Certain Subordination Results for a Class of Analytic Functions Defined by the Generalized Derivative Operator

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Abstract: ***In this paper, we discuss several interesting subordination results for a class of analytic functions defined by using a generalized derivative operator which was introduced and studied by Al-Abbadi and Darus[1]. A number of interesting consequences of some of these results are also discussed, 2000 Mathematics Subject Classification. 30C45, 30C80.***

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1. Introduction

Let A be a class of functions f(z) analytic in the unit disk U = { z : |z| < 1} and normalized by

(1.1) $$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

Let A(s) denote the class of functions of the form

(1.2) $$f(z) = z + \sum_{k=s+1}^{\infty} a_k z^k$$

and s $\in$ N = {1, 2, …}, which areanalytic in the open unit disk U on the complex plane $\mathbb{C}$. We further let c_α (s) be the class consisting of functions g which are convex of order α in U. i.e. c_α (s) =

$$\left\{ g \in A(s) : \operatorname{Re}\left(1 + \frac{zg''(z)}{g'(z)} > \alpha, \quad z \in U\right)\right\}$$

for $0 \le \alpha < 1$

Al-Abbadi and Darus[1], introduce the class $\wp^{n,m}_{\lambda_1,\lambda_2}$ (s, α) consisting of functions f(z) satisfying

(1.3) $$\operatorname{Re}\left\{ \frac{z(\mu^{n,m}_{\lambda_1,\lambda_2} f(z))}{\mu^{n,m}_{\lambda_1,\lambda_2} f(z)} \right\} > \alpha, \quad (z \in U),$$

Where for f $\in$ A = A(1), the generalized derivative operator $\mu^{n,m}_{\lambda_1,\lambda_2}$: A $\to$ A is defined by

(1.4) $$\mu^{n,m}_{\lambda_1,\lambda_2} f(z)$$

$$= z + \sum_{k=s+1}^{\infty} \frac{(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1))^m} C(n,k) a_k z^k,$$

$$o \le \alpha < 1, \quad \lambda_2 \ge \lambda_1 \ge 0, \quad (n, m \in \mathrm{N}_0 = \{0,1,2,...\},$$

$$C(n,m) = \binom{n+k-1}{n} = (n+1)_{k-1}/(1)_{k-1},$$

Note that the series expansion (1.2) is equivalent to (1.1) with omitted coefficient as follows:

s = 1 : no coefficient is omitted
s = 2 :a_2 = 0 omitted
s = 3 : $a_2 = a_3 = 0$ omitted and so on.
Consequently, A(1) = A and A(s) $\subseteq$ A(1).

2. Definitions and preliminaries

Theorem2.1. [1] If f(z)$\in$ A(s) given by (1.2), satisfies the coefficient inequality:

(2.1) $$\sum_{k=s+1}^{\infty} \frac{(k-\alpha)(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1))^m} C(n,k)\,|a_k| \le 1-\alpha$$

(s $\in$ N ={1,2,3,…}), n$\in$ N , m$\in$ N $\cup$ {0} and C(n,k) = $\binom{n+k-1}{n}$, then f(z)$\in$ $\wp^{n,m}_{\lambda_1,\lambda_2}$ (s, α) .

$$for \quad 0 \le \alpha < 1, \quad \lambda_1 \ge \lambda_2 \ge 0.$$

Let us denote by $\wp^{n,m^*}_{\lambda_1,\lambda_2}$ (s, α) the class of functions f(z) defined by (1.2) whose coefficients satisfies the condition (2.1).

Definition 1. (Hadamard product or convolution)

If f, g$\in$A(s), where f(z) is as defined in (1.2) and g(z) is given by

$$g(z) = z + \sum_{k=s+1}^{\infty} b_k z^k$$

the Hadamard product (or convolution) $f * g$ of $f(z)$ and g(z) is defined by

(2.2) $$(f*g)(z) = z + \sum_{k=s+1}^{\infty} a_k b_k z^k = (g*f)(z)$$

Definition 2. (Subordination Principle.)

