

# Development of Space Vector PWM control Technique for an Eleven Phase Voltage Source Inverter

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**Abstract:** *Multiphase Variable Speed Drives have found widespread applications in recent times, especially in the area of ship propulsion systems. Such drives almost invariably require converters with a suitable number of output phases. Pulse Width Modulation techniques have been the preference in recent times for the control of power electronics converters. Space Vector Pulse Width Modulation technique applied on multi-phase converters allows us an efficient implementation of such converters. This paper concentrates on the implementation of the Space Vector Modulation (SVM) in an 11-phase voltage source inverter (VSI) system. There are a total of  $2^{11} = 2048$  space vectors for such an inverter. But out of these only 22 large vectors and 2 zero vectors are necessary for SVM implementation. The symmetrical sequence type of implementation is chosen for the SVM and its implementation method is discussed.*

**Keywords:** Space Vector Modulation, multiphase, 11-phase VSI, Pulse Width Modulation.

## 1. Introduction

Conventionally three-phase machines and their corresponding machine drives have served as the electric drive systems preferred for most applications. This is because three-phase systems had the advantage of being able to provide higher power when compared to their single or two-phase counterparts. Further, rotating magnetic field, an important aspect in many electrical machines, is possible only with poly-phase supply systems. But most three-phase motors are interfaced with the supply-system via power-electronic converters. This offers an advantage wherein the number of machine phases does not have to be restricted to three; the only constraint is that the converter must be able to feed the machine. This resulted in a great interest being shown in multiphase machines although utility lines were generally designed only for single or three-phase. A five-phase induction motor was suggested way back in 1969 [1]. Following this multi-phase motors and their drives have seen significant improvements, primarily due to their advantages over three-phase systems. A multi-phase machine drive can have lower space-harmonic content and greater fault-tolerance. For example, a fault on one-phase of a three-phase machine can render the machine inoperable whereas this is not the case for a fault on one-phase of a 15-phase machine. Following the development of multi-phase machines, their drive systems have improved by leaps and bounds. Various modulation strategies are being researched in order to arrive at an efficient implementation of multi-phase converters. Applications such as more-electric aircraft, electric ship propulsions and traction require high-power multi-phase ac systems and their efficiency requirement is also considerably

high. In-order to achieve this, nine-phase, twelve-phase and fifteen-phase drives have been explored in the past.

Generally, eleven-phase motor drives have been largely overlooked since, among high number of phases, the above-mentioned systems are preferred as they are more common and possess a phase-number that is a multiple of 3. Due to this eleven-phase induction machines lag behind their counterparts when it comes to an efficient modulation strategy.

Traditionally different techniques such as ramp comparison, hysteresis current control and other strategies have been followed. Space Vector Modulation (SVM) concentrates on decoupling the stator components and allows the whole inverter to be addressed as a single entity. The benefits of SVM have often been discussed in sufficient literature, especially when pertaining to machine requiring high power and highly efficient variable speed drives. In this paper a modulation strategy for an inverter feeding an 11-phase machine has been developed.

## 2. Eleven-phase Voltage Source Inverters

### 2.1 Circuit

An eleven-phase inverter is shown in Fig.1. Consider the phase voltages to be  $V_a, V_b, V_c, V_d, V_e, V_f, V_g, V_h, V_i, V_j$  and  $V_k$ . The general modeling of an n-phase system is discussed in [2].

The symmetrical system can be represented in an 11-dimensional space or alternatively may be represented by five two-dimensional sub-spaces and a single-dimensional sub-space.

