Denoising and Reconstruction of Very Low Frequency Signal with Wavelet Thresholding

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Abstract: ***Over past few decades ambient environmental noises increased tremendously due to man made and natural factors. There are various reasons for these noises such as noisy heavy engine, pumps, spacecraft, lighting and earthquake. These noises often degrade the quality of very low frequency (VLF) signal observed by satellite. Due to this lot of information associated with these signals are lost. To retrieve signal with significant geologic information many types of denoising methods are used, but these traditional methods are based on linear pass band filter. These families of filters are useful according to phase properties but their efficiency is reduced when we used these filters for denoising of VLF signals. This is due to the fact that these signals are nongaussian, which contains gaussian background and narrow pulses due to lighting discharge, Power Line Harmonic Radiation (PLHR) and some time due to earthquake and volcanic eruptions. In this work we developed a new method for signal denoising which is based on wavelet threshold algorithms. We show that thresholding the wavelet coefficients of a VLF signal allows to restore the complete shape of the original signal. In this approach we substantially improve the performance of classical wavelet denoising algorithms, both in terms of SNR and of visual artifacts***.

Keywords: Very Low Frequency Signal, denoising, Wavelet thresholding

1. Introduction

Very Low Frequency (VLF) signals observed by satellite surveys are known to posses a higher resolution power than observed by traditional methods due to enhanced high frequency content of stored signal. However high frequency noise varying in intensity and frequency often contaminates the signal and need to be filtered. Unfortunately this type of noise may have a frequency contents similar to that signal. For the filtering of these types of noises frequency domain filtering [1] does not always work well because it globally removes frequencies causing generalized smoothing effect that substantially broadens features of interest. In order to derive the important characteristics of VLF Signal and geophysical data the usual denoising procedure involve the use of a linear band pass filter [2]. This family of filter is based on phase properties, so it is very effective for the stationary signals but its efficiency reduced when signals are Non-stationary in nature like Seismic and VLF signal. So the alternative solution of this problem is the Weigner filter [3], which focuses on the elimination of mean square error between observed and de noised signals. These assumptions does not hold good for the VLF signals, because they are highly non-stationary in nature the solutions based on wavelet transform proved effective for denoising problems across several areas such as Biomedical, Speech enhancement and image processing [4-6].

For the denoising of all types of non-stationary signal observed with additive noise wavelet thresholding was used [7]. The thresholding rules (which differ in choice of threshold) remove noise from a signal by explicitly setting small wavelet coefficients to zero, which is a form of high level compression [8]. A single threshold parameter determine the behavior of these procedure setting both the level below which coefficients are eliminated as well as determining how the remaining coefficients are to be estimated. Since the efficiency of this estimate depends on the rate of decay of stored decomposition coefficients, it is well known that the choice of Fourier or cosine series in representation of piecewise smooth signal [9] tends to the wavelet time localization the decay of wavelet coefficients in the neighborhood of discontinuity is faster than the decay of Fourier coefficients. However wavelet thresholding method is still regularization process and the estimator presents oscillation in the vicinity of signals discontinuities. These arises Gibbs phenomena in Wavelet coefficient, which produce error in the reconstruction process. To over comes these difficulties, we use translational invariant denoising.

2. Theory of Wavelet Denoising

In this analysis a finite support function called mother wavelet $\psi(t)$ is used. The scaling and shifting of mother wavelet with factor s and τ generates a family of function called wavelets given by [10]:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \quad \text{with} \quad s > 0 \tag{1}$$

This function provide finite temporal and spectral supports given as

$$\int |\psi(t)| dt < \infty$$

$$\text{and} \quad \int |\psi(f)| df < \infty \tag{2}$$

Where $\psi(t)$ is the Fourier transform of function $\psi(f)$

In the equation (1) the term $\left(\frac{1}{s}\right)^{1/2}$ guaranties the energy normalization

$$\int |\psi(t)|^2 dt = \int |\psi(t)|^2 dt = 1 \tag{3}$$

these equation implies that all the wavelets $\psi_{s,\tau}(t)$ of same family $\psi(t)$ retains same energy and shape.