Let f(z) and g(z) be analytic in the unit disk U. Then f(z) is said to be subordinate to g(z) in U and we write

$$f(z) \prec g(z), \quad z \in U,$$

if there exist a Schwarz function $w(z)$, analytic in U with $w(0)=0, \quad |w(z)| \quad <1$ such that

(2.3) $$f(z) = g(w(z)), \quad z \in U$$

In particular, if the function g(z) is univalent in U, then f(z) is subordinate to g(z) if

(2.4) $$f(0) = g(0), \quad f(u) \subset g(u)$$

Definition 3. (Subordinating factor sequence)

A sequence $\{C_k\}_{k=1}^{\infty}$ of complex number is said to be a subordinating factor sequence if whenever f(z) of the form (1.1) is analytic, univalent and convex in U, the subordination is given by

$$\sum_{k=1}^{\infty} a_k c_k z_k \prec f(z), \quad z \in U, \quad a_1 = 1.$$

We have the following theorem:

Theorem A: (Wilf[2])

The sequence $\{C_k\}_{k=1}^{\infty}$ is a subordinating factor sequence if and only if

(2.5) $$\mathrm{Re}\left\{1 + 2\sum_{k=1}^{\infty} c_k z^k\right\} > 0, \qquad (z \in U).$$

The object of this present work is to derive subordination results for functions in the classes $\wp_{\lambda_1,\lambda_2}^{n,m^*}$ (s, α), and to also consider some interesting consequences of our result.

3. Main Result

Subordination result for the class

Theorem 3.1. Let f(z)$\in \wp_{\lambda_1,\lambda_2}^{n,m^*}$ (s,α) $\subset \wp_{\lambda_1,\lambda_2}^{n,m}$ (s,α)

Where

$\wp_{\lambda_1,\lambda_2}^{n,m^*}$ (s, α)

= {f $\in$ A(s):

$$\sum_{k=s+1}^{\infty} \frac{(k-\alpha)(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1))^m} C(n,k)\,|\,a_k\,| \le 1-\alpha \ \}$$

Then

(a)

(3.1)

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}(f*g)(z) \prec g(z)$$

$$(z \in U, \quad 0 \le \alpha < 1; \quad \lambda_2 \ge \lambda_1 \ge 0; \quad n \in \mathrm{N}; m \in \mathrm{N}_0),$$

$$for \quad every \quad g \in C_\alpha(s).$$

And the constant factor

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}$$ is best possible.

(b)

(3.2)

$$\mathrm{Re}(f(z)) > -\frac{[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}, \quad (z \in U).$$

PROOF OF THEOREM 3.1

Let f(z) defined by (2.1) be any member of the class $\wp_{\lambda_1,\lambda_2}^{n,m^*}$ (s,α) and suppose that

$$g(z) = z + \sum_{k=s+1}^{\infty} b_k z^k \in C_\alpha(s).$$

Then

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}(f*g)(z) =$$

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\left(z+\sum_{k=s+1}^{\infty} a_k b_k z^k\right)$$

Thus, by definition (2.3) the subordination (3.1) will hold if the sequence,

(3.4)

$$\left\{\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}a_k\right\}_{k=1}^{\infty}$$

is a subordinating factor sequence with $a_1 = 1$.

Therefore by Theorem A, it is sufficient to show that

(3.5)

$$\mathrm{Re}\left\{1+\sum_{k=1}^{\infty}\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}a_k z^k\right\}$$

$$> 0; \ (z \in U)$$

Now,

$$\mathrm{Re}\left\{1+\sum_{k=1}^{\infty}\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}a_k z^k\right\}=$$

$$\mathrm{Re}\{1+\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}z+$$

$$\frac{1}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}\times$$

(3.6) $$\sum_{k=s+1}^{\infty}(2-\alpha)(n+1)(1+\lambda_1)^{m-1}a_k z^k \},$$

(s= 1,2,3,…)

$$>\left\{1-\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}r\right.$$

$$-\frac{1}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}$$

$$\left.\times\sum_{k=s+1}^{\infty}\frac{(k-\alpha)(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1))^m}C(n,k)\,|\,a_k\,|\,r^k\right\}$$

$(because \quad (k-\alpha)(1+\lambda_1(k-1))^{m-1}C(n,k))$ is an increasing function of k.

$$>\left\{1-\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}r\right.$$

$$-\frac{(1+\lambda_2)^m(1-\alpha)}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}r$$

$$1-\left\{1-\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}+(1+\lambda_2)^m(1-\alpha)}{(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}\right\}r$$

$= 1 - r > 0$; $(|z| = r > 1)$.