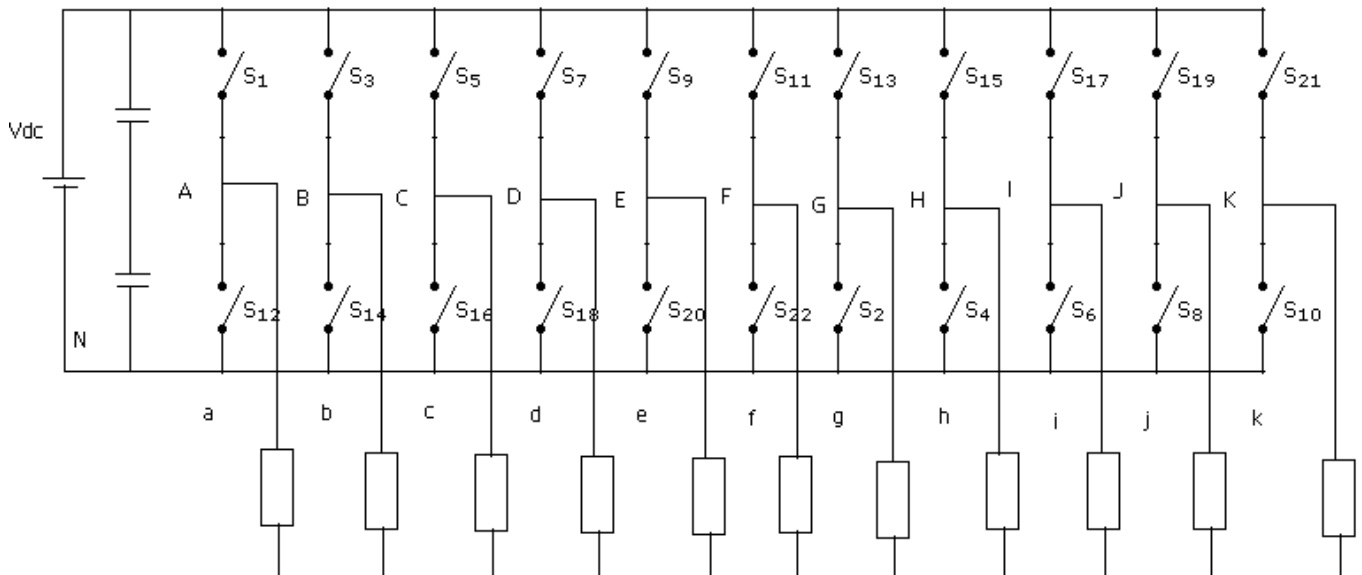


Figure 1: Eleven-phase Voltage Source Inverter

$$C = \begin{bmatrix} \alpha & 1 & \cos \alpha & \cos 2\alpha & \cos 3\alpha & \cos 4\alpha & \cos 5\alpha & \cos 5\alpha & \cos 4\alpha & \cos 3\alpha & \cos 2\alpha & \cos \alpha \\ \beta & 0 & \sin \alpha & \sin 2\alpha & \sin 3\alpha & \sin 4\alpha & \sin 5\alpha & -\sin 5\alpha & -\sin 4\alpha & -\sin 3\alpha & -\sin 2\alpha & -\sin \alpha \\ x_1 & 1 & \cos 2\alpha & \cos 4\alpha & \cos 6\alpha & \cos 8\alpha & \cos 10\alpha & \cos 10\alpha & \cos 8\alpha & \cos 6\alpha & \cos 4\alpha & \cos 2\alpha \\ y_1 & 0 & \sin 2\alpha & \sin 4\alpha & \sin 6\alpha & \sin 8\alpha & \sin 10\alpha & -\sin 10\alpha & -\sin 8\alpha & -\sin 6\alpha & -\sin 4\alpha & -\sin 2\alpha \\ x_2 & 1 & \cos 3\alpha & \cos 6\alpha & \cos 9\alpha & \cos 12\alpha & \cos 15\alpha & \cos 15\alpha & \cos 12\alpha & \cos 9\alpha & \cos 6\alpha & \cos 3\alpha \\ y_2 & 0 & \sin 3\alpha & \sin 6\alpha & \sin 9\alpha & \sin 12\alpha & \sin 15\alpha & -\sin 15\alpha & -\sin 12\alpha & -\sin 9\alpha & -\sin 6\alpha & -\sin 3\alpha \\ x_3 & 1 & \cos 4\alpha & \cos 8\alpha & \cos 12\alpha & \cos 16\alpha & \cos 20\alpha & \cos 20\alpha & \cos 16\alpha & \cos 12\alpha & \cos 8\alpha & \cos 4\alpha \\ y_3 & 0 & \sin 4\alpha & \sin 8\alpha & \sin 12\alpha & \sin 16\alpha & \sin 20\alpha & -\sin 20\alpha & -\sin 16\alpha & -\sin 12\alpha & -\sin 8\alpha & -\sin 4\alpha \\ x_4 & 1 & \cos 5\alpha & \cos 10\alpha & \cos 15\alpha & \cos 20\alpha & \cos 25\alpha & \cos 25\alpha & \cos 20\alpha & \cos 15\alpha & \cos 10\alpha & \cos 5\alpha \\ y_4 & 0 & \sin 5\alpha & \sin 10\alpha & \sin 15\alpha & \sin 20\alpha & \sin 25\alpha & -\sin 25\alpha & -\sin 20\alpha & -\sin 15\alpha & -\sin 10\alpha & -\sin 5\alpha \\ x_5 & 1 & \cos 6\alpha & \cos 12\alpha & \cos 18\alpha & \cos 24\alpha & \cos 30\alpha & \cos 30\alpha & \cos 24\alpha & \cos 18\alpha & \cos 12\alpha & \cos 6\alpha \\ y_5 & 0 & \sin 6\alpha & \sin 12\alpha & \sin 18\alpha & \sin 24\alpha & \sin 30\alpha & -\sin 30\alpha & -\sin 24\alpha & -\sin 18\alpha & -\sin 12\alpha & -\sin 6\alpha \\ 0_0 & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0_1 & -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Figure 2: Clarke’s decoupling transformation matrix for 11-phase