These wavelet also satisfy two additional procedure that are admissibility and regularity here admissibility condition allows the signal reconstruction, while regularity condition implies a quick decrease of wavelet coefficients with decrease of scale [11].

2.1 Continuous Wavelet Transform (CWT)

The CWT-$Wf(s,\tau)$ is the inner product of a time varying signal $x(t)$ and the sets of wavelets $\psi_{s,\tau}(t)$ given by:

$$W_x(s,\tau) = \left\langle x, \psi_{s,\tau} \right\rangle = \frac{1}{\sqrt{s}} \int x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \qquad (4)$$

Using Parseval's identity CWT can also be written as

$$W_x(s,\tau) = \left(\frac{1}{2\pi}\right)\left\langle x(\omega), \psi(\omega) \right\rangle \qquad (5)$$

Where $x(f)$ and $\psi(f)$ are the Fourier Transform of $x(t)$ and $\psi(t)$ respectively.

2.2 Discrete Wavelet Transform

In practice discrete wavelet transform is used in which the dilation parameter S and translation parameter τ are discrete. These procedure become much more efficient, if dyadic values of parameter s and τ are used

$$s = 2^{j,}, b = 2^{j}k \qquad j,k \in Z \qquad (6)$$

Where Z = set of positive integer

It is a special case of $\psi(t)$, corresponds to discredited wavelets $\psi_{jk}(t)$ is used and is given by

$$\psi_{jk}(t) = 2^{\frac{-1}{2}}\psi\left(2^{-j}t - k\right) \qquad (7)$$

Which constitute an orthogonal basis for $L^2(R)$ [12]. Using $L^2(R)$ the wavelet expansion for a function $f(t)$ and the wavelet expansion coefficients are defined as

$$f(t) = \sum_j \sum_k \alpha_{jk}\psi_{jk}(t)$$

$$\alpha_{jk} = \int_{-\infty}^{\infty} f(t)\bar{\psi}_{jk}(t)dt \qquad (8)$$

The DWT consists of applying the discrete signal to bank of octave and band filters based on low and high pass filter $L(n) = H(n)$ respectively. More precisely the function $f(t)$ would be expressed as follows [13]:

$$f(t) = \sum_{k=Z} a_L(k)\phi_{L,K}(t) + \sum_{j=1}^{L}\sum_{k=Z} d_j(k)\psi_{jk}(t) \qquad (9)$$

With

$$d_j(n) = \left\langle f, \psi_{j,n} \right\rangle = \sum kg'(2n-k)a_{j-1}(n)$$

$$a_L(n) = \left\langle f, \psi_{j,n} \right\rangle = \sum kh'(2n-k)a_{L-1}(n)$$

Where $\phi(t)$ is called scaling function associated to the wavelet function $\psi(t)$ governed by following condition:-

$$\int \phi(t).dt = 1 \qquad (10)$$

2.3 Wavelet De noising algorithm

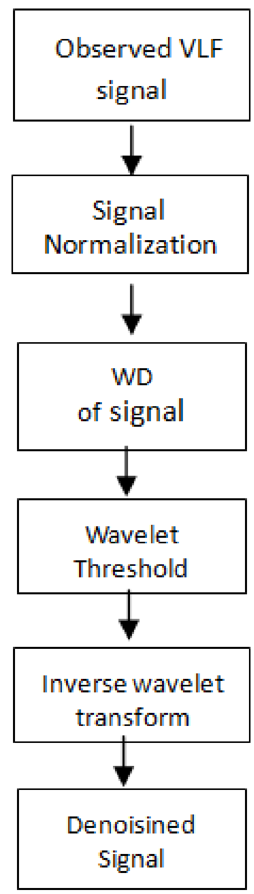

Figure 1: Wavelet Denoising Algorithm Block Diagram

Denoising algorithm scheme is shown in Figure 1. Inverse DWT is applied to get de noise time domain signal. Denoising algorithm is summarized as follow:

- Normalization and filtering the observed VLF signal by Quadratic mirror filter (QMF).
- Decompose the signal by wavelet transform here we can used Haar wavelet function for this purpose.
- Estimate the noise level and use it to threshold the wavelet coefficients.