Thus, (3.5) holds true in U and consequently proves (3.1).

To show that the constant

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}$$

Is best possible,we need to show that

(3.8)

$$Min\left\{\mathrm{Re}\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}f_0\right\}$$

$$=-\frac{1}{2}(z\in U).$$

To do this, we consider the function $f_0(z)$ defined by

$$f_0(z)=\frac{z(2-\alpha)(1+\lambda_1)^{m-1}(n+1)-(1-\alpha)(1+\lambda_2)^m z^2}{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}$$

which is a member of the class $\wp^{n,m^*}_{\lambda_1,\lambda_2}(s,\alpha)$. Thus from relation (3.1), we obtain

(3.9)

$$\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\prec\frac{z}{1-z}$$

Now, by using the fact that

(3.10) $|\mathrm{Re}\, z| \le |z|$

we have that

$$\left|\mathrm{Re}\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\right|$$

$$\le\left|\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\right|$$

$$=\frac{|z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2|}{|2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]|}$$

$$=\frac{|z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z|}{|2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]|}$$

$$\le\frac{|z||[(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z]|}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}$$

$$\le\frac{|[(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z]|}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\quad(3.11)$$

$$\le\frac{|[(2-\alpha)(n+1)(1+\lambda_1)^{m-1}+(1-\alpha)(1+\lambda_2)^m z]|}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}$$

$$\le\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}+(1-\alpha)(1+\lambda_2)^m}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}$$

$$=\frac{1}{2},\quad (|z|=1).$$

This implies that,

$$\left|\mathrm{Re}\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\right|\le\frac{1}{2}$$

i.e,

$$-\frac{1}{2}\le\left|\mathrm{Re}\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\right|\le\frac{1}{2}.$$

(3.12)

Hence,

$$Min\left\{\mathrm{Re}\frac{z(2-\alpha)(n+1)(1+\lambda_1)^{m-1}-(1-\alpha)(1+\lambda_2)^m z^2}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}f_0(z)\right\}$$

$$=-\frac{1}{2}(z\in U).$$

Next we show that

$$\mathrm{Re}(f(z))>-\frac{[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}},$$

$(z\in U)$

Now taking

$$g(z)=\frac{z}{1-z}\in C_\alpha(s)$$

in (3.1) we have the following:

(3.13)

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}f(z)\prec\frac{z}{1-z}$$

Therefore,

(3.14)

$$\mathrm{Re}\left\{\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\right\}f(z)>-\frac{1}{2}$$

Since

(3.15) $\mathrm{Re}\left(\frac{z}{1-z}\right)>-\frac{1}{2},\quad |z|<r$

which implies that

(3.16)

$$\frac{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}}{2[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}\mathrm{Re}(f(z))>-\frac{1}{2}$$

Hence, we have

$$\mathrm{Re}(f(z))>-\frac{[(1-\alpha)(1+\lambda_2)^m+(2-\alpha)(n+1)(1+\lambda_1)^{m-1}]}{(2-\alpha)(n+1)(1+\lambda_1)^{m-1}},$$

$(z\in U)$.

which is (3.2) require to complete the proof of theorem 3.1 .

SOME APPLICATION OF THE THEOREM 3.1

Taking n = 1 in theorem 3.1; we obtain the following:

Corollary 1. If the function f(z) defined by (1.2) satisfies

(4.1) $$\sum_{k=s+1}^{\infty}\frac{(2k-2\alpha)(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1))^m}|a_k|\le\rho$$

$(s\in N=\{1,2,3,...\});\ m\in N\cup\{0\};\ 0\le\alpha<1;$
$\lambda_2\ge\lambda_1\ge 0;\rho\ge 0;$ *then for every* $g\in C_\alpha(s)$,
one has