This single-dimensional sub-space represents the zero-sequence excitation which need not be applied for an isolated neutral point system. The sub-spaces may be represented by a set of equations given by Clarke’s decoupling transformation matrix [3], shown in Fig.2.

The first two rows in the above matrix provide the space vector in the  $\alpha$ - $\beta$  subspace. The stator to rotor coupling occurs only due to these components and they lead to fundamental torque and flux production. If the flux around the air gap is sinusoidal, the other x-y pairs of components do not contribute to torque production. The x-y components may produce large stator currents and correspond to current and voltage harmonics. Since these pairs of x-y components are not electromechanically related to the system unless space harmonics appear, it is sufficient to apply rotational transformation only to  $\alpha$ - $\beta$  equations [4]. The penultimate row of the matrix represents the zero-sequence which is left

unexcited for isolated neutral systems and the  $0_1$  components do not exist for systems with an odd phase number.

### 2.2 Space Vector Model

For a pure sinusoidal, balanced supply the phase voltages may be represented as

$$v_a = \sqrt{2}V \cos \omega t \tag{1}$$

$$v_b = \sqrt{2}V \cos(\omega t - \frac{2\pi}{11}) \tag{2}$$

$$v_c = \sqrt{2}V \cos(\omega t - \frac{4\pi}{11}) \tag{3}$$

$$v_d = \sqrt{2}V \cos(\omega t - \frac{6\pi}{11}) \tag{4}$$

$$v_h = \sqrt{2}V \cos(\omega t + \frac{8\pi}{11}) \tag{8}$$

$$v_i = \sqrt{2}V \cos(\omega t + \frac{6\pi}{11}) \tag{9}$$

$$v_j = \sqrt{2}V \cos(\omega t + \frac{4\pi}{11}) \tag{10}$$

$$v_e = \sqrt{2}V \cos(\omega t - \frac{8\pi}{11}) \tag{5}$$

$$v_f = \sqrt{2}V \cos(\omega t - \frac{10\pi}{11}) \tag{6}$$

$$v_g = \sqrt{2}V \cos(\omega t + \frac{10\pi}{11}) \tag{7}$$

$$v_k = \sqrt{2}V \cos(\omega t + \frac{2\pi}{11}) \tag{11}$$

By power invariant transform, this may be represented as a vector rotating in space at the speed of  $\omega t$ . This vector is termed the space vector and the transformation is represented by