- Reconstruction of signal from de noised or shrinkage wavelet coefficients.

All thresholding or shrinkage methods truncate the wavelet coefficients below certain threshold value. It is very important for signal denoising. The shrinkage method and the selection of the threshold define the purity of signal recovering. The soft and hard thresholding methods are used to estimate the wavelet coefficients in the wavelet thresholding denoising. Hard thresholding zero or truncate out small coefficients which give an efficient representation of de noised signal, while soft thresholding soften the coefficients exceeding the threshold value by lowering them by the threshold value. Analytical expression [14] for hard and soft thresholding is given below:

$$y_{hard} = \begin{cases} d, |d| > thr, thr \geq 0 \\ 0, |d \leq thr| \end{cases}$$

$$y_{soft} = \begin{cases} sign(d).(|d| - thr), |d| > thr, thr \geq 0 \\ 0, |d| < thr \end{cases} \quad (11)$$

Where d is the wavelet coefficient from the decomposed VLF signal. The variable *thr* is the threshold selected. The function if y_{hard} and y_{soft} is the wavelet coefficient processed with hard and soft threshold function. *Sign*( ) is the signum function [12]

The selection of proper threshed is very important a very small threshold does not remove the noise. On the other hand large threshold remove all valuable information form the signal. In general selection is based on the standard deviation of noise or more robust Mean Absolute Deviation (MAD). In this work Universal visu-shrink threshold is used. Analytically it is given by:

$$Thr = \sigma\sqrt{2.\log(N)} \quad (12)$$

Where N is the numbers of samples and σ is calculated by median mirror filter. Traditional denoising of the wavelet coefficients some times in the neighborhood of discontinuities excites Gibbs phenomena due to the lack of translation invariance of wavelet basis. To overcome this difficulty we used another method of denoising called Translation invariant denoising [16-18]. This method produces a reconstructed signal which exhibited much weaker Gibbs phenomena.

3. Results and discussion

To evaluate the performance of our Denoising algorithm for VLF signal, we utilized the VLF signal observed by French Micro satellite DEMETER (Detection of Electromagnetic Emission Transmitted through Earthquake Region). DEMETER ICE payload collects the VLF signal at the sampling rate of 40 kHz when it operated at burst mode. It measure the electric field between four spherical sensors located on its booms. The desired components of electric fields are chosen by selection of these experiments. The full vectors of electric field i.e. three components of electric and magnetic fields are only available in the frequency range up to 1 kHz and in the VLF range the waveform of one component of electric field and magnetic field are recorded.

In this work we used VLF whistlers and Hiss emission for testing of proposed algorithm. This was recorded during the Feb 2009 at Sumatra (Indonesia). As it has been stated in the algorithm application of our denoising algorithm, which consist quadratic mirror filter to smoothen the signal observed in the signal filtering and normalization stage. In the next stage which is the Wavelet decomposition of VLF signal we used Haar wavelet function with multi resolution level-5. In these process signal splits in to level define the high amplitude DWT coefficient represents signal and low amplitude coefficients represents noise. It considered that some samples of noisy signal contain only noise, so by selecting these coefficients advanced analysis of signal noise is possible for the better characterization of signal. So it is important to select the proper level in multi-resolution analysis.

The threshold values obtained by Global threshold function give in equation (12). We apply this threshold value to extract approximation and detail coefficients. In these work we used soft thresholding function because studies shows that it give much better performance then the hard one [19]. Finally use these coefficients to reconstruct the signal. Figure 2 and Figure 3 shows the waveform of observed VLF whistler and Hiss and its estimated (de noised) version using algorithm proposed. A comparative study between original and De noise signal shows the performance and usefulness of our denoising algorithm. In this study we used statistical and visual analysis of original and de noised signal.

