF given in (2), where the fading parameter  $q = \sigma_y/\sigma_x$ . Assuming  $X_m$  and  $X_n$  ( $Y_m$  and  $Y_n$ ) are Nakagami- $m$  with correlation coefficient  $\rho_{m,n}$ , it can be shown that RVs  $\alpha_m$  and  $\alpha_n$  are also correlated with the same correlation coefficient  $\rho_{m,n}$ . From the above modeling we can express  $\alpha^2 = X^2 + Y^2$ , where  $X^2 = X_1^2 + X_2^2 + \dots + X_L^2$  and  $Y^2 = Y_1^2 + Y_2^2 + \dots + Y_L^2$  are independent RVs while  $X_l$  ( $Y_l$ )s are correlated with correlation coefficient  $\rho_{m,n}, 1 \leq m, n \leq L$ .

Obtained by obtaining the PDF of  $\alpha^2 = \alpha_1^2 + \alpha_2^2$

#### 4. Performance of the MRC Receiver

Followed by a scaling of the PDF corresponding to the factor  $(E_b/N_0)$ . As a standard approach for the derivation of the PDF of  $\alpha^2$  we require the joint density function of  $\alpha$  which is not known for correlated Hoyt RVs  $\alpha$ s. However, using the Hoyt RV model in [8] and the expression for the sum of PDF of squared of multivariate Gaussian RVs by Garland in [9], it is possible to derive the PDF of  $\alpha^2$  for the special case of equally Nakagami- $m$   $\alpha$ s. It is presented as below

The PDF of  $X^2(Y^2)$ , for any arbitrary correlation  $\rho_{m,n}$  is not known. For the case of equal correlation, i.e.  $\rho_{m,n} = \rho, \forall m, n$  this PDF can be obtained from [9, (4)] with suitable substitutions (i.e.,  $\lambda = 1/2$  and  $k=L$ ), and then performing a scaling on the PDF corresponding to a factor  $L$ . The PDF can be given as

$$f_{X^2}(x^2) = \frac{x^{L-1} e^{-\frac{x}{2(1-\rho)}} {}_1F_1\left(\frac{1}{2}, \frac{L}{2}; \frac{L\rho x}{2\sigma_x^2(1-\rho)(1+(L-1)\rho)}\right)}{(2\sigma_x^2)^{\frac{L}{2}} (1-\rho)^{\frac{L-1}{2}} \Gamma(L/2) \sqrt{1+(L-1)\rho}}, \quad (5)$$

#### 5. Nakagami-M Fading

Hereafter, we briefly summarize the main steps followed in solving (2) for the case of two Nakagami- $m$  fading envelopes. Upon substituting (3),  $i = 1, 2$ , in (2), we complete the square in the exponential, make the change of variable

This results in the following novel expression for the pdf of the normalized sum of uncorrelated Nakagami- $m$  variables with arbitrary parameters but with  $2m$

where  $\Omega_p = \Omega_1\Omega_2$ , and  $\bar{m} = m_1\Omega_2 + m_2\Omega_1$ . The cdf  $F_R(r) = \int_0^r p_R(x)dx$  is obtained by making use of the series expansion (see [27, eq. (8.350.1)])

$$\gamma(\alpha, x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(\alpha + n)} x^{\alpha+n} \tag{6}$$

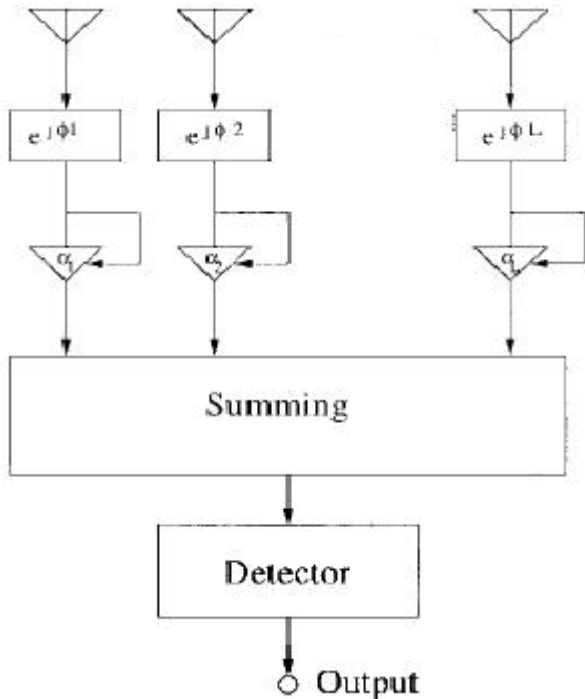


Figure 2: Maximal ration combining system for diversity reception

Normalized sum of rician variables with arbitrary parameters is, hence, derived in infinite series form as