$$\vec{v} = \sqrt{\frac{2}{11}} \begin{pmatrix} v_a + \vec{\alpha}v_b + \vec{\alpha}^2v_c + \vec{\alpha}^3v_d \\ +\vec{\alpha}^4v_e + \vec{\alpha}^5v_f + \vec{\alpha}^*v_g \\ +\vec{\alpha}^*v_h + \vec{\alpha}^*v_i + \vec{\alpha}^*v_j \\ +\vec{\alpha}^*v_k \end{pmatrix} \tag{12}$$

This may be represented as a single complex quantity,

$$\vec{v} = \sqrt{11}V \exp(j\omega t) \tag{13}$$

It must be noted that thus for the control of a balanced, 11-phase supply, it is sufficient to control the single vector termed as space vector. As is evident from the previous discussion, the rotational transform has been applied only to the  $\alpha$ - $\beta$  components and the space vector represented in (13) is only in the d-q subspace.

Since there are 11 inverter legs and each leg may assume one of the two states (0 for lower switch in the ON state, 1 for upper switch in the ON state) at any given time, there are a total of  $2^{11} = 2048$  states. Out of these, two are zero states or zero vectors corresponding to all upper switches ON and all lower switches OFF or all upper switches OFF and all lower switches ON. Care must be taken to ensure that in no leg both the upper and lower switches are in the ON state simultaneously i.e. the upper and lower switches in each leg are complementary. Thus that leaves us with a total of 2046 active vectors to consider. According to [5], it is sufficient to use only the maximum length vectors, termed as large vectors. According to this, the switching combinations of 5-6 and 6-5 (i.e. 5 switches ON in the upper row and 6 in the lower row, or vice-versa), where the ON state switches are adjacent to each other, provide the large vectors. These large vectors form a 22-side polygon, mathematically called an icosikaidigon, as shown in Fig.3. There are four other similar polygons, not shown in figure, corresponding to the switching combinations of 1-10, 2-9, 3-8, 4-7 and 10-1, 9-2, 8-3, 7-4. These five concentric polygons enclose all the space vectors.

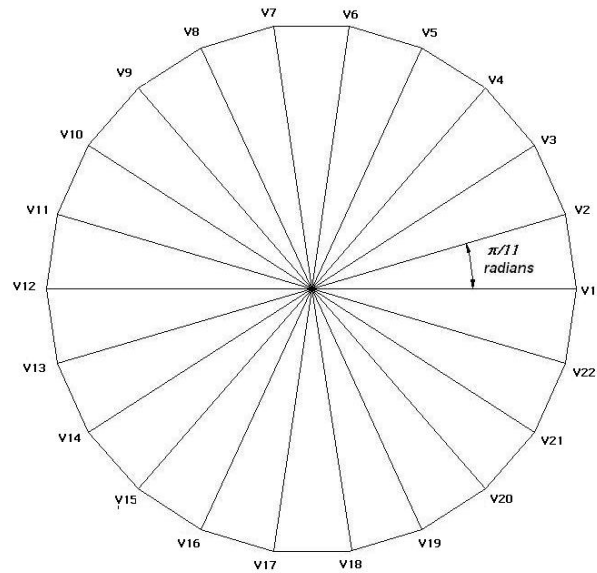


Figure 3: 22-sided space vector polygon

It is possible to analyze these vectors in a manner similar to the one followed in [6]. An analysis of the leg voltages for the large vectors alone provided in Table 1 reveals the sides of the polygon. In the table, the dc-bus voltage  $V_{DC}$  is represented by  $V$ .