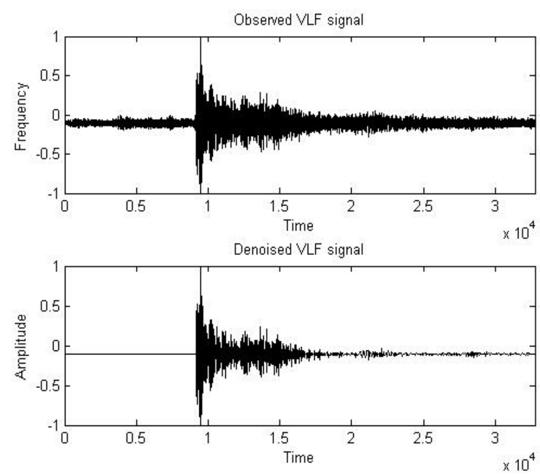

Figure 2: Waveform of VLF Signal (Whistler) observed by DEMETER satellite (Upper panel) and De noised signal (Lower panel)

SNR is a very popular and effective method in signal processing. SNR is used to quantify how much the signal has been corrupted by noise. It is defined as the ratio of signal power to noise power of corresponding signal. Analytically it is given by:

$$SNR = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = P_{signal} - P_{noise}(indb) \quad (13)$$

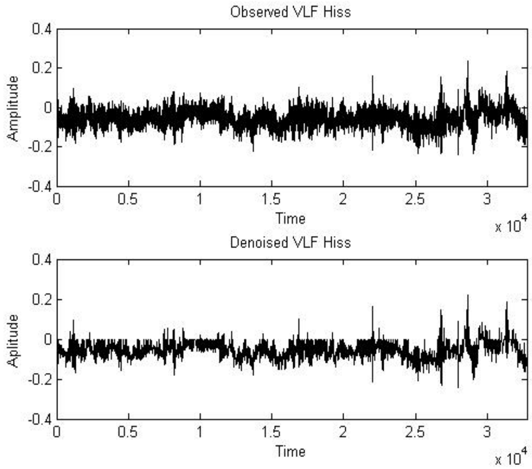

Figure 3: Waveform of VLF Signal (Hiss) observed by DEMETER satellite (Upper panel) and De noised signal (Lower panel)

For statistical analysis of original and de noised signal SNR (signal to noise ratio) and C.F (crest factor) are used. The results of analysis are summarized in Table 1 for VLF whistler and in Table 2 for VLF Hiss.

Table 1: Performance of algorithm for VLF Whistlers

Signal	*No of Samples*	*SNR (in db)*	*Crest Factor*
Observed	32768	11.89	8.21756
De noised	32768	12.73	8.23699

Table 2: Performance of algorithm for VLF Hiss

Signal	*No of Samples*	*SNR (in db)*	*Crest Factor*
Observed	32768	22.5667	3.3571
De noised	32768	25.004	3.2154

Denoising is successful when post SNR is higher then the pre SNR. Results shows that for VLF whistlers post SNR is high as compare to pre SNR. So our algorithm is good for both types of VLF signal used. It enhanced the quality of signal to study the high frequency content of VLF hiss. We used one another parameter to test the performance of our algorithm, which is the measure of waveform of signal called crest factor [20]. It is calculated from the peak amplitude of the waveform divided by the RMS value of the waveform and analytically given by:

$$C.F = \frac{|x|_{peak}}{x_{rms}} \qquad (14)$$

Where $|x|_{peak}$ = amplitude of waveform

x_{rms} =RMS value of waveform

Calculation shows that the value of C.F for original and de noised signal is same for both the signal (whistler and Hiss) its shows that our denoising algorithm not only remove the unwanted noise and enhanced the quality of signal but not affect the waveform of the signal considered.