$$p_R(r) = \frac{2\sqrt{2}}{\Omega_{r1}\Omega_{r2}} e^{-\left[\frac{A_1^2}{\Omega_{r1}} + \frac{A_2^2}{\Omega_{r2}}\right]} e^{-\frac{r^2}{\Omega_{r1} + \Omega_{r2}}} \times \sum_{k=0}^{\infty} \frac{\left(\frac{A_1}{\Omega_{r1}}\right)^{2k}}{(k!)^2} \sum_{l=0}^{\infty} \frac{\left(\frac{A_2}{\Omega_{r2}}\right)^{2l}}{(l!)^2} \left[\frac{\Omega_{r1}\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}}\right]^{k+l+\frac{3}{2}} \times \sum_{i=0}^{2k+1} \binom{2k+1}{i} \left[\frac{2\Omega_{r1}}{\Omega_{r1} + \Omega_{r2}} r^2\right]^{k+\frac{1}{2}-\frac{i}{2}} \times \sum_{j=0}^{2l+1} \binom{2l+1}{j} \left[\frac{2\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}} r^2\right]^{l+\frac{1}{2}-\frac{j}{2}} \times \left[ (-1)^i \gamma\left(\frac{i+j+1}{2}, \frac{2\Omega_{r1}}{\Omega_{r1} + \Omega_{r2}} r^2\right) + (-1)^j \gamma\left(\frac{i+j+1}{2}, \frac{2\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}} r^2\right) \right] \tag{7}$$

Note that an expression for the pdf of the sum of two Noncentral Chi variables (the rice pdf being a special Case of the No central chi pdf) with identical parameters had been previously given in [30] in terms of the Less commonly used

Hh function. Following the same Steps as in the previous section, the cdf is found as

$$F_R(r) = \frac{\sqrt{2}}{\Omega_{r1}\Omega_{r2}} e^{-\left[\frac{A_1^2}{\Omega_{r1}} + \frac{A_2^2}{\Omega_{r2}}\right]} \times \sum_{k=0}^{\infty} \frac{\left(\frac{A_1}{\Omega_{r1}}\right)^{2k}}{(k!)^2} \sum_{l=0}^{\infty} \frac{\left(\frac{A_2}{\Omega_{r2}}\right)^{2l}}{(l!)^2} \left[\frac{\Omega_{r1}\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}}\right]^{k+l+\frac{3}{2}} \times \sum_{i=0}^{2k+1} \binom{2k+1}{i} \left[\frac{2\Omega_{r1}}{\Omega_{r1} + \Omega_{r2}}\right]^{k+\frac{1}{2}-\frac{i}{2}} \times \sum_{j=0}^{2l+1} \binom{2l+1}{j} \left[\frac{2\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}}\right]^{l+\frac{1}{2}-\frac{j}{2}} \times \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \left(\frac{i+j+1}{2} + n\right)} \times \frac{\gamma\left(k+l+n+2, \frac{2}{\Omega_{r1} + \Omega_{r2}} r^2\right)}{\left[\frac{2}{\Omega_{r1} + \Omega_{r2}}\right]^{k+l+n+2}} \times \left[ (-1)^i \left(\frac{2\Omega_{r1}}{\Omega_{r1} + \Omega_{r2}}\right)^{\frac{i+j+1}{2}+n} + (-1)^j \left(\frac{2\Omega_{r2}}{\Omega_{r1} + \Omega_{r2}}\right)^{\frac{i+j+1}{2}+n} \right] \tag{8}$$

As useful side results, the LCR of the combined envelope can be obtained in closed form as

$$N_R(r) = \frac{\sqrt{\pi} f_m}{2} \sqrt{\frac{\Omega_1}{m_1} + \frac{\Omega_2}{m_2}} p_R(r) \tag{9}$$

