Significance of the Clustering Parameter and its Relation to Galaxy Clustering

Manzoor A. Malik[1]

[1]Department of Physics, University of Kashmir, Srinagar-190006, J & K, India
mmalik@kashmiruniversity.ac.in

Abstract: *Various issues pertaining to the clustering parameter in relation to galaxy clustering are outlined. Its evolution from an ``Educated Guess'' introduced in the Gravitational Quasi-Equilibrium Distribution theory to the establishment of a rigorous mathematical basis is explained. An attempt is made to give an idea of the importance of the clustering parameter without getting lost in the galaxy distribution function discussed in detail in our previous works. Spatial galaxy distribution function is outlined only in the context of the role of the clustering parameter.*

Keywords: Cosmology, Galaxy Clustering, Distribution function

1. Introduction

The galaxy clustering parameter b was introduced as an ansatz in the gravitational quasi-equilibrium distribution (GQED) theory [1] so as to complete the thermodynamic description of gravitational galaxy clustering. Later it was found to have a more fundamental basis and a significant influence on various aspects of galaxy clustering [2, 3]. The discovery of the statistical mechanical description of gravitational galaxy clustering [4], viewed as a cosmological many-body problem, gave legitimacy to the assumed physical form of the clustering parameter as it followed directly from calculations which was not possible earlier. Statistical mechanical description added a new feature to the clustering parameter by incorporating the extended nature of galaxies helping in turn to account realistically for the galaxies as non-point mass systems. It was soon realized [5] that it is possible to account for the multi-component nature of the galaxy system. Later [6], we showed that the extended nature of galaxies can also be taken into account for the multi-component description of galaxy clustering. Recently, we have extended the statistical mechanical description to a three-component system [7] which brings in an added feature to the clustering parameter.

Originally introduced as b, the clustering parameter is a measure of gravitational attraction and thus has a profound influence on the process of clustering. However, in all previous works, the focus has been more on the distribution function and in this process b has not been fairly covered. Because of its significance in the context of gravitational galaxy clustering, we intend to specifically focus on the significance of b and its various manifestations (incorporating features like extended mass, multi-component nature of the system) in this paper. In section 2, we describe the physical form of the clustering parameter as envisaged in the thermodynamic description [2] and supported by the statistical mechanical description [4]. In section 3, we describe the multi-component form of the clustering parameter and explain why development of such a theory is important. In section 4, we give a brief account of the fundamental role of b in the behavior of the spatial galaxy distribution function. In section 5, we summarize our work

in this area.

2. Physical form of b

The formulation of the thermodynamic description of galaxy clustering demanded a particular form of b given by

$$b = \frac{ax}{1+ax} \qquad (1)$$

The motivation for that form was the fact that b being a measure of clustering has to account for the transition from homogeneity to clustering; $b = 0$ corresponding to homogeneity and $b = 1$ corresponding to complete clustering. The intermediate values ($0 < b < 1$) representing an under-virialized system.

From the first and second laws of thermodynamics it followed that b has a particular dependence on x, which in turn depends on the specific combination of $\bar{n}$ and T, $x = \bar{n}T^{-3}$ where $\bar{n}$ is the average number density, $\bar{n} = \bar{N}/V$, of the average number of particles $\bar{N}$ in a cell of volume V. The same dependence follows from a scaling property of the gravitational partition function [3]. Further, a (later replaced by b_0) which appears in equation (1) may depend upon time but not on the intensive variables $\bar{n}$ and T [2]. Based on these arguments, the specific functional form of b is given as

$$b = \frac{b_0 \bar{n}T^{-3}}{1+b_0 \bar{n}T^{-3}} \qquad (2)$$

The above form had originally to be introduced as an educated guess to complete the thermodynamic description. In-fact, variants of this form were shown to lead to undesirable physical consequences [8] like violation of one or the other laws of thermodynamics and existence of negative probability distribution function which is absurd.

However, the work of Saslaw and Fang [3] shows that equation (2) has a more fundamental physical basis. Further, the statistical mechanical description of the cosmological many-body problem [4], wherein the physical form of b follows directly from the calculations yielding a result exactly equivalent to equation (2), made the results more rigorous. As a bonus, the new statistical mechanical derivation, yielding a form

$$b_\varepsilon = \frac{b_0 \bar{n} T^{-3} \alpha(\varepsilon / R_1)}{1 + b_0 \bar{n} T^{-3} \alpha(\varepsilon / R_1)}, \qquad (3)$$

accounted for an extra feature: the extended nature of galaxies. Here, ε is a measure of the average sized halo surrounding each galaxy and R_1 is the limit of the spatial integration where the expansion of the universe cancels the gravitational mean field. It is easy to see that in the point-mass limit $\varepsilon \to 0$ such that $\alpha(\varepsilon / R_1) = 1, b_\varepsilon \to b$, as given by equation (2). Thus, equation (2) is just a special case of equation (3) for $\varepsilon \to 0$.