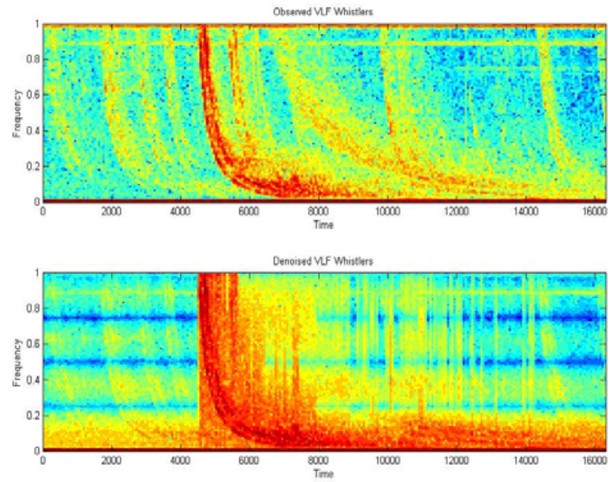

Figure 4: Spectrogram of VLF Signal (Whistler) observed by DEMETER satellite (Upper panel) and De noised signal (Lower panel)

3.1 Visual analysis

We used spectrogram [21] for the visual analysis of original and de noised signals. Spectrograms are the visual representation of signal in time-frequency plane. Figure 4 and Figure 5 shows the spectrogram of observed and de noised VLF whistlers and Hiss respectively. It shows that our denoising algorithm not only provide improved value of SNR but also reveal new phenomena's which are obscured by the traditional analysis. Our denoising algorithm itself leads to more detail picture of VLF signal under study. In Figure 4 many frequencies of whistlers are unseen in the spectrogram of observed signal, which are clearly seen in the de noised signal spectrogram similarly Figure 5 shows clear picture of Hiss emission.

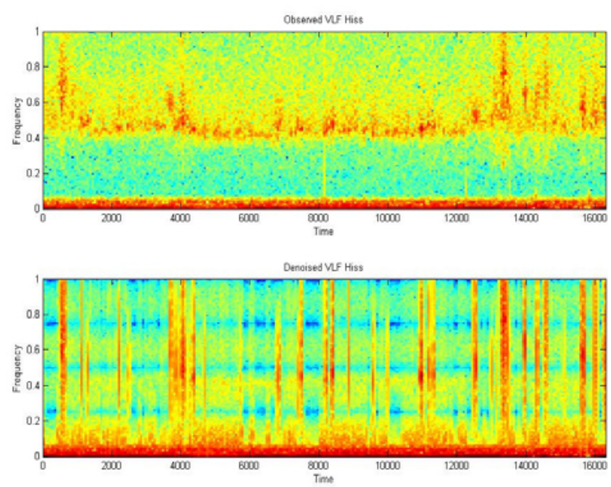

Figure 5: Spectrogram of VLF Signal (Hiss) observed by DEMETER satellite (Upper panel) and De noised signal (Lower panel)

4. Conclusion

We present here a denoising algorithm based on wavelet thresholding applied to the different VLF signal observed by

DEMETER satellite. For this purpose soft thresholding is used. Our key idea was to eliminate the noise of observed signal below a certain level to enhance the quality of signal, which helps us to study important phenomena related to these signal. In these work translation invariant denoising is used, it is particularly used when the underlying set of wavelet coefficient that describe the observed signal excites the Gibbs phenomena in the neighborhood of discontinuities due to lack of invariance. Using these algorithms our experience has been that as the resolution increased the chosen threshold function changed therefore our denoising algorithm depends on resolution level. From practical point of view this method is particularly use when signal contain same type of change like VLF hiss. Our analysis shows that our algorithms very suitable for the denoising of VLF Hiss. Also visual inspection of VLF whistlers mode signal shows that our method shows the fine structure of this, it clear that by this method fine analysis of phenomena related to this is possible. So conclude that our method works as a microscope which shows the high resolution patters of VLF signal.

In these paper we test the performance of our algorithm over two type of VLF signal (Whistlers, Hiss), but it can be easily extended to other type of VLF signals. A slight modification of constraint may also be performed in order to achieve restoration of VLF signal that have been compared within an orthogonal basis.

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