Where  $f_m$  is the Doppler frequency [1],  $p_R(r)$  is given by and the AFD is given as,

$$\tau_R(r) = N_R(r)/F_R(r) \tag{10}$$

Where is given by in the case of equal parameters

The pdf and cdf of r are given in respectively, for any real  $m \geq 0.5$ , and the LCR and AFD reduce to that in

### 6. Probability Distributions

Let  $r = (r_1+r_2)/2$  be the normalized amplitude of the signal at the output of dual-branch EGC, with  $r_1$  being the envelope of the fading that affects the signal on the  $i$ th antenna. Then, the exact pdf of  $r$  is obtained as

$$p_R(r) = \sqrt{2} \int_0^{r\sqrt{2}} p_{R1,R2}(r_1, r_2 = r\sqrt{2} - r_1) dr_1 \tag{11}$$

Where  $p_{R1}, p_{R2}(r_1, r_2)$  is the joint pdf of  $r_1$  and  $r_2$ . for independent diversity branches, which is the case considered in this paper, (1) readily reduces to

$$p_{R_1}(r) = \sqrt{2} \int_0^{r\sqrt{2}} p_{R_1}(r_1) p_{R_2}(r_2 = r\sqrt{2} - r_1) dr \quad (12)$$

The pdfs  $p_{R_1}(r)$  and  $p_{R_2}(r)$  are chosen among three fading distributions, the nakagami-m distribution

$$p_{R_i}(r) = \frac{2}{\Gamma(m_i)} \left(\frac{m_i}{\Omega_i}\right) r^{2m_i-1} \exp\left(-\frac{m_i}{\Omega_i} r^2\right), \quad r > 0 \quad (13)$$

where  $m_i \geq 0.5$  and  $\Omega_i = E[R_i^2]$  ( $E[\cdot]$

is the expectation),  $i=1,2$ , are the Nakagami-m fading parameter and the average fading power of channel  $i$ , respectively, and  $\Gamma(\cdot)$  is the gamma function the rice distribution is

$$p_{R_i}(r) = \frac{2r}{\Omega_{ri}} \exp\left(-\frac{r_i^2 + A_i^2}{\Omega_{ri}}\right) I_0\left(\frac{2rA_i}{\Omega_{ri}}\right), \quad r > 0 \quad (14)$$

Where  $A_i$  is the amplitude of the LOS component, and  $\Omega_{ri}$  is the average power of the scatter component such that  $E[R_i^2] = A_i^2 + \Omega_{ri}$ , and  $I_0(\cdot)$  is the modified Bessel function of the first kind of order 0. The Hoyt distribution is

$$p_{R_i}(r) = \frac{2r}{\Omega_{1i}\Omega_{2i}} \exp\left(-\frac{r_i^2}{2}\left(\frac{1}{\Omega_{1i}} + \frac{1}{\Omega_{2i}}\right)\right) \times I_0\left(\frac{r^2}{2}\left(\frac{1}{\Omega_{2i}} - \frac{1}{\Omega_{1i}}\right)\right), \quad r > 0 \quad (15)$$

Where  $\Omega_{1i}$  and  $\Omega_{2i}$  are the average powers of the underlying in-phase and Gaussian quadrature components with  $\Omega_{1i} \geq \Omega_{2i}$  such that  $E[R_i^2] = \Omega_{1i} + \Omega_{2i}$ ,  $i=1,2$ . The significance of the above three distributions in the context of fading channel modeling is well distributed, For example, in and hence, is not elaborated upon here for brevity. By substituting the combinations of the pdfs presented above into, we are able to obtain several new pdfs for the sums of two independent nakagami-m, rician, and Hoyt random variables, and combinations thereof, as detailed below.

### 7. HOYT Fading

In the case of fading envelopes, we substitute and  $i=1,2$  in and follow a development similar to that of the previous section, which leads to the following infinite series expressions for the pdf and cdf of the normalized sum of two Hoyt variables with arbitrary parameters.

$$P_s(\gamma) = \sum_{l=1}^K \int_0^{\Theta_l} a_l(\theta) e^{-\phi_l(\theta)\gamma} d\theta \quad (16)$$