### 3. Space Vector PWM Implementation

#### 3.1 Sequence strategy

Due to the availability of a large number of space vectors, there is flexibility in choosing the vectors to be controlled. There are some switching rules [7] followed for the implementation of SVM:

- There must be only one switching per transition
- The final state of one sample and the initial state of the next sample must be the same.
- The trajectory of the space vector must be a circle.
- Two switches in the same limb must never be in the ON state simultaneously.

In this paper, a simple adaptation of the Space Vector PWM technique is chosen. The reference vector may be at any position within the polygon at a given instant of time. In order to define the position of the space vector, the active vectors adjacent to it, along with suitable zero vectors, are applied at the right instants for certain intervals of time such that the trajectory of the vector is a circle.

There are a number of switching schemes for Space Vector Modulation. There are a number of different control strategies for three-phase matrix converters discussed in [8]. Most of these approaches may be extended for 11-phase inverters. The different SVM implementation techniques include symmetrical sequence, edge-aligned sequence, bus-clamped sequence and alternating sequence.

The simplest scheme of SVM, followed often in three-phase inverters, is to apply a zero vector at the start of a sample, followed by the two adjacent active vectors and then the other zero vector, for half a sampling period. The other half is a mirror image of the same for symmetrical sequence implementation. This also has the advantage of having only one switching per transition and thus minimizes switching losses. But an implementation of the same scheme for five-

Table 1: Leg voltages

Vect or no.	Binary representation	Switches in the ON state	Leg voltages (represented phase letter)											
			A	B	C	D	E	F	G	H	I	J	K	
v <sub>1</sub>	1110000011	18,19,20,21,22,1,2,3,4,5,6	V	V	V	0	0	0	0	0	0	0	V	V
v <sub>2</sub>	1111000011	19,20,21,22,1,2,3,4,5,6,7	V	V	V	V	0	0	0	0	0	0	V	V
v <sub>3</sub>	1111000001	20,21,22,1,2,3,4,5,6,7,8	V	V	V	V	V	0	0	0	0	0	0	V
v <sub>4</sub>	1111100001	21,22,1,2,3,4,5,6,7,8,9	V	V	V	V	V	0	0	0	0	0	0	V
v <sub>5</sub>	1111100000	22,1,2,3,4,5,6,7,8,9,10	V	V	V	V	V	V	0	0	0	0	0	0
v <sub>6</sub>	1111110000	1,2,3,4,5,6,7,8,9,10,11	V	V	V	V	V	V	0	0	0	0	0	0
v <sub>7</sub>	0111110000	2,3,4,5,6,7,8,9,10,11,12	0	V	V	V	V	V	V	0	0	0	0	0
v <sub>8</sub>	0111111000	3,4,5,6,7,8,9,10,11,12,13	0	V	V	V	V	V	V	0	0	0	0	0
v <sub>9</sub>	0011111000	4,5,6,7,8,9,10,11,12,13,14	0	0	V	V	V	V	V	V	0	0	0	0
v <sub>10</sub>	0011111100	5,6,7,8,9,10,11,12,13,14,15	0	0	V	V	V	V	V	V	0	0	0	0
v <sub>11</sub>	0001111100	6,7,8,9,10,11,12,13,14,15,16	0	0	0	V	V	V	V	V	V	0	0	0
v <sub>12</sub>	0001111110	7,8,9,10,11,12,13,14,15,16,17	0	0	0	V	V	V	V	V	V	0	0	0
v <sub>13</sub>	0000111110	8,9,10,11,12,13,14,15,16,17,18	0	0	0	0	V	V	V	V	V	V	0	0
v <sub>14</sub>	0000111111	9,10,11,12,13,14,15,16,17,18,19	0	0	0	0	V	V	V	V	V	V	0	0
v <sub>15</sub>	0000011111	10,11,12,13,14,15,16,17,18,19,20	0	0	0	0	0	V	V	V	V	V	V	0
v <sub>16</sub>	0000011111	11,12,13,14,15,16,17,18,19,20,21	0	0	0	0	0	0	V	V	V	V	V	0
v <sub>17</sub>	0000001111	12,13,14,15,16,17,18,19,20,21,22	0	0	0	0	0	0	0	V	V	V	V	0
v <sub>18</sub>	1000001111	13,14,15,16,17,18,19,20,21,22,1	V	0	0	0	0	0	0	V	V	V	V	0
v <sub>19</sub>	1000000111	14,15,16,17,18,19,20,21,22,1,2	V	0	0	0	0	0	0	0	V	V	V	0
v <sub>20</sub>	1100000111	15,16,17,18,19,20,21,22,1,2,3	V	V	0	0	0	0	0	0	V	V	V	0
v <sub>21</sub>	1100000011	16,17,18,19,20,21,22,1,2,3,4	V	V	0	0	0	0	0	0	0	V	V	0
v <sub>22</sub>	1110000011	17,18,19,20,21,22,1,2,3,4,5	V	V	V	0	0	0	0	0	0	V	V	0