3. The Multi-Component Clustering Parameter

From the analysis of a two-component system of galaxies [5] emerged a quantity b_m given by

$$b_m = \frac{b}{(1 + N_2 / N_1)}\left[1 + \frac{(m_2 / m_1)^3 (N_2 / N_1)}{1 - b + (m_2 / m_1)^3 b}\right] \qquad (4)$$

The advantage of equation (4) is that it incorporates the mass ratios and the number density ratios in the clustering parameter itself. Earlier comparisons with N-body simulation results [9, 10] were tedious and could not cover the whole range of masses and number densities because of the non-existence of a theory for the two-component system.

Table 1: The two-component clustering parameter b_m for various values of b, m_2/m_1, N_1/N_2.

Less Massive Galaxies Dominant ($N_1: N_2 = 7:1$)						
b	0.2		0.4		0.8	
$m_2 : m_1$	2	5	2	5	2	5
$b_m - b$	.058	.096	.055	.074	.021	.025

Equal Number of Galaxies of each kind ($N_1: N_2 = 1:1$)						
b	0.2		0.4		0.8	
$m_2 : m_1$	2	5	2	5	2	5
$b_m - b$	.233	.384	.221	.294	.085	.099

Massive Galaxies Dominant ($N_1: N_2 = 1:7$)						
b	0.2		0.4		0.8	
$m_2 : m_1$	2	5	2	5	2	5
$b_m - b$	.408	.637	.387	.515	.148	.173

Table (1) shows as to how the mass ratios and number density ratios effect the behavior of the multi-component clustering parameter. The top panel where we take the mass ratios of less massive to massive galaxies as 7 : 1 is motivated by the N-body simulation results of Itoh, Inagaki and Saslaw [9, 10]. One can notice that for this case, b_m differs from b by a small amount. This is reasonable as the system is dominated by less massive galaxies and the small percentage of more massive galaxies ought to have a

minimal effect. The middle panel considers equal number of galaxies of each kind. One can see that for low and intermediate values of b, b_m differs from b by an appreciable amount. However, for higher values of b (here 0.8) the difference is marginal. This is because b approaching unity corresponds to a complete virialization of the system and hence the mass of galaxies becomes less relevant. The lower panel, wherein now the more massive galaxies become dominant shows a similar behavior.

The statistical mechanical description has now been extended to the three-component system [7]. It follows from the analysis of Malik et al [7] that the three-component clustering parameter b_3 behaves in the same fashion as b_m. Though, it provides an additional information on the multi-component system, it does not lead to any surprising results. On that basis, we conclude that the clustering parameter for an N-component system should behave in much the same way as a two-component system.

4. Role in Galaxy Clustering

Taken alone, the clustering parameter determines the strength of gravitational attraction. Its role in galaxy clustering manifests itself largely in the distribution function. Distribution functions, in particular, the spatial galaxy distribution function, have been a subject of our interest for a long time. Some recent works include [5, 6, 7, 11] in which various aspects of the spatial galaxy distribution function are discussed in detail along with comparisons with N-body simulation results and observations. Here, we briefly comment on the galaxy distribution functions only in relation with the clustering parameter.
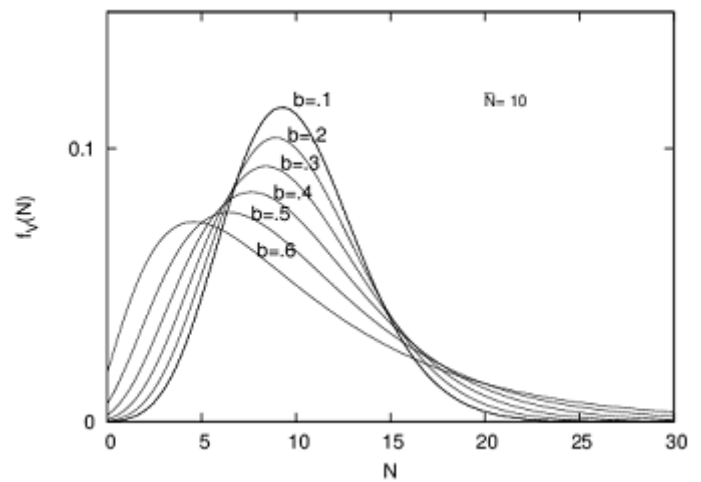