### 8. Procedure For Nakagami-M RVs

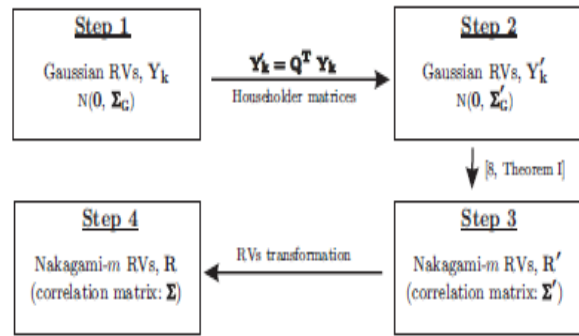


fig:3 The four-step procedure for generating Nakagami-m RVs with an arbitrary correlation matrix. ( $\Sigma$  and  $\Sigma'$  are the desired and its similar power correlation matrices, respectively.)

$$p_R(r) = \frac{\sqrt{2}a_1a_2}{2[b_1 + b_2]^{\frac{3}{2}}} e^{-2r^2 \frac{b_1b_2}{b_1+b_2}} \times \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left[\frac{d_1}{2(b_1 + b_2)}\right]^{2k} \times \sum_{l=0}^{\infty} \frac{1}{(l!)^2} \left[\frac{d_2}{2(b_1 + b_2)}\right]^{2l} \times \sum_{i=0}^{4k+1} \binom{4k+1}{i} \left[\frac{2b_2^2}{b_1 + b_2} r^2\right]^{2k+\frac{1}{2}-\frac{i}{2}} \times \sum_{j=0}^{4l+1} \binom{4l+1}{j} \left[\frac{2b_1^2}{b_1 + b_2} r^2\right]^{2l+\frac{1}{2}-\frac{j}{2}} \times \left[(-1)^i \gamma\left(\frac{i+j+1}{2}, \frac{2b_2^2}{b_1 + b_2} r^2\right) + (-1)^j \gamma\left(\frac{i+j+1}{2}, \frac{2b_1^2}{b_1 + b_2} r^2\right)\right] \quad (17)$$

$$F_R(r) = \frac{\sqrt{2}a_1a_2}{4[b_1 + b_2]^{\frac{3}{2}}} \times \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left[\frac{d_1}{2(b_1 + b_2)}\right]^{2k} \times \sum_{l=0}^{\infty} \frac{1}{(l!)^2} \left[\frac{d_2}{2(b_1 + b_2)}\right]^{2l} \times \sum_{i=0}^{4k+1} \binom{4k+1}{i} \left[\frac{2b_2^2}{b_1 + b_2} r^2\right]^{2k+\frac{1}{2}-\frac{i}{2}} \times \sum_{j=0}^{4l+1} \binom{4l+1}{j} \left[\frac{2b_1^2}{b_1 + b_2} r^2\right]^{2l+\frac{1}{2}-\frac{j}{2}} \times \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \left(\frac{i+j+1}{2} + n\right)} \times \frac{\gamma\left(2(k+l) + n + 2, 2 \frac{b_1b_2}{b_1+b_2} r^2\right)}{\left[2 \frac{b_1b_2}{b_1+b_2}\right]^{2(k+l)+n+2}} \quad (18)$$



$$\times \left[ (-1)^i \left( \frac{2b_2^2}{b_1 + b_2} \right)^{\frac{i+j+1}{2}+n} + (-1)^j \left( \frac{2b_1^2}{b_1 + b_2} \right)^{\frac{i+j+1}{2}+n} \right] \quad (19)$$

where  $a_i = 2/\sqrt{\Omega_{1i}\Omega_{2i}}$ ,  $b_i = 1/2(1/\Omega_{1i} + 1/\Omega_{2i})$ , and  $d_i = 1/2(1/\Omega_{2i} - 1/\Omega_{1i})$ ,  $i = 1, 2$ .