phase inverters in [6] shows that it is not possible to limit the number of active vectors to two per sampling period while also conforming to the constraint of a single switching per transition, for multi-phase inverters. Thus it becomes important to strike an optimization between the complexity of the algorithm, the ease of physical implementation and the losses endured. In an 11-phase system, conforming to a single switching per transition may require the choice of other vectors in between the zero vectors and the large active vectors. Due to the large number of states possible, this may effectively rule out high frequency applications due to the difficulty of practical implementation or might significantly increase the complexity of the algorithm.

Although a large number of implementation strategies have been in use, many strategies prove to be difficult for practical implementation. The feasibility of on-line computation of switching times using a processor might become dubious. Although high-end processors which may allow the fast implementation of a complex strategy involving a large number of vectors do exist, it comes at a cost relatively far higher and the very practice becomes a tedious and economically denting affair.

In order to maintain simplicity of the strategy, realizable at a reasonable cost, the rule of maintaining only one switching per state transition is dropped. This rule is followed only between the two active vectors and is not followed for

transitions from a zero vector to an active vector or vice-versa.

### 3.2 Switching time calculation

The symmetric sequence implementation strategy is followed in this paper. Here the second half of a sampling period is a mirror image of the first half. The strategy of applying a zero vector  $v_0$ , followed by the two active vectors  $v_a$  and  $v_b$ , and then the other zero vector,  $v_{0/2047}$ , is followed. The two active vectors are ordered in such a way that the number of switching between the active vector and the zero vector preceding or succeeding it is minimum. While, as discussed, there is a lot of flexibility in choosing the vector sequence, it is essential to calculate the time each vector must be applied. The volt-second balancing criterion must be satisfied while calculating the switching times.

$$\vec{v} * T_s = \vec{v}_a * T_a + \vec{v}_b * T_b + v_{0/2047} * T_{0/2047} \quad (14)$$

Here  $T_s$  represents the sampling period while  $\vec{v}_a$  and  $\vec{v}_b$  are applied for  $T_a$  and  $T_b$  times respectively. For the sake of simplicity the sampling period mentioned here is assumed to address only half a sample. In practice the times calculated here would have to be halved in order to accommodate the mirror image of the first half of the sampling period in the next half. In essence, it means that each vector is applied for half of its corresponding time in the first half of the sampling period and again for the same time in the next half.

It may be seen that

$$\vec{v} = \vec{v}_a * \frac{T_a}{T_s} + \vec{v}_b * \frac{T_b}{T_s} + v_{0/2047} * \frac{T_{0/2047}}{T_s} \quad (15)$$

Thus it is necessary to calculate the times  $T_a$  and  $T_b$  in order to apply the corresponding vectors for a suitable time. Consider Fig.4 where the space vector is in between large vectors  $\vec{v}_a$  and  $\vec{v}_b$  at an angle  $\theta$  from  $\vec{v}_a$ .