Figure 1. Effect of clustering parameter on the spatial galaxy distribution function

The thermodynamic description of galaxy clustering [1, 2] yielded the following expression for the spatial galaxy distribution function:

$$f(N) = \frac{\bar{N}(1-b)}{N!}[\bar{N}(1-b) + Nb]^{N-1} e^{-\bar{N}(1-b)-Nb} \qquad (5)$$

Note that in the limit $b \to 0$, the above equation reduces to a simple Poisson distribution:

$$f(N) = \frac{\overline{N}^N e^{-\overline{N}}}{N!} \qquad (6)$$

Figure (1) shows the effect of the clustering parameter b on the distribution function. For $b=0.1$, the distribution is close to a simple Poisson. As b increases, the deviations from a simple Poisson become more and more obvious. These deviations (departures from homogeneity) are a measure of clustering and hence an increase in b leads to enhanced clustering. In later work [4], under the framework of statistical mechanics, a similar expression for the distribution function is obtained with b replaced by b_ε. ε is a measure of an average sized halo surrounding a galaxy and has been shown to have a minimal effect on clustering [4, 6]. In another work [11], wherein the triplet contributions are included, another form of the clustering parameter appears but that too does not effect the distribution function appreciably. Domination of massive galaxies in multi-component systems [5, 6, 7] enhances clustering without altering the overall features of the distribution function.

5. Conclusion

In this paper, we have attempted to look at all aspects of the clustering parameter starting from its assumed form in the original thermodynamic description of galaxy clustering till its present established rigorous mathematical basis.

In the cosmological context, particularly with regard to gravitational galaxy clustering, it is an important parameter as its value determines the rate of clustering. Value of b close to zero corresponds to homogeneity while its value close to unity corresponds to clustering. Many aspects of galaxy clustering can be understood from the physical form of the clustering parameter itself. For example, the observation that the extended nature of galaxies does not have a profound influence on b leads us to infer that the overall distribution function may also not be affected if the extended nature of galaxies is taken into account. This has been shown to be faithfully true [4], [6]. Similarly, the observation that galaxies with different mass profiles have a marked influence (depending upon the dominance of massive galaxies) on the clustering parameter leads us to infer that the clustering should be enhanced by dominance of massive galaxies. This is again consistent with the theoretically obtained results [5]-, [7].

The originally assumed physical form of b has now a rigorous mathematical basis. The form of b (given by equation 2) and its variants incorporating other features like the extended mass [4], multi-component nature of the system [5]-[7], inclusion of higher order contributions [11] etc. follow directly from the statistical mechanical considerations. The recent analysis [12] from Riemannian Geometric considerations supports it even further.

References

[1] Saslaw W. C. and Hamilton A. J. S, (1984), Astrophysical Journal, 276, 13

[2] Saslaw W. C., (2000), The Distribution of the Galaxies: Gravitational Clustering in Cosmology, (New York: Cambridge University Press)

[3] Saslaw W. C. and Fang F. (1996), Astrophysical Journal. 460, 16.

[4] Ahmad F., Saslaw W. C. and Bhat N. I., (2002), Astrophysical Journal 571, 576.

[5] Ahmad F., Malik M. A. and Masood S., (2006), Int. J. Mod. Phys. D, 15, 8, 1267.

[6] Malik M. A., Ahmad F., Shakeel Ahmad and Sajad Masood, (2009), Int. J. Mod. Phys. D, 18, 6, 1.

[7] Malik M. A., Ali R. N. and Ahmad F., (2011), Astro. Sp. Sc. 336, 447

[8] Masood S, (2002) Ph. D. thesis, University of Kashmir, Srinagar, India

[9] Itoh M., Inagaki S. and Saslaw W.C., (1988), Astrophysical Journal. 331, 45

[10] M. Itoh, S. Inagaki and W. C. Saslaw, (1990), Astrophysical Journal, 356, 315.

[11] Ahmad F., Saslaw W. C. and Malik M. A., (2006), Astrophysical Journal, 645, 940.

[12] Ahmad F., Malik M. A. and Hameeda (2013), Astrophysis and Space Science, 343, 763

Author Profile

Dr. Manzoor A. Malik is an associate professor in the department of Physics, University of Kashmir, Srinagar. He received his B.Sc. and M.Sc. degrees in Physics from the University of Kashmir, Srinagar. Dr. Malik did his Ph. D. in Astrophysics and has a research experience of more than a decade. His work in Astrophysics is published in some of the leading journals that include Astrophysical Journal, Monthly Notices of the Royal Astronomical Society, International Journal of Modern Physics D, Astrophysics and Space Sciences.