### 9. Performance Analysis

We use the classical approach to derive the error probabilities For fading channels by directly averaging the conditional SER  $P_s(r)$  over the pdf of the SNR after combining, which leads to the unconditional SER?

$$P_s = \int_0^\infty P_s(\gamma) p_\Gamma(\gamma) d\gamma \quad (20)$$

Where  $r=r^2E_s/N_0$  is the instantaneous SNR per symbol with pdf of  $p_r(r) = PR(\sqrt{r}/(E_s/N_0))/(2\sqrt{r}E_s/N_0)$ .  $E_s=E_b \log_2 M$  is the energy per symbol (with  $E_b$  as the energy per bit and  $M$  as The constellation size), and  $N_0$  is the single-sided power spectral density of the additive white Gaussian noise. The conditional SER for several different M-ary modulations, such as M-PSK, M-PSK ( $M=2, 4$ ) can be expressed in a unified manner Where the parameter  $K$ ,  $a_1(\theta) = (3\cos(\theta)-1)/(2\cos^3(\theta)-1)$ . The SER<sub>s</sub> for all the cases considered are thus obtained by substituting the corresponding pdf<sub>s</sub> derived earlier along with and using the equation to solve the integral. The resulting explicit expressions for the SER<sub>s</sub> are listed below

### 9.1. Nakagami-m fading

$$P_s = \frac{2\sqrt{2}\Gamma(m_1 + m_2)}{\Gamma(m_1)\Gamma(m_2)} \left( \frac{m_1}{\bar{\gamma}_1} \right)^{m_1} \left( \frac{m_2}{\bar{\gamma}_2} \right)^{m_2} \left[ \frac{\bar{\gamma}_p}{\bar{m}} \right]^{m_1+m_2-\frac{1}{2}} \times \sum_{i=0}^{2m_1-1} \binom{2m_1-1}{i} \alpha_1^{m_1-\frac{1+i}{2}} \times \sum_{j=0}^{2m_2-1} \binom{2m_2-1}{j} \alpha_2^{m_2-\frac{1+j}{2}} \times \frac{1}{i+j+1} \sum_{l=1}^K \int_0^{\Theta_l} d\theta a_c(\theta) \times \left[ \frac{(-1)^i \alpha_1^{\frac{i+j+1}{2}} {}_2F_1\left(1, m_1+m_2; \frac{i+j+3}{2}; \frac{\alpha_1}{\alpha_1+\beta(\theta)}\right)}{(\alpha_1 + \beta(\theta))^{m_1+m_2}} \right] + \frac{(-1)^j \alpha_2^{\frac{i+j+1}{2}} {}_2F_1\left(1, m_1+m_2; \frac{i+j+3}{2}; \frac{\alpha_2}{\alpha_2+\beta(\theta)}\right)}{(\alpha_2 + \beta(\theta))^{m_1+m_2}} \quad (21)$$

### 10. Outage Probability

Probability  $P_{out}$  is defined as the percentage of time that the SER  $P_s$  exceeds a certain threshold  $P_{s,t}$  or alternatively, that the

instantaneous SNR per symbol is below a certain threshold  $r_t = P_s^{-1}(P_{s,t})$  (which usually needs to be solved for numerically)

Which

$$P_{out} = P(\gamma < \gamma_t) = P\left(r < \left[\frac{\gamma_t}{E_b/N_0}\right]^{\frac{1}{2}}\right) \quad (22)$$

can be alternatively expressed as?

$$P_{out} = F_R\left(\left[\frac{\gamma_t}{E_b/N_0}\right]^{\frac{1}{2}}\right) \quad (23)$$

where  $F_R(r)$  is the cdf. Hence, the outage probabilities for all the cases considered above can be deduced by simply substituting the appropriate cdf derived earlier.

### 10.1. Nakagami-m Architecture

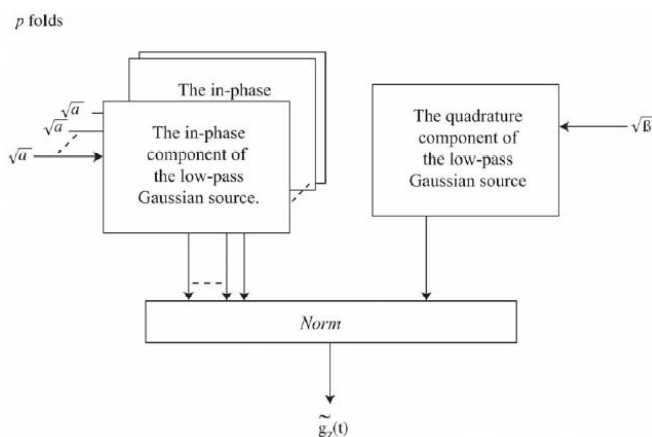


Figure 4: Nakagami Simulator Architecture

### 10.2. Algorithm

Given cross-Nakagami-m process, variance vector  $P_z=R_z(i,i)$  the following steps are taken to generate correlated Nakagami-m fading channel.