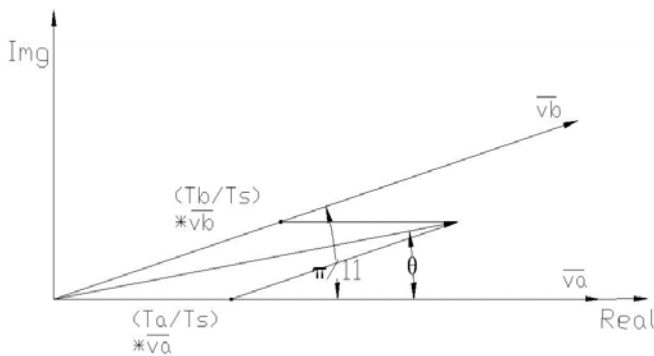


Figure 4. Reference vector in between  $v_a$  and  $v_b$

$$T_a = m * T_s * \sin \sin \left( n \frac{\pi}{11} - \theta \right) \quad (16)$$

$$T_b = m * T_s * \sin \sin \left[ \theta - \left( \frac{n-1}{11} \right) \pi \right] \quad (17)$$

where  $n$  is the sector number ranging from 1 to 22 and  $m$  is the modulation index. The modulation index depends on the length of the reference vector,  $\vec{v}$ , and the dc-bus voltage,  $V_{dc}$ . It represents a measure of the utilization of the dc-bus voltage and appropriate selection of the same is extremely important to ensure efficient operation.

$$T_0 = T_{2047} = \frac{T_s - T_a - T_b}{2} \quad (18)$$

Upon appropriate selection of switching times the inverter may be made to make fair use of the dc-bus voltage. Fig.5 shows the gating signal graphs in sector one. Note that only the pulses for upper row switches are shown since their complement provide the signals for the lower row switches.

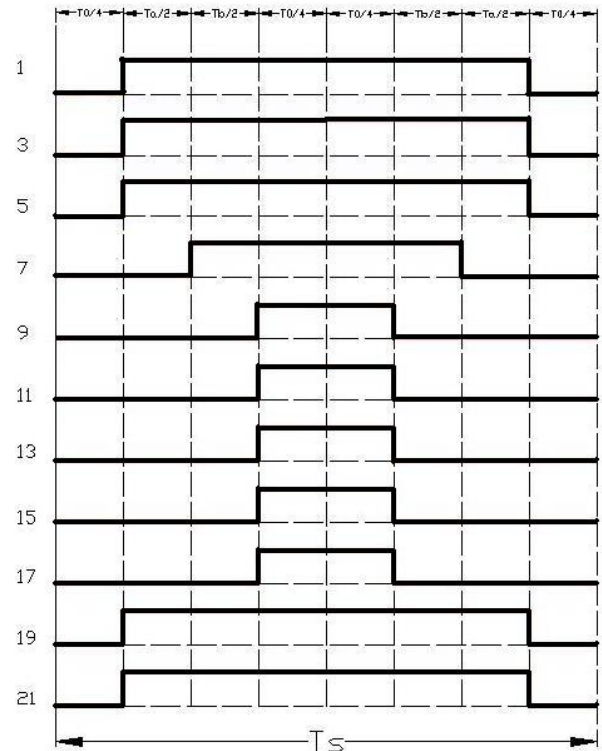


Figure 5: Gating diagram in Sector I

### 4. Conclusion

Literature [9] exists to represent the existence of interest in eleven-phase machines. However, interest in eleven-phase machines is relatively low since their other multi-phase counterparts have efficient control strategies. Space Vector Modulation technique is a kind of PWM which can be applied to converters having more than 2 phases. Applications such as electric ship propulsion aim at highly efficient converter systems and require high power. Space Vector Modulated multi-phase converters provide an apt control circuitry for such applications. Although multiple strategies of SVM exist, a simple symmetric sequence strategy which strikes a balance between switching losses and ease of implementation has been presented in this paper. These 11-phase machine drives may be considered for high power, application specific systems such as in electric propulsion, more-electric aircraft, traction and other applications where the phase number need not mandatorily be a multiple of three. Further improvement in the switching sequence might lead to an extremely efficient drive and provide an extremely viable alternative between nine and twelve phase drives.

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