(4.2)
$$\frac{(4-2\alpha)(1+\lambda_1)^{m-1}}{2[(4-2\alpha)(1+\lambda_1)^{m-1}+\rho(1+\lambda_2)^m]}(f*g)(z)\prec g(z)$$
$(z\in U;\ \ 0\le\alpha<1; g\in C_\alpha(s);\ \ \lambda_2\ge\lambda\ge 0;$
$m\in N\cup\{0\};\rho\ge 0)$

and,

(4.3)
$$\mathrm{Re}(f(z))>-\frac{[(4-2\alpha)(1+\lambda_1)^{m-1}+\rho(1+\lambda_2)^m]}{(4-2\alpha)(1+\lambda_1)^{m-1}},$$
$(z\in U).$

The contact factor
$$\frac{(4-2\alpha)(1+\lambda_1)^{m-1}}{2[(4-2\alpha)(1+\lambda_1)^{m-1}+\rho(1+\lambda_2)^m]}$$
cannot be replaced by any larger one .

Remark 1: When $\alpha=0;\lambda_1=\lambda_2=0;\rho=1;m=o$ in corollary 1, we have the result obtained by Selvaraj and Karthikeyan[3].

Taking $\lambda_1=\lambda_2=0$; and m = 0in Theorem 3.1; we obtain the following:

Corollary 2. If the function $f(z)$ defined by (1.2) satisfies

(4.4) $$\sum_{k=s+1}^{\infty}(k-\alpha)C(n,k)|a_k|\le\rho,$$

$(\rho>0,n\in N;0\le\alpha<1);$ then for every function , $g\in C_\alpha(s)$, one has

(4.5) $$\frac{4-2\alpha}{2(4-2\alpha+\rho)}(f*g)(z)\prec g(z)$$
$(z\in U;\ \ 0\le\alpha<1;\rho>0)$

and,

(4.6) $$\mathrm{Re}\,f(z)>-\left(\frac{(4-2\alpha)+\rho}{4-2\alpha}\right),\ \ (z\in U).$$

The constant factor
$$\frac{(4-2\alpha)+\rho}{4-2\alpha}$$
cannot be replaced by any larger one.

Remark 2: When $\alpha=\rho=\frac{1}{2}$ in corollary 2, we have the result obtained by Aouf et al [4].

Taking $\lambda_1=\lambda_2=1,\alpha=0,m=2,$ in Theorem 3.1; we obtain the following:

Corollary 3. If the function $f(z)$ defined by (1.2) satisfies

(4.7) $$\sum_{k=s+1}^{\infty}C(n,k)|a_k|\le\rho,$$

$(\rho>0,n\in N;)$ then for every function , $g\in C_\alpha(s)$, one has

(4.8) $$\frac{n+1}{2(n+1+\rho)}(f*g)(z)\prec g(z)$$
$(z\in U;\ \ 0\le\alpha<1;\rho>0)$

and,

(4.9) $$\mathrm{Re}\,f(z)>-\left(1+\frac{\rho}{n+1}\right),\ \ (z\in U).$$

The constant factor
$$\frac{n+1}{2(n+1+\rho)}$$
cannot be replaced by any larger one.

Remark 3: When $\rho=m(m>0),$ in the corollary 3, we have the result obtained by Attiya et al [5].

Taking $\alpha=0$ in Theorem 3.1; we obtain the following:

Corollary 4. If the function $f(z)$ defined by (1.2) satisfies

(4.10) $$\sum_{k=s+1}^{\infty}\frac{k(1+\lambda_1(k-1))^{m-1}}{(1+\lambda_2(k-1)^m}C(n,k)|a_k|\le 1,$$

$(s,n\in N;m\in N\cup\{0\};\lambda_2\ge\lambda_1\ge 0)$

and,

(4.12) $$\mathrm{Re}\,f(z)>-\frac{[(1+\lambda_2)^m+2(n+1)(1+\lambda_1)^{m-1}]}{(n+1)(1+\lambda_1)^{m-1}},$$
$(z\in U).$

The constant factor
$$\frac{(n+1)(1+\lambda_1)^{m-1}}{(1+\lambda_2)^m+2(n+1)(1+\lambda_1)^{m-1}}$$
cannot be replaced by any larger one.

Remark 4: When $m=n=1,\lambda_1=0,\lambda_2=1$ in the corollary 4, we have the result obtained by Sukhjit[6] and Selvara et al [3].

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