Determine the cross-Nakagami-m of gamma RV using algorithm

- Determine  $R_x$  using equations
- Determine  $X_k$  using method of cholskey decomposition
- Determine gamma RV  $Y$  as

$$Y = X_{k2k=12m} \sum_{2m=integer} \alpha X_{k2k=1p} \sum + \beta X_p + 12 \text{ otherwise}$$

- The desired Nakagami-m vector  $Z$  is given by

$$Z = Y (1/2)$$

### 11. Proposed System

The severity of fading on mobile communication channels calls for the combining of multiple diversity sources to achieve acceptable error rate performance. Traditional approaches perform the combining of the different diversity sources using either: the conventional selective diversity combining (CSC), equal-gain combining (EGC), or maximal-ratio combining

(MRC) schemes. CSC and MRC are the two extremes of compromise between performance quality and complexity. This paper presents a generalized diversity selection combining (GSC) scheme in which only those diversity branches whose energy levels are above a specified threshold are combined. Doing so, the proposed scheme will have a bit error (BER) performance that is upper- and lower-bounded by those of the CSC and MRC schemes respectively. Simulation results for the performances of this scheme over Nakagami-m (1960) fading channels are shown

**12. Average Bit Error Rate**

The aber can be obtained by averaging the conditional bit error rate over the PDF of the receiver output instantaneous SNR given as

$$P_e(\bar{\gamma}) = \int_0^{\infty} p_e(\epsilon|\gamma) f_{\gamma}(\gamma) d\gamma, \tag{24}$$

Where  $P_e(\epsilon|\gamma)$  is the conditional BER corresponding to the modulation scheme used. Binary coherent modulations: for binary coherent modulations, the expression for the conditional BER can be given as  $P_e(\epsilon|\gamma) = Q(\sqrt{2a\gamma})$ , where  $a=0.5, 1$  for CFSK and CPSK modulations respectively putting  $P_e(\epsilon|\gamma)$  and into  $P_e(\bar{\gamma})$  and Solving the intergral (expressing  ${}_1F_1(\dots)$  in infinite series and using A-(6), A-(8a)], an expression for aber can be given as

$$P_{e,coh}(\bar{\gamma}) = \frac{(1-\rho)q^L \sqrt{a/\eta}}{2\pi^2 [1+(L-1)\rho]^{k_1+k_2}} \sum_{k_1, k_2=0}^{\infty} \frac{\Gamma(k_1 + \frac{1}{2})\Gamma(k_2 + \frac{1}{2})}{k_1! k_2!} \times \frac{(\frac{L}{2} + k_1)_{k_1} \Gamma(L + \lambda_{13} + \frac{1}{2}) [1-q^2]^{k_1}}{(L + \lambda_{13})! q^{2k_1} (1+a/\eta)^{L+k_1+\frac{1}{2}} [1+(L-1)\rho]^{k_1}} \times {}_2F_1\left(1, L + \lambda_{13} + \frac{1}{2}; L + \lambda_{13} + 1; -\frac{\eta}{a+\eta}\right) \tag{25}$$

Binary Non-coherent modulations: for binary non-coherent modulations, the conditional BER is given as  $P_e(\epsilon|\gamma) = \frac{1}{2} \exp(-\gamma)$ , where  $a=0.5, 1$  for NCFSK and DPSK modulations, respectively putting  $P_e(\epsilon|\gamma)$  and into  $P_e(\bar{\gamma})$  and solving the integralan expression for ABER can be obtained as

$$P_{e,noncoh}(\bar{\gamma}) = \frac{(1-\rho)q^L}{2\pi [1+(L-1)\rho]^{k_1+k_2}} \sum_{k_1, k_2=0}^{\infty} \frac{\Gamma(k_1 + \frac{1}{2})\Gamma(k_2 + \frac{1}{2})}{k_1! k_2! q^{2k_2}} \times \frac{[(1+a/\eta) - (1-q^2)]^{\frac{L}{2}-k_1}}{(1+a/\eta)^{\frac{L}{2}+k_1}} \left[ \frac{q^2 \rho L}{1+(L-1)\rho} \right]^{k_1} \tag{26}$$

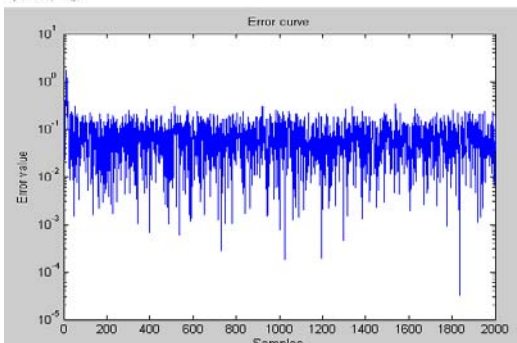


Figure 5: Hoyt fading simulation result

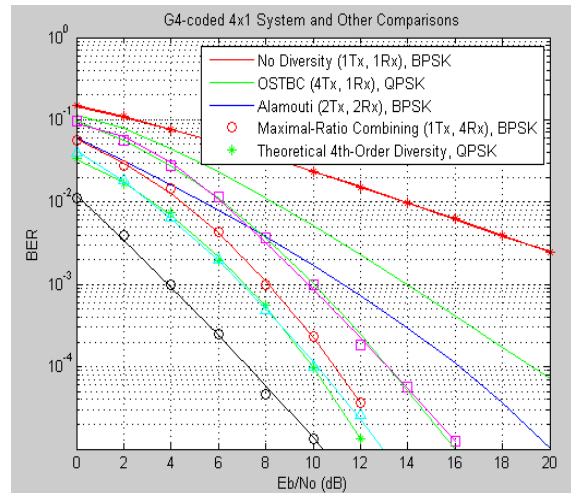


Figure 6: Nakagami-m simulation result

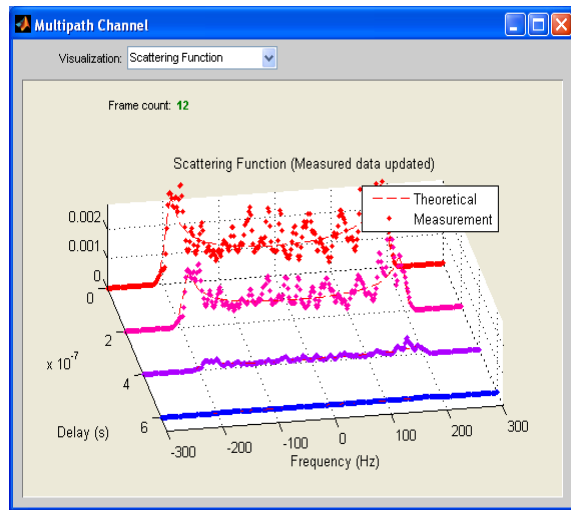


Figure 7: Nakagami-m fading scattering function

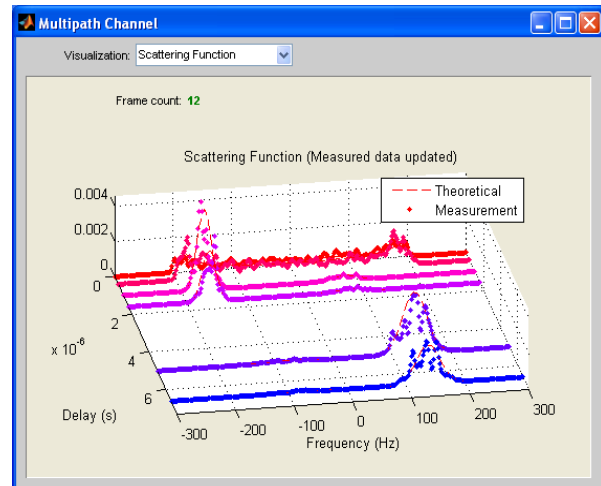


Figure 8: Hoyt fading channel scattering function

**13. Conclusions**

In this paper, we analyze the performance of anL-MRC receiver in equally Nakagami-m fading and Hoyt fading channels and obtain mathematical expressions for the PDF of the MRC output SNR, average SNR and outage probability and ABER expressions for binary, coherent, and noncoherent modulations. Numerically evaluated results have been plotted and the impact of fading Nakagami-m on the receiver

performances studied. The obtained expressions are in the form of infinite series containing hypergeometric functions. The numerical results have been compared with available special case results also Monte Carlo simulations outcome are closely matching with numerically obtained results.

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